Lecture 7

Interpolation and Curve Fitting I

Interpolating Polynomials

Ceng375 *Numerical Computations* at December 2, 2010

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Contents

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- If the function does not vary too rapidly and the tabulated points are close enough together, this linearly estimated value would be accurate enough.
- As a conclusion:
  Data can be interpolated to estimate values.

Interpolation and Curve Fitting II

- **Interpolating Polynomials:** Describes a straightforward but computationally inconvenient way to fit a polynomial to a set of data points **so that an interpolated value can be computed**. The cost of getting the interpolant with a desired accuracy is facilitated by a variant, Neville's method.

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- **Divided Differences:** These provide a more efficient way to construct an interpolating polynomial, one that allows one to readily change the degree of the polynomiall. If the data are at evenly spaced x-values, there is some simplification.

Interpolation and Curve Fitting III

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- **Least-Squares Approximations:** are methods by which polynomials and other functions can be fitted to data that are subject to errors likely in experiments. These approximations are widely used **to analyze experimental observations**.

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- We will be most interested in techniques adapted to situations where the data are far from linear.
- The basic principle is to fit a polynomial curve to the data.

Interpolation versus Curve Fitting I

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- **In interpolation**, process inherently assumes that the data have no uncertainty. The interpolation function **passes *exactly* through** each of the known data points.

Interpolation versus Curve Fitting II

- Figure 1 shows a plot of some hypothetical experimental data, a curve fit function and interpolating with piecewise-linear function.

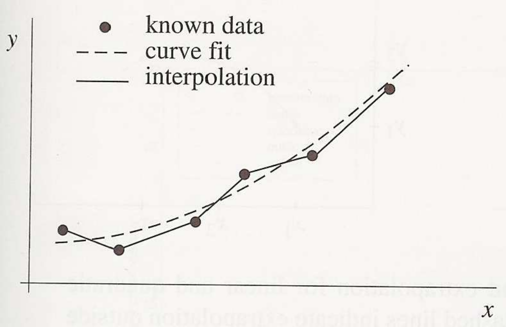

Figure: A curve fit function passes near the data points. An interpolating function passes exactly through the data points.

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  - or some other suitable set of functions.

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- The maximum degree of the polynomial is always one less than the number of points.

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- Suppose we choose the first <u>four</u> points. If the <u>cubic</u> is $ax^3 + bx^2 + cx + d$,

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- We can write four equations involving the unknown coefficients a, b, c, and d;

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- Solving these equations gives

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a &= -0.5275 \\
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- and our polynomial is

$$-0.5275x^3 + 6.4952x^2 - 16.1177x + 24.3499$$

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- Study this example in MATLAB;
  Start $\Rightarrow$ *Toolboxes* $\Rightarrow$ *CurveFitting* $\Rightarrow$ *Curve Fitting Tool*.
  » x=[3.2 2.7 1.0 4.8 5.6];
  » y=[22 17.8 14.2 38.3 51.7];

Fitting a Polynomial to Data V

- Another example;

Table: Interpolation of gasoline prices.

year	price
1986	133.5
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- Make a guess about the prices of gasoline at year of 2011.

Fitting a Polynomial to Data III

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>> format short e
>> A=[year.^5 year.^4 year.^3 year.^2 year ones(size(year))]
>> price=[133.5 132.2 138.7 141.5 137.6 144.2]'
>> a=A\price;
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>> fprintf('%12.4e \n',a)
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>> p=polyval(a,y);
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  1. the construction of the interpolating polynomials does not require the solution of a system of equations.
  2. the evaluation of the Lagrange polynomials is much less susceptible to roundoff.

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- **Linear interpolation**

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- substituting back and rearranging

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- This equation is made up of four terms, each of which is a cubic in x; hence the sum is a cubic.

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- **Example m-file:** Interpolation of gasoline prices with Lagrange Polynomials. (demoGasLag.m lagrint.m)

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- You cannot fit a function that is discontinuous or one whose derivative is discontinuous with a polynomial.

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- The Lagrange formula for linear interpolation to get $f(x)$ from two data pairs, (x_1, f_1) and (x_2, f_2), is

$$f(x) = \frac{(x - x_2)}{(x_1 - x_2)} f_1 + \frac{(x - x_1)}{(x_2 - x_1)} f_2$$

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We first *rearrange* the data pairs in order of closeness to $x = 27.5$:

i	$\lvert x - x_i \rvert$	x_i	$f_i = P_{i0}$
0	4.5	32.0	0.52992
1	5.3	22.2	0.37784
2	14.1	41.6	0.66393
3	17.4	10.1	0.17537
4	23.0	50.5	0.63608

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0	32.0	0.52992	0.46009	0.46200	0.46174	0.45754
1	22.2	0.37784	0.45600	0.46071	0.47901	
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- Thus, the value of P_{01} is computed by

$$f(x) = \frac{(27.5 - x_1)}{(x_0 - x_1)} * 0.52992 + \frac{(27.5 - x_0)}{(x_1 - x_0)} * 0.37784$$

substituting all;

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- Once we have the column of P_{i1}'s, we compute the next column.

$$P_{22} = \frac{(27.5 - 41.6) * 0.37379 + (50.5 - 27.5) * 0.44524}{50.5 - 41.6} = 0.55843$$

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- The **top line of the table** represents Lagrangian interpolates at $x = 27.5$ using polynomials of *degree equal to the second subscript* of the $P's$.

i	x	P_{i0}	P_{i1}	P_{i2}	P_{i3}	P_{i4}
0	32.0	0.52992	0.46009	0.46200	0.46174	0.45754

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0	32.0	0.52992	0.46009	0.46200	0.46174	0.45754

- The preceding data are for sines of angles in degrees and the correct value for $x = 27.5$ is 0.46175.