Çankaya University
Mcs 331 Numerical Methods
Midterm Examination
Dec 01, 2014 13.20 – 15.10
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

DURATION: 110 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the numerical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		10
1B		10
2		20
3		20
4		30
5		20
TOTAL		110

1. A) An engineer runs *the same* FORTRAN program on two different computers, a PC and a UNIX Workstation. Neither system produces any error messages, but the resulting outputs differ by several orders of magnitude more than machine precision. What, if any, reasonable explanations are there for this phenomenon?

Answer:

All modern PC s and Unix workstations conform to the IEEE arithmetic standard, so we should assume both systems use the same accuracy arithmetic. The two most likely explanations are, either

(a) The problem being solved is somewhat ill-conditioned and therefore highly sensitive to small perturbations in data or of the calculations.

(b) The particular algorithm chosen to perform these calculations is somewhat unstable.

B) How many iterations of bisection will be required to attain an accuracy of 10^{-4} if the starting interval is $[0, 1]$?

The error in the bisection method satisfies

$$e_n = \left|\frac{Original\ Interval}{2^n}\right|$$

In this case, the original interval to be [0, 1], we would have

$$e_n = \left|\frac{1}{2^n}\right|$$

Therefore, to achieve approximately the same error as we obtained with two iteration of Newton's method here, would require sufficient iterations of bisection to ensure

$$\frac{1}{2^n} = 0.0001$$

this gives

$$\frac{1}{0.0001} = e^{nln(2)}$$

$$\frac{ln(\frac{1}{0.0001})}{ln(2)} = n$$

$$n \cong 14$$

```
>> log(1/0.0001)/log(2)
ans =  13.287712379549451
```

2. Consider the function $f(x)$, on $[0,1]$, defined by

$$f(x) = \sqrt{x} - cos(x)$$

i Describe how the secant method determine a smaller sub-interval containing a root.

> To determine a root of $f(x) = 0$, given two values, x_0 and x_1, that are near the root,
> If$|f(x_0)| < |f(x_1)|$ Then
> Swap x_0 with x_1
> Repeat
> Set $x_2 = x_1 - f(x_1) * \frac{(x_0 - x_1)}{f(x_0) - f(x_1)}$
> Set $x_0 = x_1$
> Set $x_1 = x_2$
> Until $|f(x_2)| < tolerance\ value$

ii Apply the secant method to $f(x)$ twice.

```
>> x0=1.0;
>> x1=0.0;
>> syms x;
fx='sqrt(x)-cos(x)';
>> subs(fx,x0)
ans = 0.45969769413186
>> subs(fx,x1)
ans =    -1
>> x2=x1-subs(fx,x1)*((x0-x1)/(subs(fx,x0)-subs(fx,x1)))
x2 = 0.68507335732605
>> x0=x1
x0 =   0
>> x1=x2
x1 =   0.68507335732605
>> x2=x1-subs(fx,x1)*((x0-x1)/(subs(fx,x0)-subs(fx,x1)))
x2 =   0.65039498012836
>> solve('sqrt(x)-cos(x)')
ans = .64171437087288265839856530031652
```

3. Consider the function:

$$f(x) = sin(x) - 4 * x + 2$$

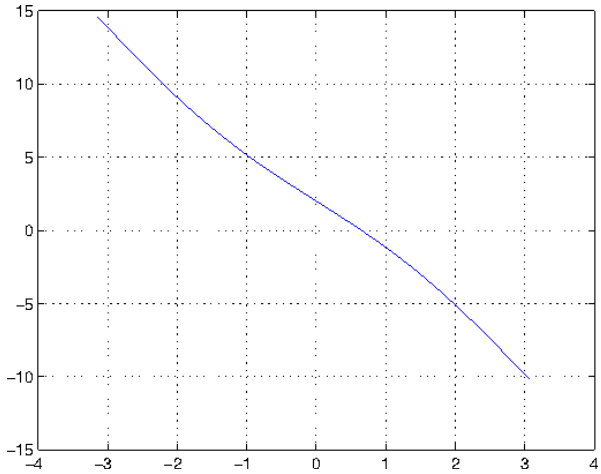

Table 1: Plot of the function, $sin(x) - 4 * x + 2$.

i Use two iterations of Newton's method to estimate the root of this function between $x = 0.0$ and $x = 1.0$ (Use four significant figures).

ii Estimate the error in your answer to part i (Use more than four significant figures).

Answer:
i) Newton's method uses the algorithm

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

where, for this function

$$f'(x) = cos(x) - 4$$

Also, in this case:

$$f(0.0) = 2, \ and \ f(1.0) = -1.1585$$

```
>> format long
>> x=-pi:0.1:pi
>> fzero('sin(x)-4*x+2',[0 1])
ans =  0.651618523135209
>> x=0;
>> sin(x)-4*x+2
ans =     2
>> x=1;
>> sin(x)-4*x+2
ans = -1.158529015192103
```

```
write the function
function fx=func(x)
fx=sin(x)-4*x+2;
save as func.m
write the function
function fx=funcdiff(x)
fx=cos(x)-4;
save as funcdiff.m
```

The best choice for x_0 is usually the value producing the smallest residual, i.e. in this case

$$x_0 = 0$$

```
>> x0=0;x0-(func(x0)/funcdiff(x0))
ans =   0.6667 (0.666666666666667)
>> x0=0.6667;x0-(func(x0)/funcdiff(x0))
ans =   0.6516 (0.651640263601115)
>> x0=0.6516;x0-(func(x0)/funcdiff(x0))
ans =   0.6516 (0.651618523167672)
```

or start with

$$x_0 = 1$$

```
>> x0=1;x0-(func(x0)/funcdiff(x0))
ans =    0.6651 (0.665135766874333)
>> x0=0.6651;x0-(func(x0)/funcdiff(x0))
ans =    0.6516 (0.651635876939131)
>> x0=0.6516;x0-(func(x0)/funcdiff(x0))
ans =    0.6516 (0.651618523167672)
```

Answer:
ii) To estimate the error, compute one more iteration (third iteration),

$$x_0 = 0$$

$$\begin{aligned} e_2 = x_3 - x_2 &= (0.651618523167672) - (0.651640263601115) \\ &= -2.174043344305154e-05 \end{aligned}$$

or start with

$$x_0 = 1$$

$$\begin{aligned} e_2 = x_3 - x_2 &= (0.651618523167672) - (0.651635876939131) \\ &= -1.735377145906103e-05 \end{aligned}$$

4. Consider the linear system ($Ax = b$);

$$A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ -1 & -3 & -3 & 5 \\ 1 & 3 & 2 & 2 \end{bmatrix} \quad and \quad b = \begin{bmatrix} 6 \\ 2 \\ 4 \\ 3 \end{bmatrix}$$

i Solve this system by Gaussian elimination with pivoting. How many row interchanges are needed?

ii What is the value of determinant?

iii Obtain the LU decomposition of the system.

iv Repeat without any row interchanges (only for the first item). Do you get the same results? Why?

Answer:

```
%*********************************************
%i) Ax=b
>> A=[1 3 1 1; 2 5 2 2; -1 -3 -3 5; 1 3 2 2]
>> b=[6 2 4 3]
>> format short
>> GEPivShow(A,b')
Begin forward elmination with Augmented system:
     1     3     1     1     6
     2     5     2     2     2
    -1    -3    -3     5     4
     1     3     2     2     3
Swap rows 1 and 2;  new pivot = 2
After elimination in column 1 with pivot = 2.000000
    2.0000    5.0000    2.0000    2.0000    2.0000
         0    0.5000         0         0    5.0000
         0   -0.5000   -2.0000    6.0000    5.0000
         0    0.5000    1.0000    1.0000    2.0000
After elimination in column 2 with pivot = 0.500000
    2.0000    5.0000    2.0000    2.0000    2.0000
         0    0.5000         0         0    5.0000
         0         0   -2.0000    6.0000   10.0000
         0         0    1.0000    1.0000   -3.0000
After elimination in column 3 with pivot = -2.000000
    2.0000    5.0000    2.0000    2.0000    2.0000
         0    0.5000         0         0    5.0000
         0         0   -2.0000    6.0000   10.0000
```

```
         0         0         0    4.0000    2.0000
ans =
  -21.0000
   10.0000
   -3.5000
    0.5000
 %**********************************************
 % ii) Determinant
>> det(A)
ans =    8
>> 2.0000*0.5000*-2.0000*4.0000 %product of the diagonal of U
ans =    8
 %**********************************************
 % iii) For LU-decomposition
>> [L,U,pv]=luPiv(A)
L =
    1.0000         0         0         0
    0.5000    1.0000         0         0
   -0.5000   -1.0000    1.0000         0
    0.5000    1.0000   -0.5000    1.0000
U =
    2.0000    5.0000    2.0000    2.0000
         0    0.5000         0         0
         0         0   -2.0000    6.0000
         0         0         0    4.0000
pv =
     2
     1
     3
     4
 % one time pivoting
 %**********************************************
 % iv) for not pivoting case;
>> GEshow(A,b')
Begin forward elmination with Augmented system:
     1     3     1     1     6
     2     5     2     2     2
    -1    -3    -3     5     4
     1     3     2     2     3
After elimination in column 1 with pivot = 1.000000
     1     3     1     1     6
     0    -1     0     0   -10
     0     0    -2     6    10
```

```
     0     0     1     1    -3
After elimination in column 2 with pivot = -1.000000
     1     3     1     1     6
     0    -1     0     0   -10
     0     0    -2     6    10
     0     0     1     1    -3
After elimination in column 3 with pivot = -2.000000
     1     3     1     1     6
     0    -1     0     0   -10
     0     0    -2     6    10
     0     0     0     4     2
ans =
  -21.0000
   10.0000
   -3.5000
    0.5000
% Solutions are the same. They are same because the system is
% not ill-conditioned.
% solution is completed
%**********************************************
```

5. Consider the linear system

$$\begin{aligned} 7x_1 - 3x_2 + 4x_3 &= 6 \\ -3x_1 + 2x_2 + 6x_3 &= 2 \\ 2x_1 + 5x_2 + 3x_3 &= -5 \end{aligned}$$

i Solve this system with the Jacobi method. First rearrange to make it diagonally dominant if possible. Use $[0, 0, 0]$ as the starting vector.

ii Repeat with Gauss-Seidel method. Compare with Jacobi method.

Answer:

```
%*******************************************
%Switching rows 2 &3 first
>> A=[7 -3 4; 2 5 3; -3 2 6]
>> B=[6  -5 2]
>> jacobi(A,B',P',0.01,20)
k =     1 P =
   0.857142857142857
  -1.000000000000000
   0.333333333333333
k =     2 P =
   0.238095238095238
  -1.542857142857143
   1.095238095238095
k =     3 P =
  -0.429931972789116
  -1.752380952380953
   0.966666666666667
k =     4 P =
  -0.446258503401361
  -1.408027210884354
   0.702494331065760
k =     5 P =
  -0.147722708130871
  -1.242993197278911
   0.579546485260771
k =     6 P =
  -0.006737933268545
  -1.288638807904114
   0.673803045027535
k =     7 P =
  -0.080161229117497
  -1.401586653709102
   0.759510636000432
k =     8 P =
  -0.177543215018434
```

```
  -1.423641889953260
   0.760448270010952
k =      9 P =
  -0.187531249986227
  -1.385251675999198
   0.719109022475203
k =     10 P =
  -0.147455873985486
  -1.356452913490631
   0.701318267006619
k =     11 P =
  -0.124947401214053
  -1.361808610609777
   0.711756367504134
k =    12 P =
  -0.133207328835124
  -1.377074860016859
   0.724795836262899
k =     13 P =
  -0.147201132157454
  -1.381594570223690
   0.725754622254725
k =     14 P =
  -0.149686028527138
  -1.376572320489853
   0.720264290662503
k =     15 P =
  -0.144396303445653
  -1.372284162986646
   0.717347759233048
k =     16 P =
  -0.140891932270305
  -1.372650134161568
   0.718563235939389
k =     17 P =
  -0.141743335177466
  -1.374781168655511
   0.720437411918703
k =     18 P =
  -0.143727593377335
  -1.375565113080236
   0.720722055296438
k =     19 P =
  -0.144226222918065
  -1.374942195826928
   0.719991241004744
 >> gseid(A,B',P',0.001,20)
k =        1   P =
      0.857142857142857
```

```
     -1.342857142857143
      1.209523809523809
k =         2   P =
     -0.409523809523810
     -1.561904761904762
      0.649206349206349
k =         3   P =
     -0.183219954648526
     -1.316235827664399
      0.680468631897203
k =         4   P =
     -0.095797430083144
     -1.369962207105064
      0.742088687326783
k =         5   P =
     -0.154034481517475
     -1.383639419789080
      0.717529232504289
k =         6   P =
     -0.145862169912057
     -1.372172671537751
      0.717793138889889
k =         7   P =
     -0.141098652881830
     -1.374236422181201
      0.720862814286152
k =         8   P =
     -0.143737217669745
     -1.375022801503793
      0.719805658333059
k =         9   P =
     -0.143470148263374
     -1.374495335694486
      0.719763371099808
% Gauss-Seidel iterates much faster
```