Ceng 375 Numerical Computing
Midterm
Nov 12, 2009 14.40–16.30
Good Luck!

Each question is 25 pts.

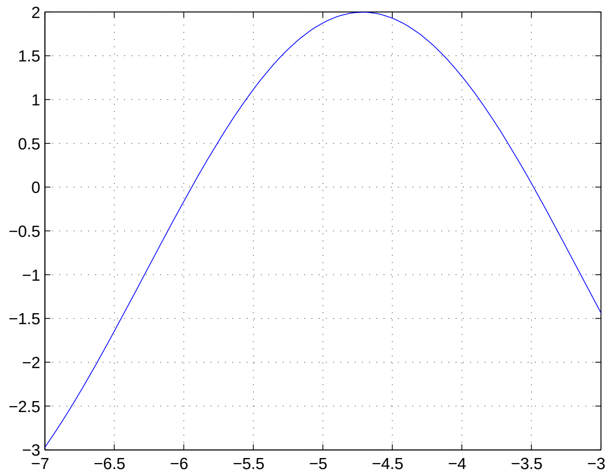
1. The following function is given

$$f(x) = 3 * sin(x) - e^x/4 - 1$$

This nonlinear equation ($f(x) = 0$) is solved by using four methods, namely *Bisection*, *Regula Falsi*, *Newton's*, *Muller's* methods. See the following MATLAB commands;

```
>> f = inline ( ' 3*sin( x) - exp ( x)/4 -1');
>> df = inline ( ' 3*cos( x) - exp ( x)/4');
>> fplot(f,[-7 -3]); grid on;
>> format short e
>> bisect(f,-7,-5,fzero(f,[-7 -5]),1e-5);
>> regula(f,-7,-5,fzero(f,[-7 -5]),1e-5,eps,20);
>> newton(f,df,-7,fzero(f,[-7 -5]),1e-5,eps,20);
>> muller(f,-7,-6,-5,fzero(f,[-7 -5]),1e-5,eps,20);
```

Plot of the function is given at the following figure;



Then, the following tables are obtained.

Table 1: Obtained **root** values at each iteration for all of four methods.

iteration	Bisection	Regula	Newton	Muller
1	-6.0000e+00	-5.5672e+00	-5.4650e+00	-5.7134e+00
2	5.5000e+00	-5.7373e+00	-5.8008e+00	-5.7604e+00
3	-5.7500e+00	-5.7575e+00	-5.7596e+00	-5.7591e+00
4	-5.8750e+00	-5.7590e+00	-5.7591e+00	-5.7591e+00
5	-5.8125e+00	-5.7591e+00	-5.7591e+00	-
6	-5.7812e+00	-5.7591e+00	-	-

Table 2: Obtained **function** values at each iteration for all of four methods.

iteration	Bisection	Regula	Newton	Muller
1	-4.4179e-01	3.1174e-01	4.5882e-01	7.8084e-02
2	4.1006e-01	3.7524e-02	-7.3051e-02	-2.2184e-03
3	1.5762e-02	2.8928e-03	-8.1042e-04	4.1882e-06
4	-2.0681e-01	2.0926e-04	-1.0968e-07	-4.0674e-11
5	-9.3753e-02	1.5061e-05	-2.2204e-15	-
6	-3.8525e-02	1.0836e-06	-	-

i Analyze these tables. Is the convergence sustained for the each methods? For the sustained ones; at which iteration and why?

ii If the exact value is given as $-5.7591e+00$, fill the following table for two methods. Choose two methods and use scientific notation with five significant figures.;

iteration	$Error_1$	$Error_2$	$Error_3$	$Error_4$
1	-2.4087e-01	1.9191e-01	2.9418e-01	4.5735e-02
2	2.5913e-01	2.1820e-02	-4.1715e-02	-1.2812e-03
3	9.1313e-03	1.6722e-03	-4.6817e-04	2.4198e-06
4	-1.1587e-01	1.2090e-04	-6.3367e-08	-2.3500e-11
5	-5.3369e-02	8.7017e-06	-1.7764e-15	-
6	-2.2119e-02	6.2607e-07	-	

iteration	$ErrorRatio_1$	$ErrorRatio_2$	$ErrorRatio_3$	$ErrorRatio_4$
1	-2.4087e-01	1.9191e-01	2.9418e-01	4.5735e-02
2	-1.0758e+00	1.1370e-01	-1.4180e-01	-2.8014e-02
3	3.5238e-02	7.6636e-02	1.1223e-02	-1.8887e-03
4	-1.2689e+01	7.2305e-02	1.3535e-04	-9.7116e-06
5	4.6060e-01	7.1972e-02	2.8033e-08	-
6	4.1445e-01	7.1948e-02	-	-

iii What can you say about the speed of convergences for each method?

iv Which method is the best one? Why?

2. Consider the same function with the previous question:

   i Find the root in the interval of $[-5, -3]$ with Newton's method.
   Hint: Newton's method uses the algorithm:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

```
>> f = inline ( ' 3*sin( x) - exp ( x)/4 -1');
>> df = inline ( ' 3*cos( x) - exp ( x)/4');
>> fzero(f,[-5 -3])
ans =  -3.484142626624529
>> newton(f,df,-3,fzero(f,[-5 -3]),1e-5,eps,20);
   1.0000e+00  -3.4814e+00  -7.7103e-03   2.7199e-03   2.7199e-03
   2.0000e+00  -3.4841e+00  -3.7332e-06   1.3176e-06   4.8442e-04
   3.0000e+00  -3.4841e+00  -8.8107e-13   3.1086e-13   2.3594e-07
>> fzero(f,[-7 -5])
ans =  -5.943116471352441
>> newton(f,df,-5,fzero(f,[-5 -3]),1e-5,eps,20);
   1.0000e+00  -7.2078e+00  -3.3953e+00  -3.7237e+00  -3.7237e+00
   2.0000e+00  -5.3280e+00   1.4480e+00  -1.8439e+00   4.9518e-01
   3.0000e+00  -6.1644e+00  -6.4514e-01  -2.6803e+00   1.4536e+00
   4.0000e+00  -5.9478e+00  -1.3352e-02  -2.4637e+00   9.1918e-01
   5.0000e+00  -5.9431e+00  -1.1028e-05  -2.4590e+00   9.9809e-01
   6.0000e+00  -5.9431e+00  -7.6171e-12  -2.4590e+00   1.0000e+00
```

or

```
***********************************
write the function
function fx=func29i(x)
fx= 3*sin( x) - exp ( x)/4 -1;
save as func29i.m
***********************************
write the function
function fx=funcdiff29i(x)
fx= 3*cos( x) - exp ( x)/4;
save as funcdiff29i.m
***********************************
```

start with

$$x_0 = -3$$

```
>> format long
>> x0=-3;x1=x0-(func29i(x0)/funcdiff29i(x0))
x1 =  -3.481422717761558
>> x2=x1-(func29i(x1)/funcdiff29i(x1))
x2 =  -3.484141309056956
>> x3=x2-(func29i(x2)/funcdiff29i(x2))
x3 =  -3.484142626624218
>> x4=x3-(func29i(x3)/funcdiff29i(x3))
x4 =  -3.484142626624529
```

same procedure for $x_0 = -5$

ii Estimate the error at the last iteration in your answer to part i. *Hint: To estimate the error, compute one more iteration.*

```
>> x5=x4-(func29i(x4)/funcdiff29i(x4))
x5 =  -3.484142626624529
```

$$e_4 = x_5 - x_4 = (-3.484142626624529) - (-3.484142626624529) = 0$$

iii Approximately how many iterations of the bisection method would have been required to achieve the error value of $1e - 5$?

The error in the bisection method satisfies

$$e_n = |\frac{Original\ Interval}{2^n}|$$

$x_0 = -3$ In this case, taking the original interval to be [-5, -3], we would have

$$e_n = |\frac{2}{2^n}|$$

Therefore, to achieve approximately the same error as we obtained with four iterations of Newton's method here, would require sufficient iterations of bisection to ensure

$$\frac{2}{2^n} = 1E - 5$$

this gives

$$\frac{2}{0.00001} = e^{nln(2)}$$

$$\frac{ln(\frac{2}{0.00001})}{ln(2)} = n$$

$$n \cong 18$$

```
>> format short
>> bisect(f,-5,-3,fzero(f,[-5 -3]),1e-5);
1.0000   -4.0000    0.5090   -0.3311   -0.3311
2.0000   -3.5000   -0.3060    0.1689   -0.5100
3.0000   -3.7500    0.1372   -0.0811   -0.4804
4.0000   -3.6250   -0.0771    0.0439   -0.5409
5.0000   -3.6875    0.0321   -0.0186   -0.4244
6.0000   -3.6562   -0.0220    0.0126   -0.6780
7.0000   -3.6719    0.0052   -0.0030   -0.2375
8.0000   -3.6641   -0.0084    0.0048   -1.6056
9.0000   -3.6680   -0.0016    0.0009    0.1886
10.0000   -3.6699    0.0018   -0.0010   -1.1511
11.0000   -3.6689    0.0001   -0.0001    0.0656
12.0000   -3.6685   -0.0007    0.0004   -6.1168
13.0000   -3.6687   -0.0003    0.0002    0.4183
14.0000   -3.6688   -0.0001    0.0001    0.3046
15.0000   -3.6689    0.0000   -0.0000   -0.1417
16.0000   -3.6689   -0.0000    0.0000   -3.0289
17.0000   -3.6689   -0.0000    0.0000    0.3349
18.0000   -3.6689   -0.0000    0.0000    0.0071
```

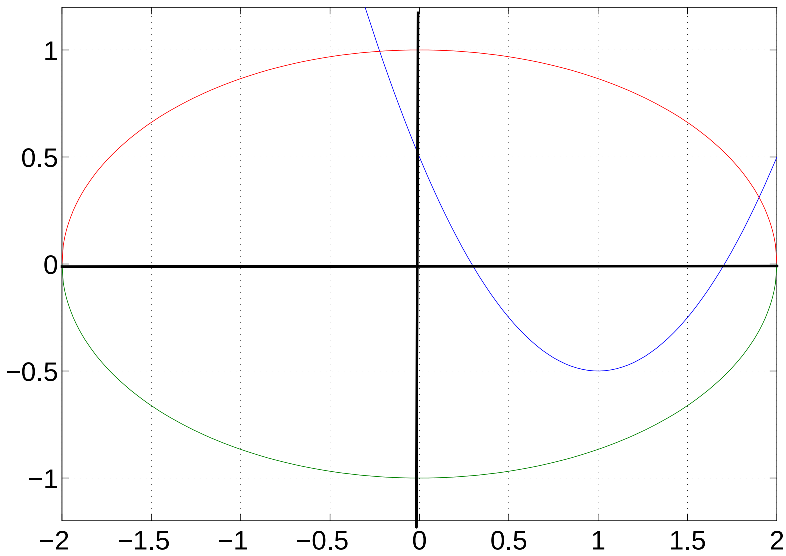
same result.

3. Consider this pair of equations:

$$x^2 + 4y^2 = 4$$

$$y - x^2 + 2x = 0.5$$

Plot of the system is given at the following figure;



Solve this system by <u>iteration</u>. *Hint: Start with something like* $y = ...$ *and proceed only two iterations.*

```
>> fplot(@(x)[x^2-2*x+0.5,-sqrt(1-x^2/4),sqrt(1-x^2/4)],[-2 2 -1.2 1.2]);
    grid on;axis xy
>> y=0.7;
>> x=sqrt(4-4*y^2);yy=x^2-2*x+0.5;y=yy;D=[x,y];disp(D);
1.4283   -0.3166   %xy-pair after first iteration
>> x=sqrt(4-4*y^2);yy=x^2-2*x+0.5;y=yy;D=[x,y];disp(D);
1.8971    0.3049   %xy-pair after second iteration
>> x^2+4*y^2
ans = 3.970877751497605 %should be equal to 4, see the pair of equations
>> y-x^2+2*x
ans = 0.500000000000000 %should be equal to 0.5, see the pair of equations
```

4. Consider the linear system;

$$\begin{aligned} x_1 - 2x_2 + 4x_3 &= 6 \\ 8x_1 - 3x_2 + 2x_3 &= 2 \\ -1x_1 + 10x_2 + 2x_3 &= 4 \end{aligned}$$

i Solve this system by Gaussian elimination with pivoting. How many row interchanges are needed?

ii What is the value of determinant?

iii Obtain the LU decomposition of the system.

iv Repeat without any row interchanges (only for the first item). Do you get the same results? Why?

```
>> A=[1 -2 4; 8 -3 2; -1 10 2]
>> b=[6 2 4]
>> GEPivShow(A,b')
Begin forward elimination with Augmented system:
     1    -2     4     6
     8    -3     2     2
    -1    10     2     4
Swap rows 1 and 2;  new pivot = 8
After elimination in column 1 with pivot = 8.000000
    8.0000   -3.0000    2.0000    2.0000
         0   -1.6250    3.7500    5.7500
         0    9.6250    2.2500    4.2500
Swap rows 2 and 3;  new pivot = 9.625
After elimination in column 2 with pivot = 9.625000
    8.0000   -3.0000    2.0000    2.0000
         0    9.6250    2.2500    4.2500
         0         0    4.1299    6.4675
ans =   -0.1132     0.0755     1.5660 %these are x1, x2, x3
>> det(A)
ans =   318
>> 8.0000*9.6250 *4.1299 %product of the diagonal of U
ans =  318.0023
% For LU-decomposition
>> [L,U,pv] = luPiv(A)
L =    1.0000         0         0
       -0.1250    1.0000         0
        0.1250   -0.1688    1.0000
U =   8.0000   -3.0000    2.0000
           0    9.6250    2.2500
```

```
         0         0    4.1299
pv =

     2
     3
     1
% two times pivoting
% solution is completed
%*******************************************
% For proving purpose
>> A=[1 -2 4; 8 -3 2; -1 10 2]
A =
     1    -2     4
     8    -3     2
    -1    10     2
% our Idendity matrix becomes for swaping rows 1 and 2
>> pivoting1=[0 1 0; 1 0 0; 0 0 1]
pivoting1 =
     0     1     0
     1     0     0
     0     0     1
% apply this pivoting to our original matrix
>> A=pivoting1*A
A =
     8    -3     2
     1    -2     4
    -1    10     2
% our Idendity matrix becomes for swaping rows 2 and 3
>> pivoting2=[1 0 0; 0 0 1; 0 1 0]
pivoting2 =
     1     0     0
     0     0     1
     0     1     0

% also apply this pivoting to our pivoted matrix
>> A=pivoting2*A
A =
     8    -3     2
    -1    10     2
     1    -2     4
>> [L,U,pv] = luPiv(A)
L =
   1.000000000000000                   0                   0
```

```
  -0.125000000000000   1.000000000000000                   0
   0.125000000000000  -0.168831168831169   1.000000000000000
U =
   8.000000000000000  -3.000000000000000   2.000000000000000
                   0   9.625000000000000   2.250000000000000
                   0                   0   4.129870129870130
pv =
     1
     2
     3
% it is proved.
%**********************************************
% for not pivoting case;
>> GEshow(A,b')
Begin forward elimination with Augmented system:
     1    -2     4     6
     8    -3     2     2
    -1    10     2     4
After elimination in column 1 with pivot = 1.000000
     1    -2     4     6
     0    13   -30   -46
     0     8     6    10
After elimination in column 2 with pivot = 13.000000
    1.0000   -2.0000    4.0000    6.0000
         0   13.0000  -30.0000  -46.0000
         0         0   24.4615   38.3077
ans =    -0.1132     0.0755    1.5660
>> 1.0000*13.0000*24.4615
ans =   317.9995
% Solutions are the same. They are same because the system is
% not ill-conditioned.
```