

1 Hands-on–Numerical Differentiation and Integration with MATLAB

Composite Trapezoidal Rule. To approximate the integral

$$\int_a^b \approx \frac{h}{2}(f(a) + f(b)) + h \sum_{k=1}^{M-1} f(x_k)$$

by sampling $f(x)$ at the $M + 1$ equally spaced points $x_k = a + kh$, for $k = 0, 1, 2, \dots, M$. Notice that $x_0 = a$ and $x_M = b$. You are given the

```
function s=traprl(f,a,b,M)
%Input    - f is the integrand input as a string 'f'
%         - a and b are upper and lower limits of integration
%         - M is the number of subintervals
%Output   - s is the trapezoidal rule sum

h=(b-a)/M;
s=0;
for k=1:(M-1)
    x=a+h*k;
    s=s+feval(f,x);
end
s=h*(feval(f,a)+feval(f,b))/2+h*s;
```

function $f(x) = 2 + \sin(2\sqrt{x})$ for the interval $[1, 6]$.

1. Plot the function.
2. Use the composite trapezoidal rule with 11 sample points to compute an approximation to the integral of $f(x)$ taken over $[1, 6]$ by using the MATLAB program given above.
3. Do the error analysis. Error term for the composite trapezoidal rule is given as;

$$E(f, h) = -\frac{(b-a)}{12}h^2 f''(\xi) = O(h^2)$$

4. Calculate the exact value of the integration by using MATLAB. Compare your results for the aspects of integration and error analysis.
5. Repeat the procedure with increased number of sample points.

```

function week11lsg(fs,a,b,vec)
% input fs as string
fplot(fs,[a b])
f=inline(fs); % inline
format short;
N=length(vec);
disp('
           N           Error           Exact
           Approximated           Minerr');
           Maxerror
for i=1:N
approximated=num2str(traprl(f,a,b,vec(i)));
fx2 = diff(fs,2);
h=(b-a)/vec(i);
fs = '2 + sin (2*(x^.5))';
x=a; value=eval(fx2); x=((b-a)/12)*h^2*value;
errmax=num2str(x);
%errmin=num2str('((b-a)/12)*h^2*fx2(a)');
x=b; value=eval(fx2); x=((b-a)/12)*h^2*value;
errmin=num2str(x);
%errmin=num2str('((b-a)/12)*h^2*fx2(b)');
exact=int(fs,a,b);
D=[vec(i),exact,approximated,approximated-exact,errmax,errmin];
disp(D);
end

```

Solution:

save with the name *week12lsg.m*. Then;

```

>> vec=[10 20 40 50 100 500 1000]';
>> week12lsg(?,?,?,vec);

```