1 Hands-on–Chebyshev Polynomials; Approximation of Functions with MATLAB I

1. The Chebyshev series and Maclaurin series for e^x are given as the following;

$$e^x = 0.9946 + 0.9973x + 0.5430x^2 + 0.1772x^3$$
$$e^x = 1 + x + 0.5x^2 + 0.1667x^3$$

- Tabulate the error values for the interval [-1,1].
- Plot the error values for the interval.

Solution:

```
function week9litem1(ll,ul,s)
format short;
%format long;
disp('     x        e^x    Chebyshev   Error   Maclaurin   Error')
x =(ll:s:ul)';
taylor=exp(x);
max=(ul-ll)/s+1;
for i=1:max
chebyshev=(0.9946 + 0.9973*x(i) + 0.5430*x(i)^2 + 0.1772*x(i)^3);
errorchebyshev(i)=taylor(i)-chebyshev;
maclaurin=(1+x(i)+0.5*x(i)^2+0.1667*x(i)^3);
errormaclaurin(i)=taylor(i)- maclaurin;
D=[x(i),taylor(i),chebyshev,errorchebyshev(i),maclaurin,
                                                  errormaclaurin(i)];
disp(D);
end
plot(x,errorchebyshev,'o',x,errormaclaurin,'-')
```

save with the name *week9litem1.m*. Then;

```
>> week9litem1(-1,1,0.1)
```

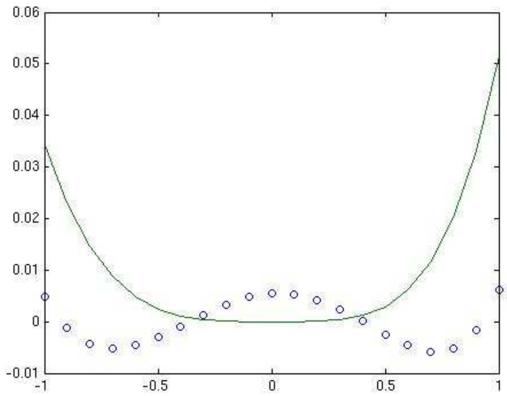

Figure 1: plot(x,errorchebyshev,'o',x,errormaclaurin,'-').

2. Write a MATLAB program using Chebyshev polynomials to economize a Maclaurin series for e^x in the interval [0,1] with a precision of 0.0001.
   - Tabulate the error values.
   - Plot the behaviors of the exact and approximated functions and errors for the interval.
   - Utilize the following code segment;

```
>> syms x
>> ts=taylor(exp(x),8)
ts =1+x+1/2*x^2+1/6*x^3+1/24*x^4+1/120*x^5+1/720*x^6+1/5040*x^7
>> cs=collect(Tch(7))
cs = 64*x^7-112*x^5+56*x^3-7*x
>> es=ts-cs/factorial(7)/2^6
es =
1+46081/46080*x+1/2*x^2+959/5760*x^3+1/24*x^4+5/576*x^5+1/720*x^6
>> vpa(es,7)
ans = 1.+1.000022*x+.5000000*x^2+.1664931*x^3+.4166667e-1*x^4
+.8680556e-2*x^5+.1388889e-2*x^6
```

3. **Exercise:** Modify the program so that it uses Chebyshev polynomials to economize a Maclaurin series for e^{2x} in the interval [0,1] with a precision of 0.008.
   - Tabulate the error values.
   - Plot the behaviors of the exact and approximated functions and errors for the interval.