1 Hands-on– Solving Sets of Equations with MATLAB II

1. **Factorization with Pivoting**, $PA = LU$ (A is a non-singular matrix, P is permutation matrix).

   (a) Solve the following linear system by LU-decomposition of coefficient matrix;

$$\begin{aligned} x_1 + 2x_2 + 4x_3 + x_4 &= 21 \\ 2x_1 + 8x_2 + 6x_3 + 4x_4 &= 52 \\ 3x_1 + 10x_2 + 8x_3 + 8x_4 &= 79 \\ 4x_1 + 12x_2 + 10x_3 + 6x_4 &= 82 \end{aligned}$$

   Solution:
   Download the file lufact.m.
   LU-decomposition

   - LU-decomposition

     ```
     >> [X,Y]=lufact(A,B')
     ```

     The obtained X and Y are the following x and y.

     – $Ax = b$
     – $LUx = b$
     – defining $y = Ux$ then solving two systems:
     1 solve by hand $Ly = b$ for y by using **forward-substitution** method
     2 solve by hand $Ux = y$ for x by using **back-substitution** method

   - Check and compare your result with

     ```
     >> GEPivshow(A,B')
     ```

   (b) Download the files luNopiv.m, luPiv.m.

   – Try the following commands to see L and U explicitly.

     ```
     >> [L,U] = luNopiv(A)
     >> [L,U,pv] = luPiv(A)
     ```

     are the results different? Why?

   (c) Now, assume that B is changed as $BB = 2 * B$. We are given L and U. Find new X and Y **by hand**. Check your results by

   ```
   >> BB=2*B
   >> [X,Y]=lufact(A,BB')
   >> GEPivshow(A,BB')
   ```

2. The jacobi.m MATLAB code is given for *Jacobi Iteration*.

   - To solve the linear system $Ax = b$ by starting with an initial guess $x = P_0$ and generating a sequence P_k that converges to the solution.
   - A sufficient condition for the method to be applicable is that A is strictly diagonally dominant.
   - Analyze the given MATLAB code, then solve the following linear system by Jacobi iterations;

$$\begin{aligned} 4x - y + z &= 7 \\ -2x + y + 5z &= 15 \\ 4x - 8y + z &= -21 \end{aligned}$$

   - Start by $P_0 = (1, 2, 2)$; then answer: $x = 2, y = 4, z = 3$ and number of iterations $k = 19$.

```
>> A=[? ? ?; ? ? ?; ? ? ?]
>> B=[? ? ?]'
>> P=[? ? ?]
>> [k,X]=jacobi(A,B,P,10^-9,20)
```

   - Try some other starting sets such as $P_0 = (0, 0, 0)$, $P_0 = (2, 2, 2)$ and $P_0 = (?, ?, ?)$, compare them. Which one hast the smallest value of iterations (k)?

3. Modify the code given in the previous item for Gauss-Seidel method. Solve the same linear system and compare your results. Is convergence accelerated?