İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Midterm Examination
November 20, 2024 10:20 – 11:50
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		10
1B		10
2		20
3		20
4		20
5		20
TOTAL		100

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1. A) i Given two masses, $m_1 = (100.0 \pm 0.4)\ g$ and $m_2 = (49.3 \pm 0.3)\ g$, what is their sum, $m_1 + m_2$, and what is their difference, $m_1 - m_2$, both expressed with uncertainties.

ii What is the absolute and percentage uncertainty in the calculated area of a circle whose radius is determined to be $r = (14.6 \pm 0.5)\ cm$? (Hint: $\Delta A = 2\pi r \Delta r$)

You should be using the correct number of significant figures in your result.

i) $m_1 + m_2 = 100.0 + 49.3 = 149\ g$

$m_1 - m_2 = 100.0 - 49.3 = 50.7\ g$

$\Delta m = \sqrt{(0.4)^2 + (0.3)^2} = 0.5\ g$ (2) (1)

$m_1 + m_2 = 149 \pm 0.5\ g$ (1)

$m_1 - m_2 = 50.7 \pm 0.5\ g$ (1)

ii) area of the circle: $A = \pi r^2 = 3.14 * 14.6^2 = 670\ cm^2$ (1)

uncertainity: $\Delta A = 2\pi r \Delta r = 2 * 3.14 * 14.6 \times 0.5 = 46\ cm^2$ (2)
(absolute)

percentage uncertainty: $\frac{\Delta A}{A} \times 100 = 2 \frac{\Delta r}{r} \times 100 = 2 * \frac{0.5}{14.6} \times 100$

$\approx 6.85\%$ (2)

B) The position of a particle moving along an x-axis is given by $x(t) = 12t^2 - 2t^3$, where x is in meters and t is in seconds.

i Determine the acceleration of the particle at $t = 3.0\ s$.

ii What are the maximum positive coordinate reached by the particle and the acceleration of the particle at that instant?

i) $x(t) = 12t^2 - 2t^3$

$v(t) = \frac{dx}{dt} = 24t - 6t^2$ (1)

$a(t) = \frac{dv}{dt} = 24 - 12t$ (1) $\rightarrow a(t=3s) = 24 - 12\times 3 = -12\ m/s^2$ (1) (1) (1)

ii) maximum positive coordinate $\Rightarrow v(t) = \frac{dx}{dt} = 0$ (2)

$24t - 6t^2 = 0 = 6t(4-t) = 0 \Rightarrow t = 4s$ (1)

$\rightarrow x(t=4s) = 12(4)^2 - 2(4)^3 = 192 - 128 = 64m$ (1)

$a(t=4s) = 24 - 12\times 4 = -24\ m/s^2$ (1)

2.

Three vectors are given as:

$$\vec{A} = \hat{\imath} - 5\hat{k},$$
$$\vec{B} = 3\hat{\imath} - 2\hat{\jmath},$$
$$\vec{C} = 5\hat{\imath} + \hat{\jmath} + \hat{k}$$

Find:

i $\vec{A}\cdot(\vec{B}+\vec{C})$,

ii $\vec{A}\cdot(\vec{B}\times\vec{C})$,

iii The angle between $\vec{A}$ and $\vec{B}$,

iv The angle between $\vec{A}$ and $\vec{A}\times\vec{B}$,

i. $\vec{A}\cdot(\vec{B}+\vec{C})$ should be scalar

$\vec{B}+\vec{C} = 8\hat{\imath}-\hat{\jmath}+\hat{k} \rightsquigarrow \vec{A}\cdot(\vec{B}+\vec{C}) = 8-0-5 = 3$ (3)

(2)

ii. $\vec{A}\cdot(\vec{B}\times\vec{C})$ should be scalar

$\vec{B}\times\vec{C} = (B_y C_z - B_z C_y)\hat{\imath} + (B_z C_x - B_x C_z)\hat{\jmath} + (B_x C_y - B_y C_x)\hat{k}$

$= ((-2)(1)-(0)(1))\hat{\imath} + ((0)(5)-(3)(1))\hat{\jmath} + ((3)(1)-(-2)(5))\hat{k}$ (2)

$= -2\hat{\imath} - 3\hat{\jmath} + 13\hat{k}$ (1)

$\rightsquigarrow \vec{A}\cdot(\vec{B}\times\vec{C}) = -2-0-65 = -67$ (2)

iii. $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \rightsquigarrow |\vec{A}| = \sqrt{1^2+(-5)^2} = 5.10$

$|\vec{B}| = \sqrt{3^2+(-2)^2} = 3.61$ (2)

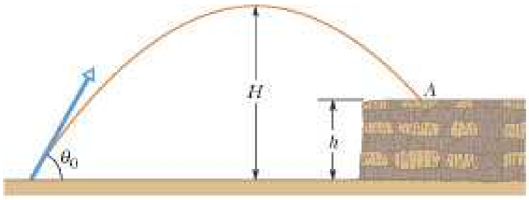
$\rightsquigarrow 3 = 5.10 * 3.61 * \cos\theta$ (2) $\vec{A}\cdot\vec{B} = 3+0+0 = 3$

$\rightsquigarrow \cos\theta = \frac{3}{5.10 * 3.61} = 0.16 \rightsquigarrow \theta = \cos^{-1} 0.16 = 80.6°$ (1)

iv. $\vec{A}$ and $\vec{A}\times\vec{B}$ is peperdicular

$\rightarrow 90°$ (5)

3. In figure given below, a stone is projected at a cliff of height h with an initial speed of $42.0\ m/s$ directed at angle $\theta_0 = 60°$ above the horizontal. The stone strikes point A in $5.50\ s$ after launching.



Find

i the height h of the cliff,

ii the speed of the stone hit at point A,

iii the maximum height H reached above the ground.

$v_0 = 42.0\ m/s$

$\theta_0 = 60°$

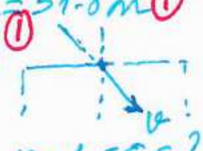
$t_A = 5.50\ s$

i) $y - y_0 = v_0 \sin\theta\, t - \frac{1}{2} g t^2$ (2)

$h - 0 = (42\ m/s)\sin 60°(5.5\ s) - \frac{1}{2}(9.8\ m/s^2)(5.5\ s)^2$ (2)

$\rightarrow h = 51.8\ m$ (1) (1)

ii)



v at $5.5\ s$?

$v = \sqrt{v_x^2 + v_y^2}$ (2)

$v_x = v_{0x} = v_0 \cos\theta_0$

$v_y = v_0 \sin\theta_0 - gt$

$\rightarrow \vec{v} = v_x \hat{i} + v_y \hat{j} = v_0 \cos\theta_0 \hat{i} + (v_0 \sin\theta_0 - gt)\hat{j}$ (2)

$v(t = 5.5\ s) = \sqrt{v_0^2 \cos^2\theta_0 + (v_0 \sin\theta_0 - gt)^2}$ (2)

$= \sqrt{(42\ m/s \cos 60)^2 + [42\ m/s \sin 60 - (9.8\ m/s^2)(5.5\ s)]^2} = 27.35\ m/s$ (1) (1)

iii) H ? at maximum height $v_y = 0$

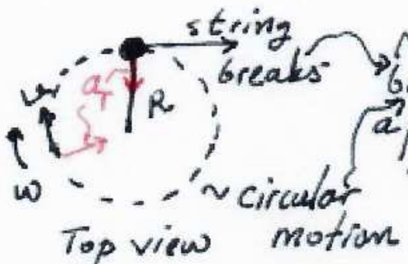
$v_y = v_0 \sin\theta_0 - gt = 0$ (1) (1) $\rightarrow t_H = \frac{v_0 \sin\theta_0}{g} = \frac{(42\ m/s)(\sin 60°)}{9.8\ m/s^2} = 3.71\ s$

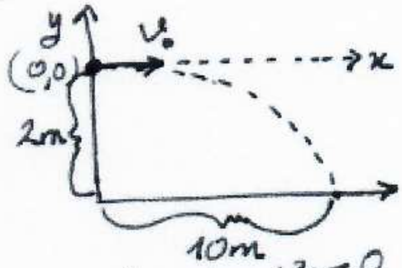
$\Rightarrow y - y_0 = H = v_0 \sin\theta_0\, t_H - \frac{1}{2} g t_H^2$ (2)

$= 42\ m/s \sin 60°\ 3.71\ s - \frac{1}{2}\ 9.8\ m/s^2\ 3.71\ s^2$

$= 67.5\ m$ (1) (1)

4. A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks, and the stone flies off horizontally and strikes the ground after traveling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone during the circular motion?





$R = 1.5\,m$

(3) $a_r = \frac{v^2}{R}$, $v = v_0 = ?$

$\Rightarrow x - x_0 = v_0 t$ (2)

$y - y_0 = -\frac{1}{2} g t^2$ (2)

$10\,m = v_0 t$ (2)

$-2m = -\frac{1}{2}(9.8\,m/s^2)\,t^2$ (2)

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2m}{9.8\,m/s^2}} = 0.64\,s$ (2) (1)

$\Rightarrow v_0 = \frac{10m}{0.64s} = 15.65\,m/s$ (2) (1)

$x - x_0 = v_{0x} t - \frac{1}{2} a_x t^2$ ($\frac{1}{2} a_x t^2 \to 0$)

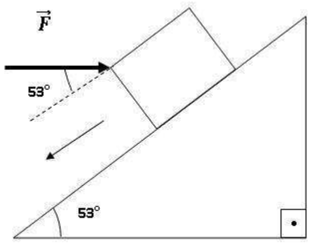
$y - y_0 = v_{0y} t - \frac{1}{2} g t^2$ (a_y)

$v_{0x} = v_0 \cos\theta_x = v_0 \sim \theta_x = 0$

$v_{0y} = v_0 \cos\theta_y = 0 \sim \theta_y = 90°$

$a_r = \frac{(15.65\,m/s)^2}{1.5\,m} = 163.\bar{3}\,m/s^2$ (2) (1)

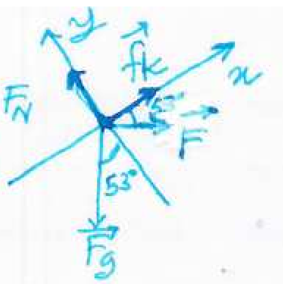
5. The block shown below moves down at a constant speed. If the block has a mass of 26 kg and the coefficients of kinetic (μ_k) and static (μ_s) frictions are 0.3 and 0.4, respectively;



i Draw free body diagram for the block,

ii Determine the magnitude of applied force.

i)



(5)

ii) Equations of motion
Newton's 2nd Law
$\Sigma F_x = ma_x$ & $\Sigma F_y = ma_y$
1 mass 2 directions: 2 eqns

(1) x: $f_k + F\cos 53 - mg\sin 53 = ma_x = 0$ (3) (1) constant speed, $a_x = 0$

(2) y: $F_N - F\sin 53 - mg\cos 53 = ma_y = 0$ (3) (1) no motion at y, $a_y = 0$

(3) $f_k = \mu_k F_N$ (2)

⇒ (2) → $F_N = F\sin 53 + mg\cos 53$

(2) & (3) → $\mu_k(F\sin 53 + mg\cos 53) + F\cos 53 - mg\sin 53 = 0$

$$F = \frac{mg\sin 53 - \mu_k mg\cos 53}{\mu_k \sin 53 + \cos 53} = \frac{26\,kg\,(9.8\,m/s^2)\sin 53 - (0.3)\,26\,kg\,(9.8\,m/s^2)\cos 53}{(0.3)\sin 53 + \cos 53}$$ (3)

$\cong 187.2\ N$ (1) (1)

İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Midterm Examination
December 01, 2023 14:30 – 16:00
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
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◇ **Calculator is allowed.**
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Question	Grade	Out of
1A		15
1B		10
1C		10
2		20
3		20
4		20
5		20
TOTAL		115

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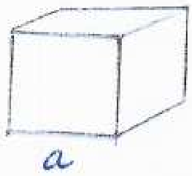
1. A) The side of a cube of metal is measured to be (1.60±0.05) cm and its mass is measured to be (30.1±0.4) g

   i Find the perimeter of one face of the cube with the uncertainty.

   ii Find the volume and uncertainty in the volume.

   iii Determine the density of the solid in kilograms per cubic meter and the uncertainty in the density.

   You should be using the correct number of significant figures in your result.



$a = (1.60 \pm 0.05)\,cm = (1.60 \pm 0.05)\times10^{-2}\,m$

$m = (30.1 \pm 0.4)\,g = (30.1 \pm 0.4)\times10^{-3}\,kg$

→ 3 sig figs

i) perimeter: $4a$ ② $= 4(1.60 \pm 0.05)\times10^{-2}\,m = (6.40 \pm 0.20)\times10^{-2}\,m$ ① ①

ii) volume: $V = a^3$ ① $\rightarrow C = A^n,\ \Delta C = C|n|\frac{\Delta A}{A}$ ②

$\Rightarrow V = a^3 = (1.60\times10^{-2}\,m)^3 = 4.10\times10^{-6}\,m^3$.

$\Delta V = a^3|3|\frac{\Delta a}{a} = 4.10\times10^{-6}|3|\frac{0.05}{1.60} = 0.38\times10^{-6}\,m^3$

$\Rightarrow$ Volume: $(4.10 \pm 0.38)\times10^{-6}\,m^3$ ① ①

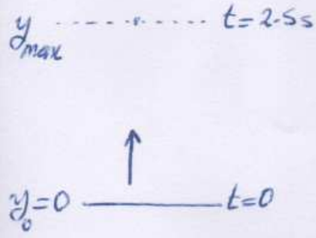
iii) density: $\rho = m/V$ ① $\rightarrow C = \frac{A}{B},\ \Delta C = |C|\sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$

$\rho = \frac{m}{V} = \frac{30.1\times10^{-3}\,kg}{4.10\times10^{-6}\,m^3} = 7341\,kg/m^3 \sim 7.34\times10^{3}\,kg/m^3$ ①

$\Delta\rho = \left|\frac{30.1\times10^{-3}\,kg}{4.10\times10^{-6}\,m^3}\right|\sqrt{\left(\frac{0.4}{30.1}\right)^2 + \left(\frac{0.38}{4.10}\right)^2} = 687\,kg/m^3$

$\Rightarrow$ density: $\rho = (7.34 \pm 0.69)\times10^{3}\,kg/m^3$ ① ① ② Results in 3 sig figs

B) A rock is thrown vertically upward from ground level at time $t = 0\ s$. At $t = 1.5\ s$ it passes the top of a tall tower, and $1.0\ s$ later it reaches its maximum height. What is the height of the tower?



at $y_{max} \Rightarrow v_y = 0$ (2)

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ (1)

$v_y = v_{0y} - gt$ (1)

$\rightarrow 0 = v_{0y} - 9.8\,m/s^2(2.5\,s)$ (1)

$v_{0y} = 24.5\,m/s$ (0.5) (0.5)

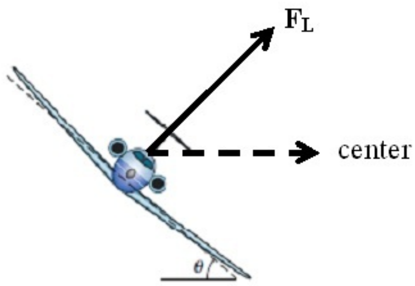
$\Rightarrow y(t=1.5s)$: height of the tower

$y(t=1.5s) = y_0 + v_{0y}(1.5s) - \frac{1}{2}g(1.5s)^2$ (3)

$= 0 + (24.5\,m/s)(1.5s) - (4.9\,m/s^2)(1.5s)^2$

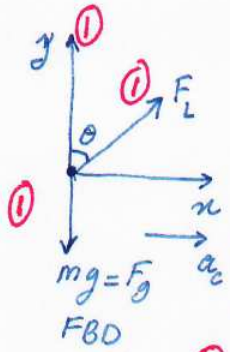
$= 25.725\,m$ (0.5) (0.5) $\rightsquigarrow$ **height $\cong 26\,m$**

C) An airplane is flying in a horizontal circle at a speed of 480 km/h as given in the figure below. Its wings are tilted at angle $\theta = 40°$ to the horizontal. Assume that the required force is provided entirely by an "aerodynamic lift" ($\mathbf{F_L}$) that is perpendicular to the wing surface.



i What is the radius of the circle in which the plane is flying ?

ii What is the magnitude of $\mathbf{F_L}$ if the air plane has a mass of 240 $\times 10^3$ kg ?



x: (1) $F_L \sin\theta = m a_x = m \frac{v^2}{R}$ (1)

y: (1) $F_L \cos\theta - mg = m a_y = 0$ (2)

$v = 480 \frac{km}{h} \frac{1000 m}{1 km} \frac{1 h}{3600 s} = 133 m/s$ (1)

$\theta = 40°$

(2) $\rightarrow F_L = \frac{mg}{\cos\theta}$

$\rightarrow$ (1) $\frac{mg}{\cos\theta} \sin\theta = m \frac{v^2}{R}$

i) $R = \frac{v^2}{(\tan\theta) g} = \frac{(133 m/s)^2}{(9.8 m/s^2)\tan 40} = \boxed{2151 m}$ (1) (1)

ii) $F_L = \frac{mg}{\cos\theta} = \frac{(240 \times 10^3 kg)(9.8 m/s^2)}{\cos 40} = \boxed{3.07 \times 10^6 N}$ (1) (1)

2. Three vectors are given by $\vec{a} = 3.0\hat{i}+3.0\hat{j}-2.0\hat{k}$, $\vec{b} = -1.0\hat{i}-4.0\hat{j}+2.0\hat{k}$, and $\vec{c} = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$. Find (a) $\vec{a}.(\vec{b} \times \vec{c})$, (b) $\vec{a}.(\vec{b} + \vec{c})$, and (c) $\vec{a} \times (\vec{b} + \vec{c})$.

Three vectors:

$\vec{a} = 3.0\hat{i}+3.0\hat{j}-2.0\hat{k}$

$\vec{b} = -1.0\hat{i}-4.0\hat{j}+2.0\hat{k}$

$c = 2.0\hat{i}+2.0\hat{j}+1.0\hat{k}$

i) $\vec{a}.(\vec{b}\times\vec{c}) = ?$ (scalar) (4)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4 & 2 \\ 2 & 2 & 1 \end{vmatrix} \Rightarrow \vec{b}\times\vec{c} = (b_y c_z - b_z c_y)\hat{i} + (b_z c_x - b_x c_z)\hat{j} + (b_x c_y - b_y c_x)\hat{k}$$

$$\vec{b}\times\vec{c} = ((-4)(1)-(2)(2))\hat{i} + ((2)(2)-(-1)(1))\hat{j} + ((-1)(2)-(-4)(2))\hat{k} = -8\hat{i}+5\hat{j}+6\hat{k}$$ (2)

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (3\hat{i}+3\hat{j}-2\hat{k})\cdot(-8\hat{i}+5\hat{j}+6\hat{k}) = -24+15-12 = \boxed{-21}$$ (4)

ii) $\vec{a}.(\vec{b}+\vec{c}) = ?$ (scalar)

$\vec{b}+\vec{c} = 1\hat{i}-2\hat{j}+3\hat{k}$ (2)

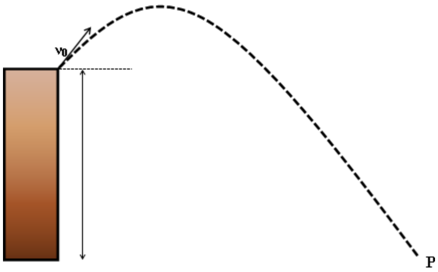
$$\vec{a}.(\vec{b}+\vec{c}) = (3\hat{i}+3\hat{j}-2\hat{k}).(1\hat{i}-2\hat{j}+3\hat{k}) = 3-6-6 = \boxed{-9}$$ (4)

iii) $\vec{a}\times(\vec{b}+\vec{c}) = ?$ (vector)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & -2 \\ 1 & -2 & 3 \end{vmatrix} \rightsquigarrow ((3)(3)-(-2)(-2))\hat{i} + ((-2)(1)-(3)(3))\hat{j} + ((3)(-2)-(3)(1))\hat{k}$$

$$= \boxed{5\hat{i}-11\hat{j}-9\hat{k}}$$ (4)

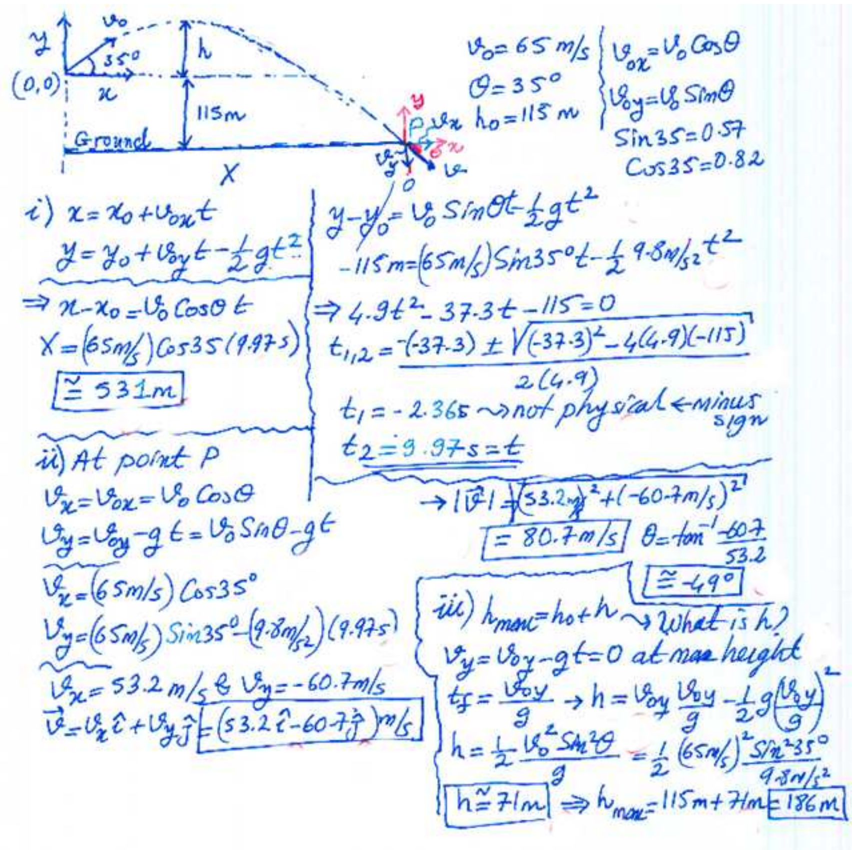
3. A projectile is shot from the edge cliff 120 m above ground level with an initial speed of 60 m/s at an angle of 30° with the horizontal.



i Determine the distance X of point P from the base of the vertical cliff.

ii What is the velocity v at point P in magnitude-angle notation and in unit-vector notation?

iii Find the maximum height reached by projectile **above ground**.



$v_0 = 65\ m/s$, $\theta = 35°$, $h_0 = 115\ m$ | $v_{0x} = v_0 \cos\theta$, $v_{0y} = v_0 \sin\theta$, $\sin 35 = 0.57$, $\cos 35 = 0.82$

i) $x = x_0 + v_{0x} t$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$\Rightarrow x - x_0 = v_0 \cos\theta\, t$

$X = (65 m/s) \cos 35 (9.97 s)$

$\boxed{\cong 531 m}$

$y - y_0 = v_0 \sin\theta t - \frac{1}{2} g t^2$

$-115 m = (65 m/s) \sin 35° t - \frac{1}{2} 9.8 m/s^2 t^2$

$\Rightarrow 4.9 t^2 - 37.3 t - 115 = 0$

$t_{1,2} = \frac{-(-37.3) \pm \sqrt{(-37.3)^2 - 4(4.9)(-115)}}{2(4.9)}$

$t_1 = -2.36 s$ ⇝ not physical ← minus sign

$t_2 = 9.97 s = t$

ii) At point P

$v_x = v_{0x} = v_0 \cos\theta$

$v_y = v_{0y} - g t = v_0 \sin\theta - g t$

$v_x = (65 m/s) \cos 35°$

$v_y = (65 m/s) \sin 35° - (9.8 m/s^2)(9.97 s)$

$v_x = 53.2\ m/s$ & $v_y = -60.7\ m/s$

$\vec{v} = v_x \hat{i} + v_y \hat{j} = \boxed{(53.2 \hat{i} - 60.7 \hat{j})\ m/s}$

$\rightarrow |\vec{v}| = \sqrt{(53.2 m/s)^2 + (-60.7 m/s)^2}$

$\boxed{= 80.7 m/s}$ $\theta = \tan^{-1} \frac{-60.7}{53.2}$

$\boxed{\cong -49°}$

iii) $h_{max} = h_0 + h$ ⇝ What is h?

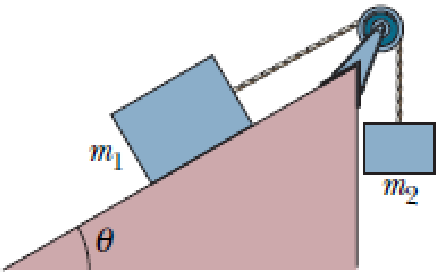
$v_y = v_{0y} - g t = 0$ at max height

$t_f = \frac{v_{0y}}{g} \rightarrow h = v_{0y} \frac{v_{0y}}{g} - \frac{1}{2} g \left(\frac{v_{0y}}{g}\right)^2$

$h = \frac{1}{2} \frac{v_0^2 \sin^2\theta}{g} = \frac{1}{2} \frac{(65 m/s)^2 \sin^2 35°}{9.8 m/s^2}$

$\boxed{h \cong 71 m} \Rightarrow h_{max} = 115 m + 71 m = \boxed{186 m}$

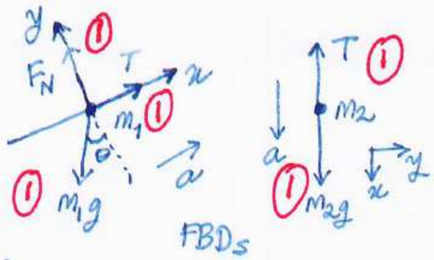
4. A block of mass $m_1 = 6\ kg$ on a frictionless plane inclined at angle $\theta = 60°$ is connected by a cord over a massless, frictionless pulley to a second block of mass $m_2 = 8\ kg$ as given in figure below.



What are

i the magnitude of the acceleration of each block?

ii the tension in the cord?



m_1: ② x: $T - m_1 g \sin\theta = m_1 a_{x,1}$ ① m_2: x: $m_2 g - T = m_2 a_{x,2}$ ③

y: $F_N - m_1 g \cos\theta = m_1 a_{y,1}$ ② ② y: ~ ②

where $a_{x,1} = a_{x,2} = a$ system acceleration

$a_{y,1} = 0$

i) ② → $F_N = m_1 g \cos\theta$

① → $T = m_1 a + m_1 g \sin\theta$ } → ③ $m_2 g - (m_1 a + m_1 g \sin\theta) = m_2 a$

$\Rightarrow a = \dfrac{m_2 g - m_1 g \sin\theta}{m_1 + m_2}$ ②

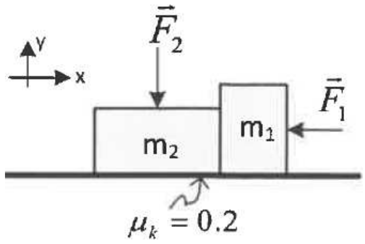
$= \dfrac{(8kg)(9.8m/s^2) - (6kg)(9.8m/s^2)\sin 60}{6kg + 8kg} = \boxed{1.96\ m/s^2}$ ① ①

ii) ③ → $T = m_2(g - a)$ ② $= 8kg(9.8m/s^2 - 1.96 m/s^2) = \boxed{62.7\ N}$ ② ①

or ① → $T = m_1(a + g \sin\theta)$

5.

Two blocks ($m_1 = 4\ kg$ and $m_2 = 1\ kg$) on a rough horizontal surface ($\mu_k = 0.2$ for both blocks) are pushed to the left by a horizontal force $F_1 = 80\ N$. Another force $F_2 = 20\ N$ is vertically pressing the block m_2 to the surface. There is no friction between the blocks. Use the coordinate system as depicted in the figure. Take $g = 10\ m/s^2$.

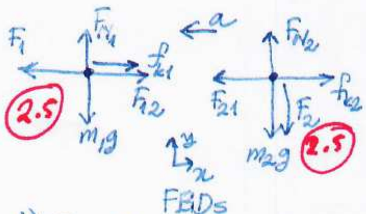


i Find the normal force **vectors** (use unit vector notation) exerted by the surface on each block

ii Find the frictional force **vectors** on each block

iii Determine the acceleration **vector** of each block.

iv Find the action-reaction force **vectors** exerted by each block on the other.



FBDs

m_1: x: ① $f_{k1} + F_{12} - F_1 = m_1 a_{x1}$; y: ② $F_{N1} - m_1 g = m_1 a_{y1}$ (3)

m_2: x: ③ $f_{k2} - F_{21} = m_2 a_{x2}$; y: ④ $F_{N2} - F_2 - m_2 g = m_2 a_{y2}$ (3)

where $a_{x1} = a_{x2} = a$ & $a_{y1} = a_{y2} = 0$

i) ② → $F_{N1} = m_1 g = (4\,kg)(10\,m/s^2) = 40N$ → $\vec{F}_{N1} = 40N(\hat{j})$ (1)

④ → $F_{N2} = F_2 + m_2 g = 20N + (1kg)(10\,m/s^2) = 30N$ → $\vec{F}_{N2} = 30N(\hat{j})$ (1)

ii) $f_{k1} = \mu_k F_{N1} = 0.2 * 40N = 8N$ → $\vec{f}_{k1} = 8N(\hat{i})$ (1)

$f_{k2} = \mu_k F_{N2} = 0.2 * 30N = 6N$ → $\vec{f}_{k2} = 6N(\hat{i})$ (1)

iii) ①&③ → $f_{k1} + F_{12} - F_1 = -m_1 a$ & $F_{21} = f_{k2} + m_2 a$ & $|F_{21}| = |F_{12}|$

→ $f_{k1} + f_{k2} + m_2 a - F_1 = -m_1 a$ ⇒ (2) $a = -\frac{f_{k1} + f_{k2} - F_1}{(m_1 + m_2)} = \frac{8N + 6N - 80N}{(1kg + 4kg)} = 13.2\,m/s^2$

⇒ $\vec{a} = 13.2\,m/s^2(-\hat{i})$ (1)

iv) Action-Reaction pair $\vec{F}_{12} = -\vec{F}_{21}$

③ → $f_{k2} + m_2 a = F_{21}$ → $F_{21} = 6N + (1kg)(13.2\,m/s^2) = 19.2N$ → $\vec{F}_{21} = 19.2N(-\hat{i})$ (1)

$\vec{F}_{12} = 19.2N(+\hat{i})$ (1)

İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Midterm Examination
April 04, 2022 08:30 – 10:00
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

INSTRUCTOR:

DURATION: 90 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		10
1C		10
2		20
3		20
4		20
5		20
TOTAL		115

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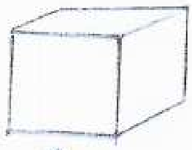
1. A) The side of a cube of metal is measured to be (1.60±0.05) cm and its mass is measured to be (30.1±0.4) g

   i Find the perimeter of one face of the cube with the uncertainty.

   ii Find the volume and uncertainty in the volume.

   iii Determine the density of the solid in kilograms per cubic meter and the uncertainty in the density.

   You should be using the correct number of significant figures in your result.



a

$a = (1.60 \pm 0.05)\,cm = (1.60 \pm 0.05)\times 10^{-2}\,m$

$m = (30.1 \pm 0.4)\,g = (30.1 \pm 0.4)\times 10^{-3}\,kg$

→ 3 sig figs //

i) Perimeter: ② $4a = 4(1.60 \pm 0.05)\times 10^{-2}\,m = (6.40 \pm 0.20)\times 10^{-2}\,m$ ① ①

ii) volume: ① $V = a^3 \rightsquigarrow C = A^n$, $\Delta C = C|n|\frac{\Delta A}{A}$ ②

$\Rightarrow V = a^3 = (1.60\times 10^{-2}\,m)^3 = 4.10\times 10^{-6}\,m^3$

$\Delta V = a^3|3|\frac{\Delta a}{a} = 4.10\times 10^{-6}|3|\frac{0.05}{1.60} = 0.38\times 10^{-6}\,m^3$

$\Rightarrow$ Volume: $(4.10 \pm 0.38)\times 10^{-6}\,m^3$ ① ①

iii) density: ① $\rho = m/V \rightsquigarrow C = \frac{A}{B}$, $\Delta C = |C|\sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$

$\rho = \frac{m}{V} = \frac{30.1\times 10^{-3}\,kg}{4.10\times 10^{-6}\,m^3} = 7341\,kg/m^3 \sim 7.34\times 10^{3}\,kg/m^3$ ①

$\Delta\rho = \left|\frac{30.1\times 10^{-3}\,kg}{4.10\times 10^{-6}\,m^3}\right|\sqrt{\left(\frac{0.4}{30.1}\right)^2 + \left(\frac{0.38}{4.10}\right)^2} = 687\,kg/m^3$ ① ①

$\Rightarrow$ density: $\rho = (7.34 \pm 0.69)\times 10^{3}\,kg/m^3$ ② Results in 3 sig figs

B) The position of a particle moving along an x-axis is given by $x(t) = 12t^2 - 2t^3$, where x is in meters and t is in seconds.

i Determine the acceleration of the particle at $t = 3.0\ s$.

ii What are the maximum positive coordinate reached by the particle and the acceleration of the particle at that instant?

i) $x(t) = 12t^2 - 2t^3$

$v(t) = \frac{dx}{dt} = 24t - 6t^2$ (1)

$a(t) = \frac{dv}{dt} = 24 - 12t$ (1) $\rightsquigarrow a(t=3s) = 24 - 12\times 3 = -12\ m/s^2$ (1) (1) (1)

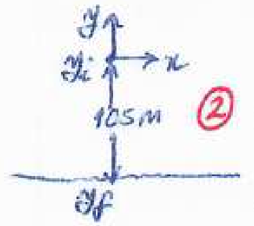
ii) maximum positive coordinate $\rightsquigarrow v(t) = \frac{dx}{dt} = 0$ (2)

$24t - 6t^2 = 0 = 6t(4-t) = 0 \Rightarrow t = 4s$ (1)

$\rightsquigarrow x(t=4s) = 12(4)^2 - 2(4)^3 = 192 - 128 = 64m$ (1)

$a(t=4s) = 24 - 12*4 = -24\ m/s^2$ (1)

C) A helicopter is ascending (move upward) vertically with a speed of 5.40 m/s. At a height of 105 m above the Earth, a package is dropped from the helicopter. How much time does it take for the package to reach the ground? [Hint: What is v_0 for the package?]



with our chosen coordinate system
$v_0 = 5.40\ m/s$ as upward & $y_i = 0$
$y_f = -105\ m$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \quad (2)$$

$$-105 - 0 = 5.4t - \frac{1}{2} 9.8 t^2 \Rightarrow 4.9t^2 - 5.4t - 105 = 0 \quad (2)$$

Quadratic equation $at^2 + bt + c = 0$

$$\Rightarrow t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5.4) \pm \sqrt{(-5.4)^2 - 4\times4.9\times(-105)}}{2\times4.9} \quad (2)$$

$< 5.21\ s$ ✓ (2)
$< -4.11\ s$ ✗

2. Vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ lie in an xy-plane. $\overrightarrow{A}$ has magnitude 8.0 and an angle 130°; B has components $B_x = -7.72$ and $B_y = -9.20$.

   i What are $5\overrightarrow{A} \cdot \overrightarrow{B}$ and $4\overrightarrow{A} \times 3\overrightarrow{B}$ in unit vector notation?

   ii What is $(3\hat{i} + 5\hat{j}) \times (4\overrightarrow{A} \times 3\overrightarrow{B})$? Find magnitude and angle of resultant vector.

$\vec{A}$ & $\vec{B}$ lie in xy-plane ⇒ only x & y components

$|\vec{A}| = 8$ with angle 130° & $B_x = -7.72$, $B_y = -9.20$

$$\Rightarrow \vec{A} = |\vec{A}|\cos 130\,\hat{i} + |\vec{A}|\sin 130\,\hat{j} = -5.14\hat{i} + 6.13\hat{j}$$

$$\vec{B} = -7.72\hat{i} + 9.20(-\hat{j})$$

i) $$5\vec{A}\cdot\vec{B} = 5(-5.14\hat{i} + 6.13\hat{j})\cdot(-7.72\hat{i} - 9.20\hat{j})$$

$$= 5[(-5.14)\times(-7.72) + (6.13\times(-9.20))] = -83.58$$

$$4\vec{A}\times 3\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -20.56 & 24.52 & 0 \\ -23.16 & -27.6 & 0 \end{vmatrix} = (24.52\times 0 - 0(-27.6))\hat{i} + (0(-23.16) - (-20.56)0)\hat{j} + ((-20.56)(-27.6) - (24.52)(-23.16))\hat{k}$$

$$= 1135.4\,\hat{k}$$

ii) $$(3\hat{i} + 5\hat{j})\times(1135.4\hat{k}) = (3\times 1135.4)\underbrace{(\hat{i}\times\hat{k})}_{-\hat{j}} + (5\times 1135.4)\underbrace{(\hat{j}\times\hat{k})}_{\hat{i}}$$

$$\hat{i}\to\hat{j}\to\hat{k}$$

$$= 5677\hat{i} + 3460.2(-\hat{j})$$

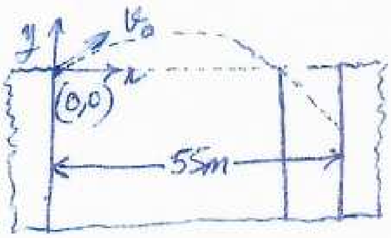
magnitude: $\sqrt{(5677)^2 + (-3460.2)^2} = 6620.5$

angle: $\theta = \tan^{-1}\dfrac{-3460.2}{5677} = -31°$ or $329°$

3. A ball is shot from the top of a building with an initial velocity of 18 m/s at an angle $\theta = 42°$ above the horizontal.

   i What are the horizontal and vertical components of the initial velocity?

   ii If a nearby building is the same height and 55 m away, how far below the top of the building will the ball strike the nearby building?



$v_0 = 18\ m/s$

$\theta = 42°$

i) $v_x = v_0 \cos\theta = 18 m/s \cos 42 = 13.38\ m/s$

$v_y = v_0 \sin\theta = 18 m/s \sin 42 = 12.04\ m/s$

ii) $v_x = v_{0x}$ & $\Delta x = v_{0x} t \rightsquigarrow$ time of flight

$$t = \frac{\Delta x}{v_{0x}} = \frac{55 m}{13.38 m/s} = 4.11 s$$

same time interval for y direction & $y_0 = 0$

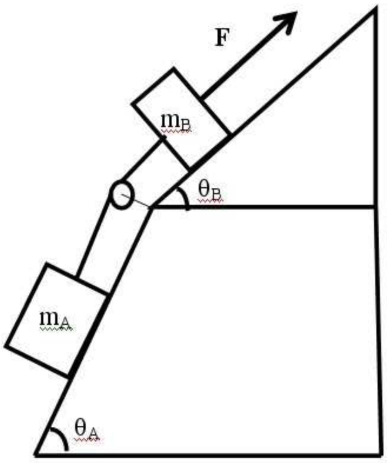
$$\Delta y = y - y_0 = v_{0y} t - \frac{1}{2} g t^2$$

$$= (12.04)(4.11) - \frac{1}{2} 9.8 (4.11)^2$$

$$= -33.3 m$$

{ minus sign shows below the top of the building

4. Consider the system shown in figure with $m_A = 9.5\ kg$ and $m_B = 11.5\ kg$. The angles $\theta_A = 59°$ and $\theta_B = 32°$.



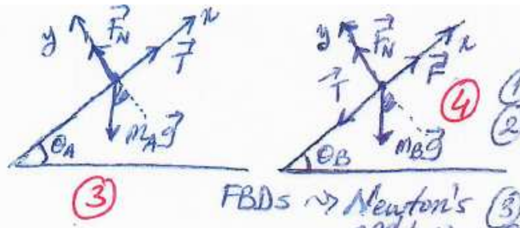
i Draw the free body diagrams for block A and block B.

ii In the absence of friction, what force F would be required to pull the masses at a **constant velocity** up?

iii The force F now is removed . What is the magnitude and direction of acceleration of the two blocks?

iv In the absence of F, what is the tension in the string?

i)



FBDs ⇝ Newton's 2nd law ⇝ equations of motion

mass A

(1) x: $T - m_A g \sin\theta_A = m_A a_{Ax}$

(2) y: $F_N - m_A g \cos\theta_A = m_A a_{Ay}$

mass B

(3) x: $F - T - m_B g \sin\theta_B = m_B a_{Bx}$

(4) y: $F_N - m_B g \cos\theta_B = m_B a_{By}$

$a_{Ay} = 0 = a_{By}$ & $a_{Ax} = a_{Bx} = a$

no motion in y-direction

ii) constant velocity ⇝ $a = 0$ (1)&(3)

(1) $T - m_A g \sin\theta_A = 0$

(1) $F - T - m_B g \sin\theta_B = 0$

$F = T + m_B g \sin\theta_B$ (1)

$= m_A g \sin\theta_A + m_B g \sin\theta_B$

$= (9.5 \sin 59 + 11.5 \sin 32)\, 9.8 = 79.8\,N + 59.7\,N \cong 140\,N$ (1)(1)

iii) now $a \neq 0$ (1) (1)&(3)

(1) $T - m_A g \sin\theta_A = m_A a$

(1) $F - T - m_B g \sin\theta_B = m_B a$

$F = 0 \Rightarrow m_A(g \sin\theta_A + a) - m_B g \sin\theta_B = m_B a$ (1)

$\Rightarrow a = \dfrac{-m_A g \sin\theta_A - m_B g \sin\theta_B}{m_A + m_B} \cong 6.7\,m/s^2\,(-\hat{i})$ (1) (1)

iv) $T - m_A g \sin\theta_A = m_A a$ (3) (1) $\sim T = m_A(a + g \sin\theta_A)$

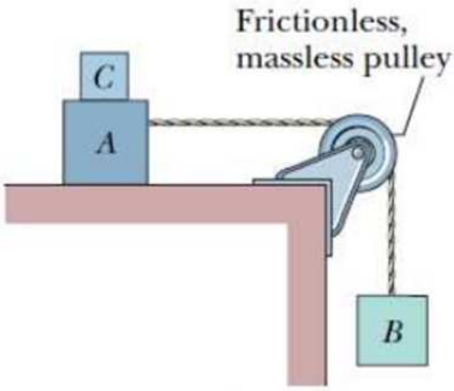
$= 9.5(-6.7 + 9.8 \sin 59°)$

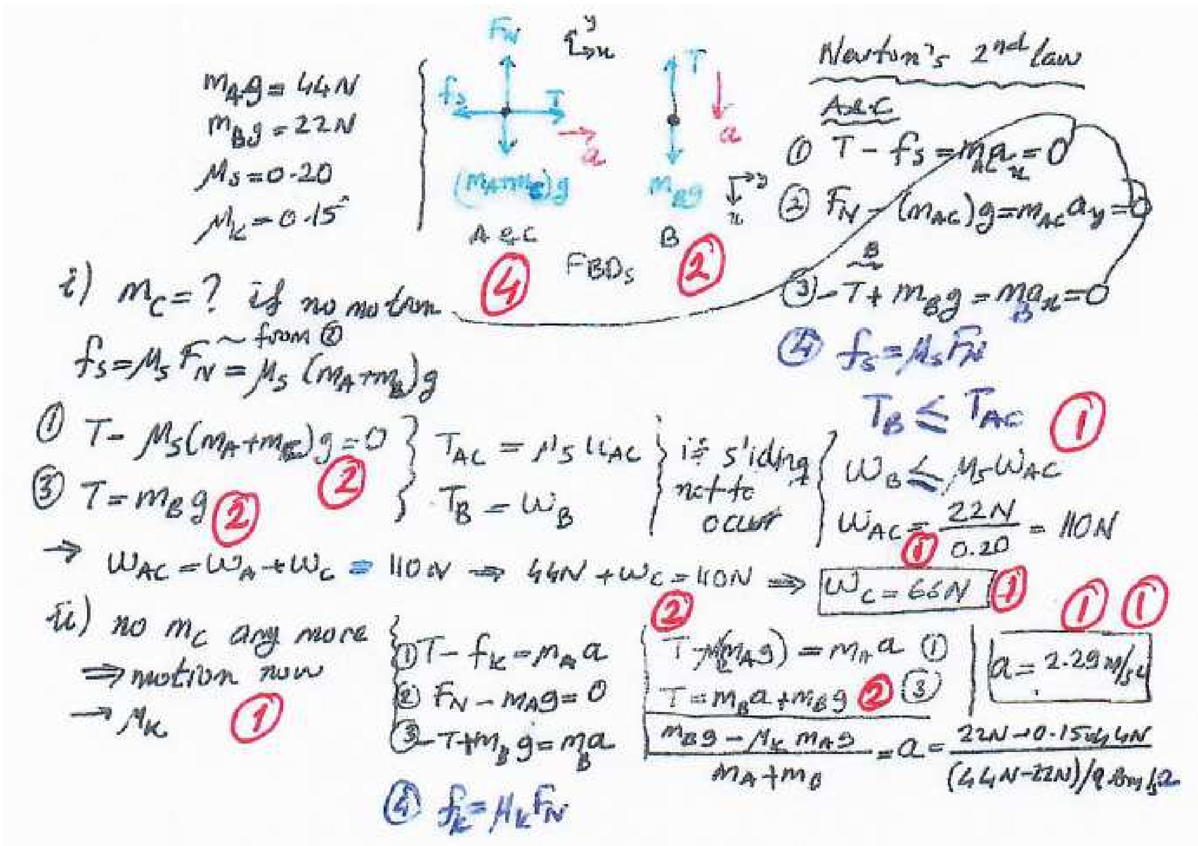
$\cong 16.2\,N$ (1) (1)

5. In Figure, blocks A and B have weights of 44 N and 22 N, respectively.

i Determine the minimum weight of block C to keep A from sliding if μ_s between A and the table is 0.20.

ii Block C suddenly is lifted off A. What is the acceleration of block A if μ_k between A and the table is 0.15?





İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Canvas Midterm Examination
April 20, 2020 13:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

DURATION: 180 minutes

◇ **Solve 10 questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology**.

Question	Grade	Out of
TOTAL		110

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1. (Measurement) Calculate z and Δz for each of the following cases:

   i $z = x - 2.5y + w$ for $x = (4.72 \pm 0.12)\ m$, $y = (4.4 \pm 0.2)\ m$, $w = (15.63 \pm 0.16)\ m$.

   ii $z = \frac{wx}{y}$ for $w = (14.42 \pm 0.03)\ m/s^2$, $x = (3.61 \pm 0.18)\ m$, $y = (650 \pm 20)\ m/s$.

   iii $z = x^3$ for $x = (3.55 \pm 0.15)\ m$.

   iv $z = A \sin y$ for $A = (1.602 \pm 0.007)\ m/s$, $y = (0.774 \pm 0.003)\ rad$.

Answer: i) $(9.4 \pm 0.5)\ m$ **ii)** $(0.080 \pm 0.005)\ m/s$ **iii)** $(44.7 \pm 5.7)\ m^3$ **iv)** $(1.120 \pm 0.006)\ m/s$

i. $2.5y = 2.5\,(4.4 \pm 0.2)m = (11 \pm 0.5)m$

$z = (4.72 - 11 + 15.63)m = 9.35m$ & $\Delta z = \sqrt{(0.12m)^2 + (0.5m)^2 + (0.16m)^2}$

$= 0.539m.$

$z \mp \Delta z = (9.4 \pm 0.5)m$ (least precise)

ii. $z = \frac{(14.42 m/s^2)(3.61m)}{650\ m/s} = 0.080\ m/s$ (2 sig. fig)

$\Delta z = |0.080\ m/s| \sqrt{\left(\frac{0.03 m/s^2}{14.42 m/s^2}\right)^2 + \left(\frac{0.18m}{3.61m}\right)^2 + \left(\frac{20m/s}{650m/s}\right)^2} = 0.00469\ m/s$

$z \pm \Delta z = (0.080 \pm 0.005)\ m/s.$

iii. $z = (3.55\ m)^3 \doteq 44.7$ & $\Delta z = |3.55m|^3$

$\Delta z = (3.55m)^3\ |3|\ \frac{(0.15m)}{(3.55m)} \doteq 5.671 m^3$

$z \pm \Delta z = (44.7 \pm 5.7)\ m^3$

iv. $\sin y \pm (\cos y\, \Delta y) = \sin(0.774 rad) \pm \cos(0.774 rad)(0.003 rad)$

$z = (1.602 m/s)(0.699) \doteq 1.11980\ m/s$

$\Delta z = (1.11980\ m/s) \sqrt{\left(\frac{0.002145}{0.699}\right)^2 + \left(\frac{0.007 m/s}{1.602 m/s}\right)^2} = 0.0059791\ m/s$

$z + \Delta z = (1.120 \pm 0.006)\ m/s.$

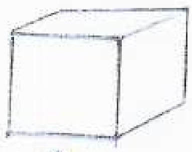
2. (Measurement) The side of a cube of metal is measured to be (1.60±0.05) cm and its mass is measured to be (30.1±0.4) g

   i Find the perimeter of one face of the cube with the uncertainty.

   ii Find the volume and uncertainty in the volume.

   iii Determine the density of the solid in kilograms per cubic meter and the uncertainty in the density.

Answer: i) 4a$=(6.40\pm0.20)\times10^{-2}\ m$ **ii) V**$=(4.10\pm0.38)\times10^{-6}\ m^3$ **iii)** $\rho=(7.34\pm0.69)\times10^{3}\ kg/m^3$



$a=(1.60\pm0.05)\,cm=(1.60\pm0.05)\times10^{-2}\,m$

$m=(30.1\pm0.4)\,g=(30.1\pm0.4)\times10^{-3}\,kg$

⤳ 3 sig figs //

i) perimeter: $4a \overset{(2)}{=} 4(1.60\pm0.05)\times10^{-2}\,m=(6.40\pm0.20)\times10^{-2}\,m$ (1) (1)

ii) volume: $V \overset{(1)}{=} a^3$ ⤳ $C=A^n,\ \Delta C=C|n|\frac{\Delta A}{A}$ (2)

$\Rightarrow V=a^3=(1.60\times10^{-2}\,m)^3=4.10\times10^{-6}\,m^3$.

$\Delta V=a^3|3|\frac{\Delta a}{a}=4.10\times10^{-6}|3|\frac{0.05}{1.60}=0.38\times10^{-6}\,m^3$

$\Rightarrow$ Volume: $(4.10\pm0.38)\times10^{-6}\,m^3$ (1) (1)

iii) density: $\rho \overset{(1)}{=} m/V$ ⤳ $C=\frac{A}{B},\ \Delta C=|C|\sqrt{\left(\frac{\Delta A}{A}\right)^2+\left(\frac{\Delta B}{B}\right)^2}$

$\rho=\frac{m}{V}=\frac{30.1\times10^{-3}\,kg}{4.10\times10^{-6}\,m^3}=7341\,kg/m^3\sim7.34\times10^{3}\,kg/m^3$ (1)

$\Delta\rho=\left|\frac{30.1\times10^{-3}\,kg}{4.10\times10^{-6}\,m^3}\right|\sqrt{\left(\frac{0.4}{30.1}\right)^2+\left(\frac{0.38}{4.10}\right)^2}=687\,kg/m^3$

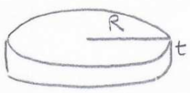
$\Rightarrow$ density: $\rho=(7.34\pm0.69)\times10^{3}\,kg/m^3$ (1) (1) (2) Results in 3 sig figs

3. (Measurement) A circular disk with a radius of (8.50±0.02) cm and a thickness of (0.050±0.005) cm.

   i Find the perimeter of the circle with the uncertainty.

   ii Find the volume and the uncertainty in the volume.

   Answer: i) $2\pi R = (53.4 \pm 0.1) \times 10^{-2}\ m$ **ii) V**$=(1.1 \pm 0.1) \times 10^{-5}\ m^3$



$R = (8.50 \pm 0.02)\ cm = (8.50 \pm 0.02) \times 10^{-2} m$ (3sf)

$t = (0.050 \pm 0.005)\ cm = (0.050 \pm 0.005) \times 10^{-2} m$ (2sf)

a) $2\pi R = 2\pi(8.50 \pm 0.02) \times 10^{-2}$

$= (53.4 \pm 0.1) \times 10^{-2} m$ //

b) $V = (\pi R^2)\, t$

1st step : Raised to a power $C = A^n$, $\Delta C = C\,|n|\,\frac{\Delta A}{A}$

$C = R^2 \rightsquigarrow \Delta C = R^2 |2| \frac{\Delta R}{R} = (8.50 \times 10^{-2} m)^2\, |2|\, \frac{0.02}{8.50}$

$= (8.50 \times 10^{-2} m)^2$

$= 7.23 \times 10^{-3} m^2$

$= 0.34 \times 10^{-4} m^2$

$\Rightarrow (7.23 \times 10^{-3} \pm 0.34 \times 10^{-4})\ m^2$

2nd step : Multiplation with a scalor $\pi R^2 = 22.7 \times 10^{-3} \pm 1.07$

$\pi R^2 = (22.7 \times 10^{-3} \pm 1.07 \times 10^{-4})\ m^2$

3rd step : Multiplation/Division

$C = AB \rightsquigarrow \Delta C = |C| \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$

$= (22.7 \times 10^{-3} m^2)(0.05 \times 10^{-2} m)$

$= 1.14 \times 10^{-5} m^3$

$= |1.14 \times 10^{-5} m^3| \sqrt{\left(\frac{1.07 \times 10^{-4}}{22.7 \times 10^{-3}}\right)^2 + \left(\frac{0.005}{0.05}\right)^2}$

$= 1.14 \times 10^{-6} m^3$

$V = (1.1 \pm 0.1) \times 10^{-5} m^3$ (2 sig figs)

4. (Motion Along a Straight Line) The position of a particle moving along an x-axis is given by $x = 12t^2 - 2t^3$, where x is in meters and t is in seconds.

   i Determine the acceleration of the particle at $t = 3.0\ s$.

   ii What are the maximum positive coordinate reached by the particle and the acceleration of the particle at that instant?

   Answer: i) $a(t = 3\ s) = -12\ m/s^2$ **ii)** $x(t = 4\ s) = 64\ m$**,** $a(t = 4\ s) = -24\ m/s^2$

i) $x(t) = 12t^2 - 2t^3$

$v(t) = \frac{dx}{dt} = 24t - 6t^2$ (1)

$a(t) = \frac{dv}{dt} = 24 - 12t$ (1) $\rightsquigarrow a(t = 3s) = 24 - 12 \times 3 = -12\ m/s^2$ (1) (1) (1)

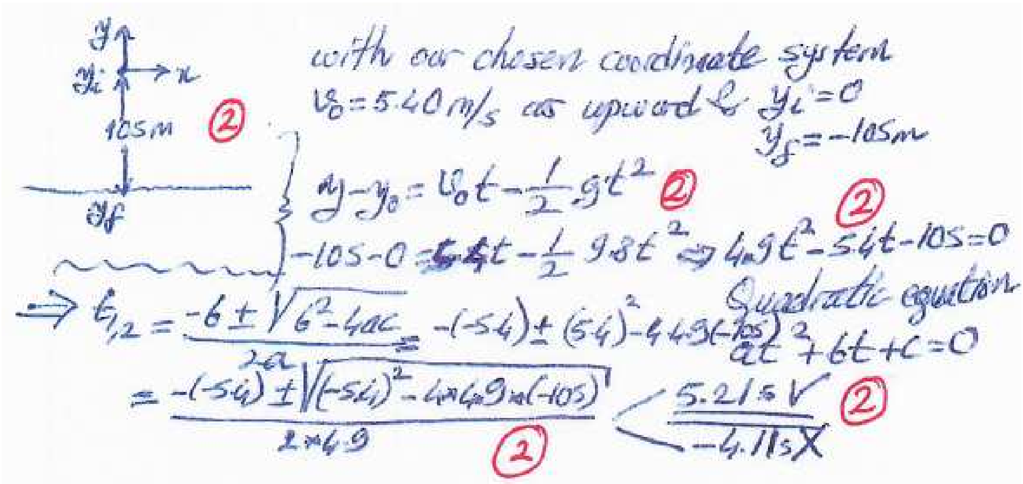
ii) maximum positive coordinate $\rightsquigarrow v(t) = \frac{dx}{dt} = 0$ (2)

$24t - 6t^2 = 0 = 6t(4 - t) = 0 \Rightarrow t = 4s$ (1)

$\rightsquigarrow x(t = 4s) = 12(4)^2 - 2(4)^3 = 192 - 128 = 64m$ (1)

$a(t = 4s) = 24 - 12 \times 4 = -24\ m/s^2$ (1)

5. (Motion Along a Straight Line) A helicopter is ascending vertically with a speed of 5.40 m/s. At a height of 105 m above the Earth, a package is dropped from the helicopter. How much time does it take for the package to reach the ground? [Hint: What is v_0 for the package?]
Answer: $t = 5.21\ s$

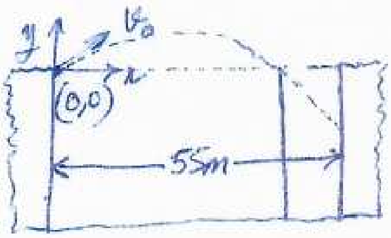


6. ((Motion in Two and Three Dimensions) A ball is shot from the top of a building with an initial velocity of 18 m/s at an angle $\theta = 42°$ above the horizontal.

   i What are the horizontal and vertical components of the initial velocity?

   ii If a nearby building is the same height and 55 m away, how far below the top of the building will the ball strike the nearby building?

Answer: i) $v_{xo} = 13\ m/s\ v_{yo} = 12\ m/s$ **ii)** $y = -33\ m$



$v_0 = 18\ m/s$

$\theta = 42°$

(3)

i) $v_x = v_0 \cos\theta = 18\ m/s \cos 42 = 13.38\ m/s$

$v_y = v_0 \sin\theta = 18\ m/s \sin 42 = 12.04\ m/s$

(3)

ii) $v_x = v_{0x}$ & $\Delta x = v_{0x} t \rightarrow$ time of flight

(3) $t = \frac{\Delta x}{v_{0x}} = \frac{55m}{13.38 m/s} = 4.11 s$ (1)

(2)

same time interval for y direction & $y_0 = 0$

(1)

(2)

$\Delta y = y - y_0 = v_{0y} t - \frac{1}{2} g t^2$ (2)

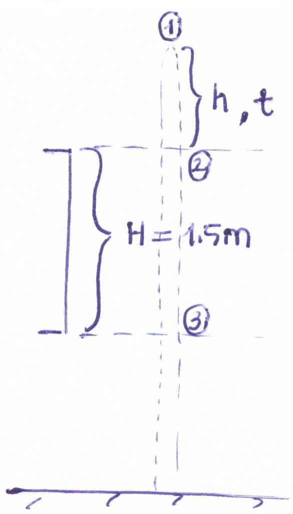
$= (12.04)(4.11) - \frac{1}{2} 9.8 (4.11)^2$

$= -33.3 m$ { minus sign shows below the top of the building

(2) (1)

7. (Motion Along a Straight Line) A boy sees a flower pot sail up and then back past a window $1.5m$ high. If the total time the pot is in sight is $1.0s$, find the height above the top pf the window that the pot rises.
Answer: $1.5\ cm$

t = time of ascent through a height 'h' = time of descent through a height



Choose downword as the positive y-direction

$$\Delta y = v_{oy} t + \frac{1}{2} a_y t^2$$

from 1 to 2 : $h = \frac{1}{2} g t^2$

from 1 to 3 : $h + H = \frac{1}{2} g \left(t + \frac{1}{2}\right)^2$

$$\frac{1}{2} g t^2 + H = \frac{1}{2} g t^2 + g \frac{t}{2} + \frac{g}{8}$$

$$1.5 = (9.8) \cdot \frac{t}{2} + \frac{9.8}{8}$$

$$t = 0.056\ s$$

$$h = \frac{1}{2} (9.8\ m/s^2)(0.056 s)^2 \doteq 1.5 cm$$

8. (Vectors) For the following three vectors, what is $3\vec{C} \cdot (2\vec{A} \times \vec{B})$

$$\begin{aligned} \vec{A} &= 2.00\hat{\imath} + 3.00\hat{\jmath} - 4.00\hat{k}, \\ \vec{B} &= -3.00\hat{\imath} + 4.00\hat{\jmath} + 2.00\hat{k}, \\ \vec{C} &= 7.00\hat{\imath} - 8.00\hat{\jmath} \end{aligned}$$

Answer: 540

$$2\vec{A} = 4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$$

$$3\vec{C} = 21\hat{\imath} - 24\hat{\jmath}$$

$$2\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 6 & -8 \\ -3 & 4 & 2 \end{vmatrix} = (12+32)\hat{\imath} - (8-24)\hat{\jmath} + (16+18)\hat{k}$$

$$= 44\hat{\imath} + 16\hat{\jmath} + 34\hat{k}$$

$$3\vec{C} \cdot (2\vec{A} \times \vec{B}) = (21\hat{\imath} - 24\hat{\jmath}) \cdot (44\hat{\imath} + 16\hat{\jmath} + 34\hat{k})$$

$$= 21 \cdot 44 - 24 \cdot 16 = \boxed{540}$$

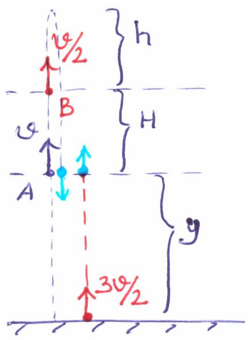
9. (Motion Along a Straight Line) A stone is thrown vertically upward. On its way up it passes point A with speed v, and point B, 3.00 m higher than A, with speed $\frac{v}{2}$.

   i Calculate the speed v and the maximum height reached by the stone above point B. At the instant when the first ball is on its way up at point B, second ball is thrown upward from the ground and with an initial speed of $\frac{3v}{2}$.

   ii How long after the second ball is thrown does it take if the two balls are to meet at the point A.

   iii What is the height of the point A above the ground if the two balls are to meet at the point A.

Answer: **i)** $v = 8.86\ m/s$ $h = 1.00\ m$ **ii)** $t = 1.36\ s$ **iii)** $y = 9.00\ m$



i) $v_y^2 = v_{oy}^2 \pm 2a_y\Delta y$

$\left(\frac{v}{2}\right)^2 = v^2 - 2gH$

$v = \sqrt{8gH/3} = \sqrt{8(9.81)(3)/3} = 8.86$ m/s

$0 = \left(\frac{v}{2}\right)^2 - 2gh$

$h = \frac{v^2}{8g} = \frac{8.86^2}{8(9.81)} = 1.00\,m$

ii) $\Delta y = v_{oy}t \pm \frac{1}{2}a_y t^2$

$-H = \frac{v}{2}t - \frac{1}{2}gt^2$

$4.91t^2 - \frac{8.86}{2}t - 3 = 0 \rightarrow t = \frac{4.43 + \sqrt{4.43^2 + 4(4.91)(3)}}{9.81}$

$t = 1.36\,s$

iii) $y = \frac{3v}{2}t - \frac{1}{2}gt^2$

$= \frac{3}{2}(8.86)(1.36) - \frac{1}{2}(9.81)(1.36)^2$

$= 9.00\,m$

10. (Motion Along a Straight Line) A juggler performs in a room whose ceiling is 3.00 m above the level of his hands. He throws a ball upward so that it just reaches the ceiling.

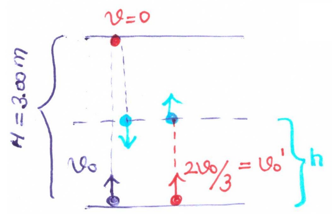
i What is the initial velocity of the ball?

ii What is the time required for the ball to reach the ceiling? At the instant when the first ball is at ceiling, the juggler throws a second ball upward with two-thirds the initial velocity of the first.

iii How long after the second ball is thrown did the two balls pass each other?

iv At what distance above the juggler's hand do they pass each other?

Answer: i) $v_o = 7.67\ m/s$ **ii)** $t = 0.782\ s$ **iii)** $t = 0.587\ s$ **iv)** $h = 1.31\ m$



i) $v_y^2 = v_{oy}^2 \pm 2a_y \Delta y$

$0 = v_o^2 - 2gH$

$v_o = \sqrt{2(9.81)3} = 7.67\ m/s.$

ii) $v_y = v_{oy} \pm a_y t$

$0 = 7.67 - (9.8)t \rightarrow t = 0.782\ s.$

iii) $\Delta y = v_{oy} \cdot t \pm \frac{1}{2} a_y t^2$

$H - h = 0 + \frac{1}{2} g (t')^2$ (for the first ball)

$3 - h = \frac{1}{2}(9.81)(t')^2$

$h = v_o' t' - \frac{1}{2} g (t')^2$

$h = \frac{2}{3}(7.67)t' - \frac{1}{2}(9.81)(t')^2$

$3 - 5.11 t' + 4.91 (t')^2 = 4.91 (t')^2 \rightarrow t' = 0.587\ s.$

$h = 1.31\ m$

11. (Vectors) Vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ lie in an xy-plane. $\overrightarrow{A}$ has magnitude 8.0 and an angle 130°; B has components $B_x = -7.72$ and $B_y = -9.20$.

  i What are $5\overrightarrow{A} \cdot \overrightarrow{B}$ and $4\overrightarrow{A} \times 3\overrightarrow{B}$ in unit vector notation?

  ii What is $(3\hat{\imath} + 5\hat{\jmath}) \times (4\overrightarrow{A} \times 3\overrightarrow{B})$? Find magnitude and angle of resultant vector.

Answer: i)$5\overrightarrow{A} \cdot \overrightarrow{B} = -83.58$**,** $4\overrightarrow{A} \times 3\overrightarrow{B} = 1135.4\ \hat{k}$
ii) $|(3\hat{\imath} + 5\hat{\jmath}) \times (4\overrightarrow{A} \times 3\overrightarrow{B})| = 6620.5$**,** $\theta = -31^\circ\ OR\ = 329^\circ$

$\vec{A}$ & $\vec{B}$ lie in xy-plane ⇒ only x & y components

$|\vec{A}| = 8$ with angle 130° & $B_x = -7.72$, $B_y = -9.20$

$\Rightarrow \vec{A} = |\vec{A}|\cos 130\,\hat{\imath} + |\vec{A}|\sin 130\,\hat{\jmath} = -5.14\,\hat{\imath} + 6.13\,\hat{\jmath}$

$\vec{B} = -7.72\,\hat{\imath} + 9.20(-\hat{\jmath})$

i) $5\vec{A}\cdot\vec{B} = 5(-5.14\hat{\imath} + 6.13\hat{\jmath})\cdot(-7.72\hat{\imath} - 9.20\hat{\jmath})$

$= 5[(-5.14)\times(-7.72) + (6.13\times(-9.20))] = -83.58$

$$4\vec{A}\times 3\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -20.56 & 24.52 & 0 \\ -23.16 & -27.6 & 0 \end{vmatrix} = (24.52\times 0 - 0(-27.6))\,\hat{\imath} + (0(-23.16) - (-20.56)0)\,\hat{\jmath} + ((-20.56)(-27.6) - (24.52)(-23.16))\,\hat{k}$$

$= 1135.4\,\hat{k}$

ii) $(3\hat{\imath} + 5\hat{\jmath})\times(1135.4\hat{k}) = (3\times 1135.4)(\hat{\imath}\times\hat{k}) + (5\times 1135.4)(\hat{\jmath}\times\hat{k})$

$\hat{\imath}\times\hat{k} = -\hat{\jmath}$, $\hat{\jmath}\times\hat{k} = \hat{\imath}$ ($\hat{\imath} \to \hat{\jmath} \to \hat{k}$)

$= 5677\,\hat{\imath} + 3460.2(-\hat{\jmath})$

magnitude: $\sqrt{(5677)^2 + (-3460.2)^2} = 6620.5$

angle: $\theta = \tan^{-1}\dfrac{-3460.2}{5677} = -31^\circ$ or 329°

12. (Vectors) Given two vectors, $\vec{A} = 5i - 6.5j$ and $\vec{B} = -3.5i + 7j$. A third vector $\vec{C}$ lies in the xy-plane. Vector $\vec{C}$ is perpendicular to vector $\vec{A}$, and the scalar product of $\vec{C}$ with $\vec{B}$ is 15.0. From this information, find the components of vector $\vec{C}$.
Answer: $C_x = 8.0$ and $C_y = 6.1$

$\vec{A}$ and $\vec{C}$ are perpendicular, so $\vec{A} \cdot \vec{C} = 0$

$$A_x C_x + A_y C_y = 0$$

$$5.0 C_x - 6.5 C_y = 0 \qquad (1)$$

$\vec{B} \cdot \vec{C} = 15.0$, so $-3.5 C_x + 7.0 C_y = 15.0 \qquad (2)$

We have two equations in two unknowns C_x any C_y.
Solving gives $C_x = 8.0$ and $C_y = 6.1$

13. (Motion in Two and Three Dimensions) The position of an object moving along an x axis is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t is in seconds.

   i What is the object displacement between $t = 3$ s and $t = 4$ s?

   ii What is its average velocity for the time interval t= 1 s and $t = 3$ s?

   iii Is there ever a time when the velocity is zero ?

   iv What is the its instantaneous acceleration at $t = 2$ s?

   Answer: i) 12 m ii) 0 m/s iii) 0.45 s, 2.21 s iv) 4 m/s^2

i) $\Delta\vec{x} = \vec{x}(t=4s) - \vec{x}(t=3s)$

$x(t=4s) = 3.4 - 4.4^2 + 4^3 = 12m$

$x(t=3s) = 3.3 - 4.3^2 + 3^3 = 0m$

(2) $\Delta x = 12m - 0m = 12m$

ii) $\vec{V}_{avg} = \frac{\Delta x}{\Delta t} = \frac{\vec{x}(t=3s) - \vec{x}(t=1s)}{3-1}$

$x(t=3s) = 3.3 - 4.3^2 + 3^3 = 0m$

$x(t=1s) = 3.1 - 4.1^2 + 1^3 = 0m$

(2) so $V_{avg} = \frac{0m}{2s} = 0m/s$

iii) $V_{ins} = \frac{dx}{dt} = 3 - 8t + 3t^2$ (1)

$V_{ins} = 0 \Rightarrow 3t^2 - 8t + 3 = 0$

$t_{1,2} = \frac{8 \mp \sqrt{28}}{2.3}$

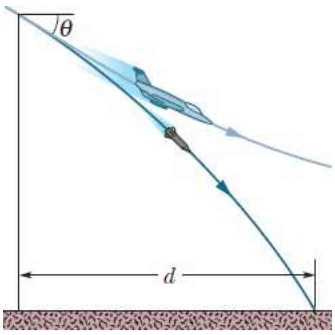
$t_1 = \frac{8 + 2\sqrt{7}}{6} = 1.77s$ (1)

$t_2 = \frac{8 - 2\sqrt{7}}{6} = 0.89s$ (1)

iii) $a_{ins} = \frac{dv}{dt} = -8 + 6t$ (1)

(2) $a_{ins}(t=2s) = -8 + 6.2 = 4m/s^2$

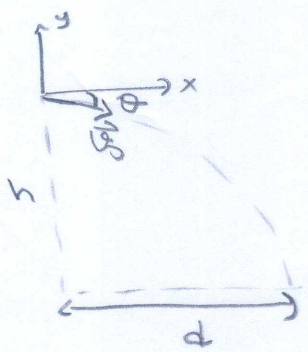
14. (Motion in Two and Three Dimensions) A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0°$ below the horizontal when the pilot releases a radar decoy (see Figure below). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700\ m$.



i How long is the decoy in the air?

ii How high was the release point?

Answer: i) 10 s ii) 893 m



a) $v_0 = 290\ km/h = \frac{290\,(1000)\,m}{(60)(60)\,s} = 80.6\ \frac{m}{s}$ (2)

$d = v_{0x} \cdot t$ (2)

$d = v_0 \cos\theta \cdot t$ (2)

$700 = (80.6)\cos 30 \cdot t \Rightarrow t = 10\ s$ (1) (1)

(2) 0.866

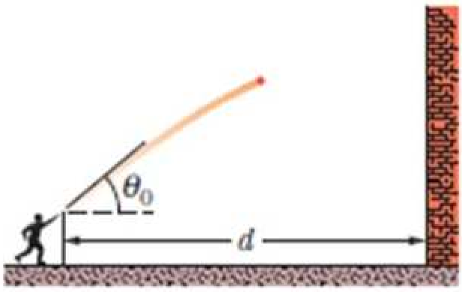
b) $-h = -v_0 \sin\theta \cdot t - \frac{1}{2} g t^2$ (2)

$h = (80.6)\sin 30\,(10) + \frac{1}{2}(9.8)(10)^2$ (6)

(2)

$= 893\ m$ (1) (1)

15. (Motion in Two and Three Dimensions) You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (as given in figure below). The wall has a distance $d = 22.0\ m$ from the release point of the ball.



i How far above the release point does the ball hit the wall?

ii What are the horizontal and vertical components of its velocity as it hits the wall?

iii When it hits, has it passed the highest point on its trajectory?

Answer: i) 12 m **ii)** $v_x = 19.2\ m/s,\ v_y = 4.8\ m/s$ **iii) NO**

i) Horizontal

$d = V_{0x}.t \Rightarrow d = V_0.\cos\theta_0.t \Rightarrow t = \frac{d}{V_0.\cos\theta_0} = \frac{22m}{(25m/s).\cos 40} = \boxed{1.15s}$ (2) (2)

Vertical ;

$y - y_0 = V_{0y}.t - \frac{1}{2}gt^2 \Rightarrow y = V_0.\sin\theta_0.t - \frac{1}{2}gt^2$ (2)

$y = (25m/s)(\sin 40).(1.15) - (4.9m/s^2).(1.15s)^2$

$= 18.48m - 6.48 = \boxed{12m}$ (2)

ii) $V_x = V_{0x} = V_0.\cos\theta_0 = \boxed{19.5m/s}$ (2) (1)

$V_y = V_{0y} - gt = V_0.\sin\theta_0 - g(t) = (25m/s)\sin 40 - (9.8m/s^2).(1.15s)$ (2)

$= 16.07 - 11.27 = \boxed{4.8m/s}$ (2)

iii) when the ball hits the ground since $V_y = 4.8m/s > 0$ (3)

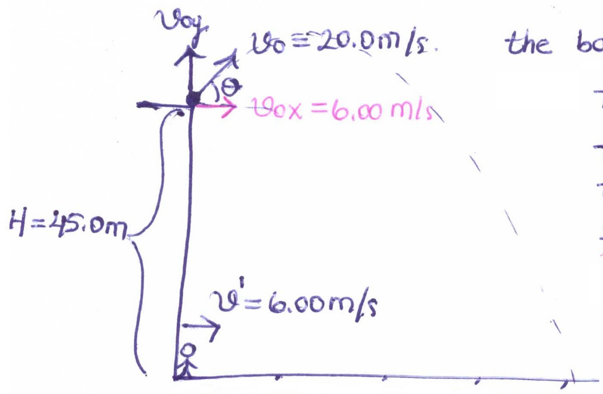
then it hasn't passed the highest point on its trajectory !! (2)

16. (Motion in Two and Three Dimensions) A 2.7 kg ball is thrown upward with an initial speed of 20.0 m/s from the edge of a 45.0 m high cliff. At the instant the ball is thrown, a woman starts running away from the base of the cliff with a constant speed of 6.00 m/s. The woman runs in a straight line on level ground. Ignore air resistance on the ball.

i At what angle above the horizontal should the ball be thrown so that the runner will catch it just before it hits the ground?

ii How far does she run before she catches the ball?

Answer: i) $theta_0 = 72.5°$ **ii)** 33.3 m



the ball moves in projectile motion.

The woman and the ball travel for the same time and must travel the same horizontal distance so for the ball $v_{ox} = 6.00 m/s$.

i. $v_{ox} = v_0 \cos\theta$

$\theta = \cos^{-1}\left(\frac{v_{ox}}{v_0}\right)$

$\theta = \cos^{-1}\left(\frac{6.00 m/s}{20.0 m/s}\right) = 72.5°$

ii. Choose upward as the positive y-direction

$\Delta y = v_{oy} t + \frac{1}{2} a_y t^2$

$-H = v_0 \sin\theta \, t - \frac{1}{2} g t^2$

$-(45.0 m) = (20.0 m/s) \sin(72.5°)\, t - \frac{1}{2}(9.81 m/s^2) t^2$

$t = 5.54 s$

$\Delta x = v_{ox} t$

$\Delta x = (20.0 m/s) \cos(72.5°)(5.54 s) = 33.3 m$

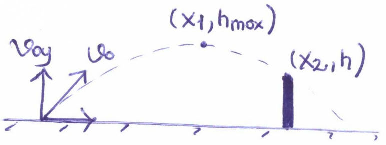
17. (Motion in Two and Three Dimensions) A ball is hit at ground level. The ball reaches its maximum height above ground level 3.0 s after being hit. Then 2.5 s after reaching its maximum height, the ball barely clears a fence that is 97.5 m from where it was hit.

i What maximum height above ground level is reached by the ball?

ii How high is the fence?

iii How far beyond the fence does the ball strike the ground?

Answer: i) 44.1 m **ii)** 13.4 m **iii)** 8.86 m



The trajectory of the baseball is shown in the figure. At $t_1 = 3.0\,s$, the ball reaches the maximum height h_{max}, and at $t_2 = t_1 + 3.0s = 5.5\,s$, it barely clears a fence at $x_2 = 97.5\,m$.

Choose downward as the positive y-direction

i) $\Delta y = v_{oy} \cdot t + \frac{1}{2} a_y t^2$

$h_{max} = 0 + \frac{1}{2} g t_1^2$

$h_{max} = \frac{1}{2}(9.8\,m/s^2)(3.0s)^2 = 44.1\,m$

ii) $(h_{max} - h) = 0 + \frac{1}{2} g (t_2 - t_1)^2$

$44.1\,m - h = \frac{1}{2}(9.8\,m/s^2)(2.5)^2 \rightarrow h = 13.48\,m \approx 13\,m$

iii) $x = v_{ox}\, t$

$97.5\,m = v_{ox}(5.5s) \rightarrow v_{ox} = 17.7\,m/s$

The total flight time of the ball is $T = 2t_1 = 2(3.0s) = 6.0s$

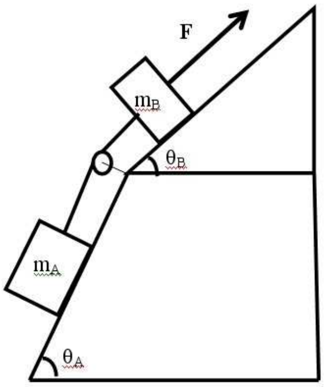
Thus, the range of baseball is

$R = v_{ox} T = (17.7\,m/s)(6.0\,s) = 106.4\,m$

which means that the ball travels an additional distance

$\Delta x = R - x_2 = 106.4\,m - 97.5\,m = 8.86\,m \approx 8.9\,m$

18. (Force and Motion - I) Consider the system shown in figure with $m_A = 9.5\ kg$ and $m_B = 11.5\ kg$. The angles $\theta_A = 59°$ and $\theta_B = 32°$.



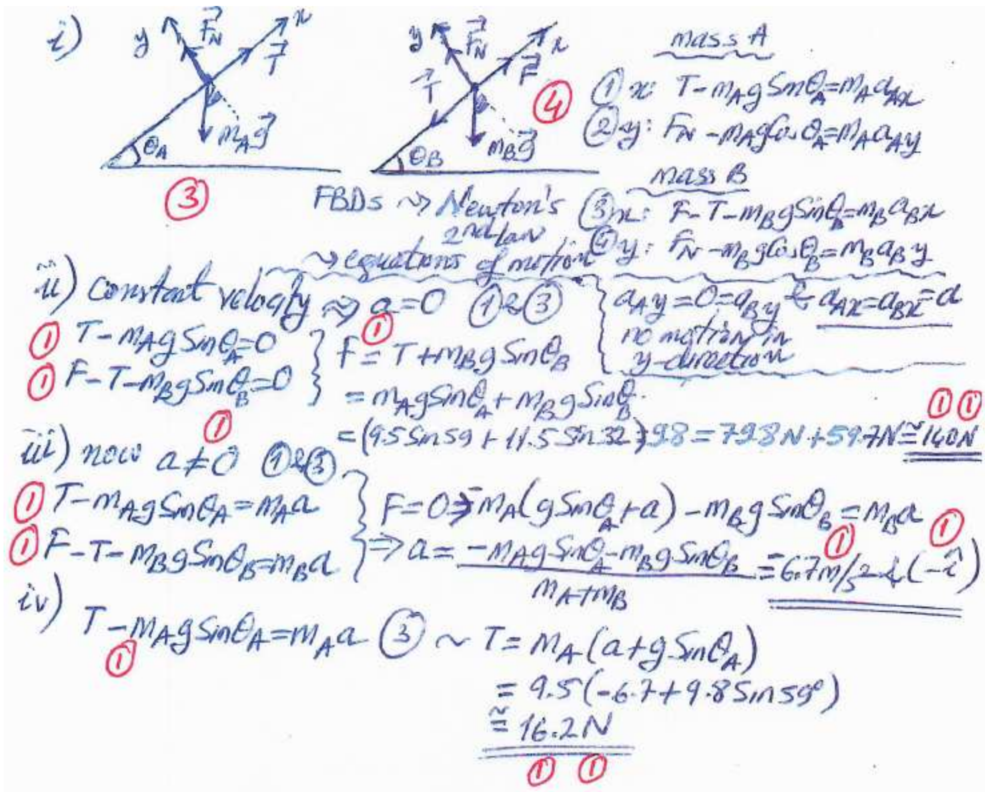
i Draw the free body diagrams for block A and block B.

ii In the absence of friction, what force F would be required to pull the masses at a constant velocity up?

iii The force F now is removed . What is the magnitude and direction of acceleration of the two blocks?

iv In the absence of F, what is the tension in the string?

Answer: ii) 140 N iii) $\vec{a} = -6.7\ m/s^2\hat{i}$ **iv) 17 N**


i) FBDs → Newton's 2nd law → equations of motion

mass A
(1) x: $T - m_A g \sin\theta_A = m_A a_{Ax}$
(2) y: $F_N - m_A g \cos\theta_A = m_A a_{Ay}$

mass B
(3) x: $F - T - m_B g \sin\theta_B = m_B a_{Bx}$
(4) y: $F_N - m_B g \cos\theta_B = m_B a_{By}$

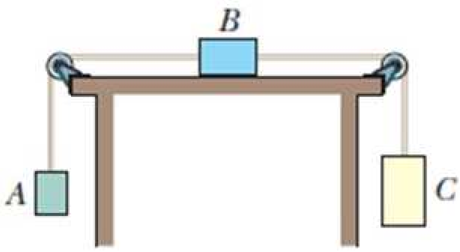
$a_{Ay} = 0 = a_{By}$ & $a_{Ax} = a_{Bx} = a$ no motion in y-direction

ii) constant velocity ⇒ $a = 0$ (1)&(3)
$T - m_A g \sin\theta_A = 0$
$F - T - m_B g \sin\theta_B = 0$
$F = T + m_B g \sin\theta_B = m_A g \sin\theta_A + m_B g \sin\theta_B$
$= (9.5 \sin 59 + 11.5 \sin 32)\, 9.8 = 79.8\,N + 59.7\,N \cong 140\,N$

iii) now $a \neq 0$ (1)&(3)
$T - m_A g \sin\theta_A = m_A a$
$F - T - m_B g \sin\theta_B = m_B a$
$F = 0 \Rightarrow m_A(g \sin\theta_A + a) - m_B g \sin\theta_B = m_B a$
$\Rightarrow a = \dfrac{-m_A g \sin\theta_A - m_B g \sin\theta_B}{m_A + m_B} = 6.7\,m/s^2$ & $(-\hat{i})$

iv) $T - m_A g \sin\theta_A = m_A a$ (3) ~ $T = m_A(a + g \sin\theta_A)$
$= 9.5(-6.7 + 9.8 \sin 59°)$
$\cong 16.2\,N$

19. (Force and Motion - I) Figure shows three blocks attached by cords that loop over **frictionless** table. The masses are $m_A = 6\ kg$, $m_B = 8\ kg$ and $m_C = 10\ kg$.

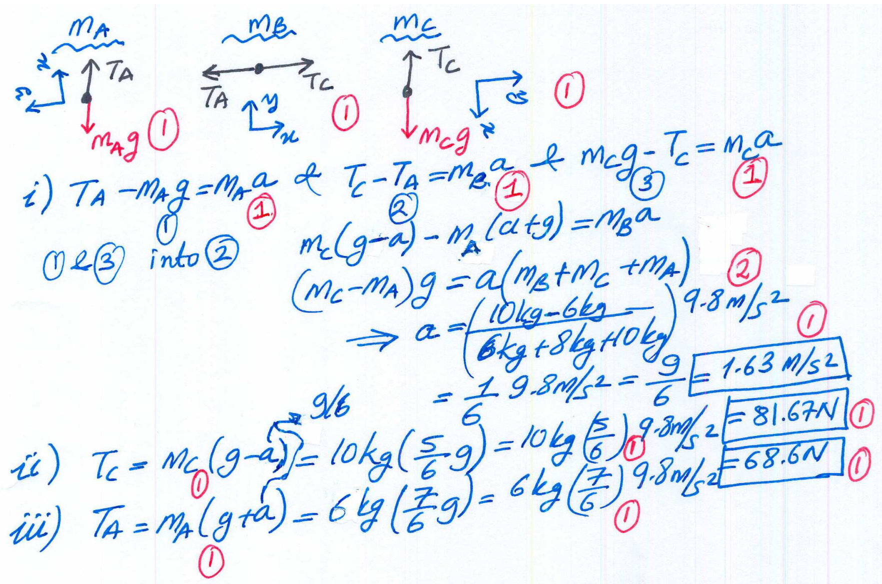


i What is the acceleration of the system?

ii When the block are released, what is the tension in the cord at the right?

iii When the block are released, what is the tension in the cord at the left?

Answer: i) $a = 1.63\ m/s^2$ **ii)** $T_C = 81.67\ N$ **iii)** $T_A = 68.6\ N$



m_A: T_A up, $m_A g$ down; m_B: T_A left, T_C right; m_C: T_C up, $m_C g$ down

i) $T_A - m_A g = m_A a$ (1) & $T_C - T_A = m_B a$ (2) & $m_C g - T_C = m_C a$ (3)

(1) & (3) into (2): $m_C(g-a) - m_A(a+g) = m_B a$

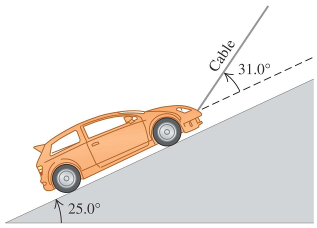
$(m_C - m_A) g = a(m_B + m_C + m_A)$

$\Rightarrow a = \left(\frac{10kg - 6kg}{6kg + 8kg + 10kg}\right) 9.8 m/s^2 = \frac{1}{6} 9.8 m/s^2 = \frac{g}{6} = 1.63\ m/s^2$

ii) $T_C = m_C(g - a) = 10kg\left(\frac{5}{6}g\right) = 10kg\left(\frac{5}{6}\right) 9.8 m/s^2 = 81.67 N$

iii) $T_A = m_A(g + a) = 6kg\left(\frac{7}{6}g\right) = 6kg\left(\frac{7}{6}\right) 9.8 m/s^2 = 68.6 N$

20. (Force and Motion - I) A 1130 kg car is held in place by a light cable on a frictionless ramp shown in the figure. The cable makes an angle of 31.0° above the surface of the ramp, and the ramp itself rises at 25.0° above the horizontal.

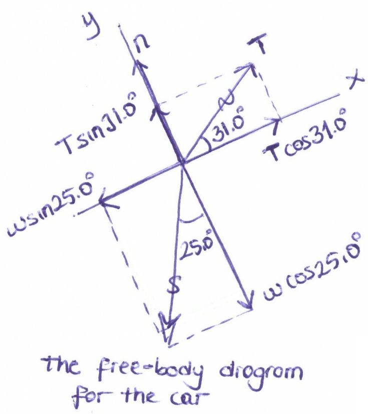


i Draw a free-body diagram for the car.

ii Find the tension in the cable.

iii How hard does the surface of the ramp push on the car?

Answer: ii) 5460 N **iii)** 7220 N



i) $\Sigma F_x = 0$

$T\cos 31.0^\circ - W\sin 25.0^\circ = 0$

$T = w \cdot \dfrac{\sin 25.0^\circ}{\cos 31.0^\circ}$

$T = (1130\ kg)(9.80\ m/s^2)\dfrac{\sin 25.0^\circ}{\cos 31.0^\circ}$

$T = 5460\ N$

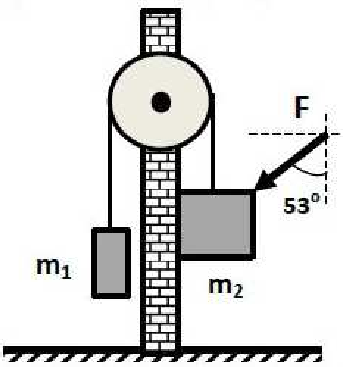
ii) $\Sigma F_y = 0$

$n + T\sin 31.0^\circ - w\cos 25.0^\circ = 0$

$n = w\cos 25.0^\circ - T\sin 31.0^\circ$

$n = (1130\ kg)(9.80\ m/s^2) - (5460\ N)\sin 31.0^\circ = 7220\ N$

21. (Force and Motion - II) Two blocks of masses $m_1 = 1\ kg$ and $m_2 = 2\ kg$ are suspended by a cord from a pulley which is attached to in front of a wall as shown in figure. A horizontal force of 8.3 N is applied to second block and the coefficients of static and kinetic frictions between the wall and the second block are 0.4 and 0.2. Cord and pulley are massless and pulley is frictionless.



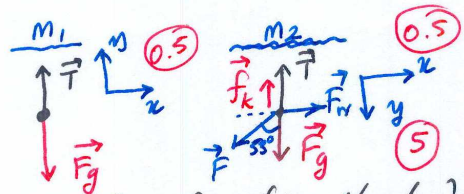
If the blocks, which are initially at rest, start moving when they are released;

i Draw the free body diagrams for both blocks.

ii Find the acceleration of the blocks.

Answer: ii) $a = 4.49\ m/s^2$

i) FBDs (2) (0.5) (0.5) (5)



ii) Equations of motion. Newton's 2nd Law (2)

m_1: y: $T - m_1 g = m_1 a \rightarrow T = m_1(g+a)$ (2)

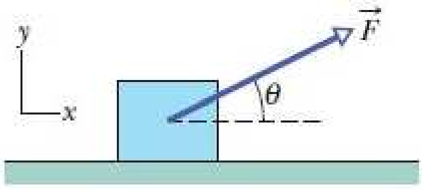
m_2: x: $F_N - F\sin 53^\circ = m_2 a_x = 0 \rightarrow F_N = F\sin 53$ (2) & $f_k = \mu_k F_N$ (1)

y: $m_2 g + F\cos 53^\circ - f_k - T = m_2 a \Rightarrow m_2 g + F\cos 53 - \mu_k F\sin 53 - m_1(g+a) = m_2 a$ (3)

$$\Rightarrow \frac{(m_2 - m_1)g + F(\cos 53 - \mu_k \sin 53)}{(m_1 + m_2)} = a \quad (2)$$

$$a = \frac{(2kg - 1kg)\, 9.8\, m/s^2 + 8.3N(\cos 53 - 0.2\sin 53)}{3kg} = 4.49\ m/s^2$$ (1) (1)

22. (Force and Motion - II) Figure shows an initially stationary block of mass m on a floor. A force of magnitude $0.500mg$ (where m is the mass and g is the gravitational acceleration) is then applied at upward angle $\theta = 20°$.



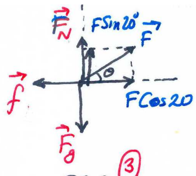
What is the magnitude of the acceleration of the block across the floor if the friction coefficients are

i $\mu_s = 0.600$ and $\mu_k = 0.500$

ii $\mu_s = 0.400$ and $\mu_k = 0.300$?

Answer: i) a=0 since no motion ii) 2.17 m/s^2

ii $\mu_s = 0.400$ and $\mu_k = 0.300$



Equations of motion

x: $|\vec{F}|Cos20 - f_k = ma_x$ ⓘ ①

y: $|\vec{F}|Sin20 + F_N - mg = ma_y = 0$ ⓘⓘ ①

from ⓘⓘ $F_N = mg - |\vec{F}|Sin20$ & $f_k = \mu_k F_N$

$\Rightarrow$ ⓘ $|\vec{F}|Cos20 - \mu_k(mg - |\vec{F}|Sin20) = ma$

$\Rightarrow a = \frac{|\vec{F}|}{m}(Cos20 + \mu_k Sin20) - \mu_k g$

i) $\mu_s = 0.600$ & $\mu_k = 0.500$ } $Sin20 = 0.342$ & $Cos20 = 0.940$

$f_{s,max} = \mu_s F_N = \mu_s(mg - |\vec{F}|Sin20) = 0.600(mg - 0.500mg\,Sin20) = 0.497mg$ ① ①

$|\vec{F}|Cos20 - f_{s,max} = ma$ } $0.500mg\,0.940 - 0.497mg = ma$ ②

① $(0.470 - 0.497)mg = ma$

negative $\Rightarrow$ no motion $a = 0$ ②

ii) $\mu_s = 0.400$ & $\mu_k = 0.300$

$f_{s,max} = 0.400(mg - 0.500mg\,Sin20) = 0.332mg$ ②

① $(0.470 - 0.332)mg = ma \rightarrow$ positive $\Rightarrow$ motion ②

if motion exists, we use μ_k

$|\vec{F}|Cos20 - f_k = ma$ } $|\vec{F}|Cos20 - \mu_k(mg - |\vec{F}|Sin20) = ma$ ②

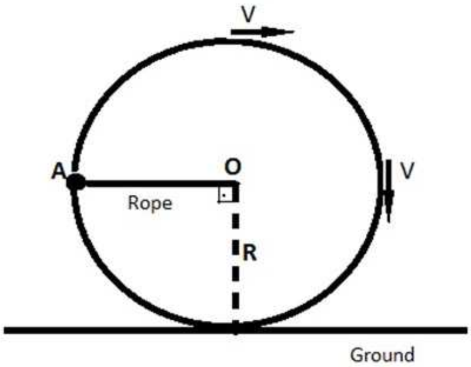
$0.500mg\,0.940 - 0.300(mg - 0.500mg\,0.342) = ma$

$mg(0.500\times0.940 - 0.300 + 0.300\times0.500\times0.342) = ma$

$\Rightarrow a = 0.221\times(9.8\,m/s^2)$

$\boxed{a = 2.17\,m/s^2}$ ①

23. (Force and Motion - II) A stone which tied to a rope is rotated in a circular path with a constant speed V in clockwise direction as given in figure below. At point A, the rope is breaking up and the stone is released. The radius R of the circular path is 0.50 m.



If the period of the motion is given as $T = \pi/20\ s$ calculate;

i Magnitude of the centripetal acceleration, a.

ii Time required that the stone hit the ground, t.

iii Maximum height of the stone with respect to ground, y_{max}.

Hint: Choose the point A as your origin of the your reference frame.

Answer: i) 800 m/s^2 ii) 4.11 s iii) 20.91 m

i) $V = \frac{2\pi R}{T} = \frac{2\pi(0.5m)}{(\frac{\pi}{20}s)} = 20m/s$ (2)

$a = \frac{V^2}{R} = \frac{(20m/s)^2}{(0.5m)} = 800m/s^2$ (3)

iii) $y_{max} = h + R$

$\boxed{h = \frac{V_{0y}^2 - V_y^2}{2g}}$ (1) where $V_{0y} = V$, $V_y = 0$ (at max. height)

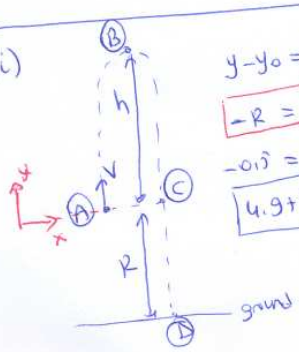
$h = \frac{(20m/s)^2}{2(9.8m/s^2)}$

$\boxed{h = 20.41m}$ (2)

$y_{max} = h + R = 20.41m + 0.5m$

$\boxed{y_{max} = 20.91m}$ (2)

ii)



$y - y_0 = V_{0y}.t - \frac{1}{2}gt^2$

$\boxed{-R = V.t - \frac{1}{2}gt^2}$ (1)

$-0.5 = 20.t - 4.9t^2$

$\boxed{4.9t^2 - 20t - 0.5 = 0}$ (2)

$t_{1,2} = \frac{-20 \mp \sqrt{409.8}}{2.(4.9)}$

$\boxed{t = 4.11s}$ (2) take positive root.

OR

ii) you can find the time required to reach from point A to point B t_{AB}, then find the time required to reach from point B to point D, t_{BD}

then $t = t_{AB} + t_{BD}$

$t = 2.04s + 2.07s$

$\boxed{t = 4.11s}$

24. (Force and Motion - II) An astronaut is rotated in a horizontal centrifuge at a radius of 5.0 m.

i What is the astronaut's speed if the centripetal acceleration has a magnitude of $6.0g$? (g is the gravitational acceleration)

ii How many revolutions per minute are required to produce this acceleration?

iii What is the period of the motion?

Answer: i) 17.15 m/s **ii)** 32.7 rev **iii)** 1.83 s

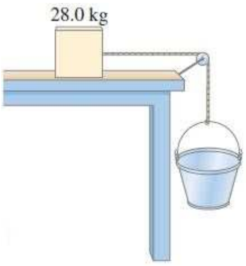
i) $a = \frac{v^2}{r}$ (2) $\Rightarrow v = \sqrt{ar} = \sqrt{6g \cdot r} = \sqrt{6 \cdot (9.8 m/s^2) \cdot 5m}$

$\boxed{v = 17.15 m/s}$ (3)

iii) $T = \frac{2\pi r}{v}$ (2) $= \frac{2\pi (5m)}{17.15 m/s} = \boxed{1.83 s}$ (3)

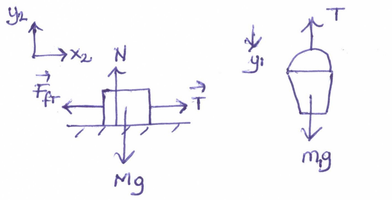
ii) $n = \frac{t}{T}$ (3) $= \frac{1 min}{1.83 s} = \frac{60 s}{1.83 s} = \boxed{32.7}$ (2)

25. (Force and Motion - II) A 28.0 kg block is connected to an empty 2.00 kg bucket by a cord running over a frictionless pulley as shown in the figure. The coefficient of static friction between the table and the block is 0.45 and the coefficient of kinetic friction between the table and the block is 0.32. Sand is gradually added to the bucket until the system just begins to move. **Answer: i)** 10.6 kg **ii)** 0.88 m/s^2



i Calculate the mass of sand added to the bucket.

ii Calculate the acceleration of the system.



Consider the free-body diagrams for both objects, initially stationary. As sand is added, the tension will increase, and the force of static friction on the block will increase until it reaches its maximum of $F_{fr} = \mu_s N$. Then the system will start to move.

i) When the static frictional force is at its maximum, but the objects are still stationary;

for the bucket: $\Sigma F_y = 0 \rightarrow m_1 g - T = 0 \rightarrow T = m_1 g$

for the block: $\Sigma F_y = 0 \rightarrow N - Mg = 0 \rightarrow N = Mg$

$\Sigma F_x = 0 \rightarrow T - F_{fr} = 0 \rightarrow T = F_{fr}\ ,\ F_{fr} = \mu_s N$

$m_1 g = \mu_s M g$

$m_1 = (0.45)(28.0\ kg)$

$m_1 = 12.6\ kg$

Thus $12.6\ kg - 2.00\ kg = 10.6\ kg \approx 11\ kg$ of sand was added.

ii) Now the objects will accelerate. $a_{y_1} = a_{x_2} = a$. The frictional force is now kinetic friction, given by $F_{fr} = \mu_k N = \mu_k M g$

for the bucket: $\Sigma F_y = m a_y \rightarrow m_1 g - T = m_1 a \rightarrow T = m_1 g - m_1 a$

for the block: $\Sigma F_x = m a_x \rightarrow T - F_{fr} = Ma \rightarrow T = F_{fr} + Ma$

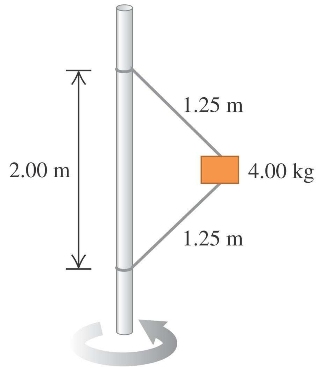
$m_1 g - m_1 a = F_{fr} + Ma$

$(12.6\ kg)(9.80\ m/s^2) - (12.6\ kg)\ a = (0.32)(28.0\ kg)(9.80\ m/s^2)$

$(12.6\ kg)(9.80\ m/s^2) - (12.6\ kg)a = (0.32)(28.0\ kg)(9.80\ m/s^2) + (28.0\ kg)\ a$

$a = 0.88\ m/s^2$

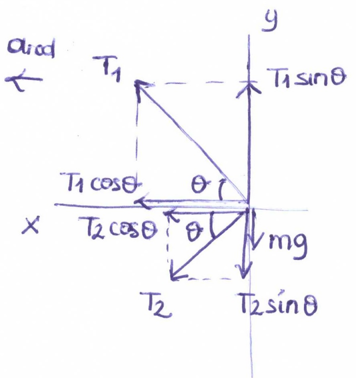
26. (Force and Motion - II) A 4.00 kg block is attached to a vertical rod by means of two strings. When the system rotates about the axis of the rod, the strings are extended as shown in the figure and the tension in the upper string is 80.0 N.



i What is the tension in the lower cord?

ii How many revolutions per minute does the system make?

Answer: i) 31.0 N **ii)** 44.9 rev/min



$$\sin\theta = \frac{1.00\,m}{1.25\,m} \rightarrow \theta = 53.1^\circ$$

i) $\Sigma F_y = 0$

$$T_1 \sin\theta - T_2 \sin\theta - mg = 0$$

$$(80.0\,N)\sin 53.1^\circ - T_2 \sin 53.1^\circ - (4.00\,kg)(9.81\,m/s^2)$$

$$T_2 = 31.0\,N$$

ii) $\Sigma F_x = ma_x$

$$T_1\cos\theta + T_2\cos\theta = m\frac{v^2}{r}, \quad r = (1.25\,m)\cos 53.1^\circ = 0.75\,m$$

$$(80.0\,N)\cos 53.1^\circ + (31.0\,N)\cos 53.1^\circ = (4.00\,kg)\frac{v^2}{0.75\,m}$$

$$v = 3.53\,m/s.$$

$$f = \frac{v}{2\pi r} = \frac{3.53\,m/s}{2\pi(0.75\,m)} = 0.749\,\frac{rev}{s}\cdot\frac{60\,s}{1\,min} = 44.9\,rev/min$$

İzmir Kâtip Çelebi University
Department of Engineering Sciences
Phy101 Physics I
Midterm Examination
April 07, 2019 10:30 – 12:30
Good Luck!

NAME-SURNAME:

SIGNATURE:

ID:

DEPARTMENT:

DURATION: 120 minutes

◇ **Answer all the questions.**
◇ **Write the solutions explicitly and clearly. Use the physical terminology.**
◇ **You are allowed to use Formulae Sheet.**
◇ **Calculator is allowed.**
◇ **You are not allowed to use any other electronic equipment in the exam.**

Question	Grade	Out of
1A		15
1B		15
2		20
3		20
4		20
5		20
TOTAL		110

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1. A) Estimate the number of breaths taken during an average life time. (Hints: YOU estimate; the typical life time, the average number of breaths that a person takes in 1 min. Use chain rule. Use scientific notation in your final result.)

Typical life span is 70 years.

2pt (estimation) ← 10 breathes per minute is the average number for all sitiations which contain exercising, angry, sleeping, serene & so forth.

Conversion by chain: 10 pt

The # of minutes per year: $1\,yr \times 400\,\frac{days}{yr} \times \frac{25\,h}{day} \times 60\,\frac{min}{h} = 6\times10^{5}\,min$

(To multiply 400×25 is simpler than 365×24 !)

$(70\,yr)(6\times10^{5}\,min/yr) = 4\times10^{7}\,min$

At a rate of 10 breathes/min, an individual would take 4×10^{8} breathes in a lifetime.

3pt

B) The radius of a solid sphere is measured to be $(13.00 \pm 0.40)\ cm$, and its mass is measured to be $(3.70 \pm 0.04)\ kg$. Determine the density of the sphere in kilograms per cubic meter and the uncertainty in the density.

$R = (6.50 \pm 0.20)\,cm = (6.50 \pm 0.20) \times 10^{-2}\,m$

$m = (1.85 \pm 0.02)\,kg$ (3 sig figs)

$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi R^3}$ (1) Three steps

1st step: Raised to a power $C = A^n$ & $\Delta C = C\,|n|\,\frac{\Delta A}{A}$ (2)

$C = R^3 = (6.50 \times 10^{-2}\,m)^3 = 2.75 \times 10^{-4}\,m^3$

$\Delta C = R^3\,|3|\,\frac{\Delta R}{R} = (2.75 \times 10^{-4}\,m^3)\,|3|\,\frac{0.20 \times 10^{-2}\,m}{6.50 \times 10^{-2}\,m} = 2.54 \times 10^{-5}\,m^3$

$\Rightarrow (2.75 \times 10^{-4} \pm 2.54 \times 10^{-5})\,m^3$ (2)

2nd step: Multiplication with a scalar (2)

$\frac{4}{3}\pi\,(2.75 \times 10^{-4} \pm 2.54 \times 10^{-5})\,m^3 = (1.15 \times 10^{-3} \pm 1.06 \times 10^{-4})\,m^3$

3rd step: Multiplication/Division $C = \frac{A}{B}$ & $\Delta C = |C|\sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$ (2)

$\frac{(1.85 \pm 0.02)\,kg}{(1.15 \times 10^{-3} \pm 1.06 \times 10^{-4})\,m^3} \Rightarrow C = \frac{1.85\,kg}{1.15 \times 10^{-3}\,m^3} = 1.61 \times 10^{3}\,kg/m^3$

& $\Delta C = |1.61 \times 10^{3}\,kg/m^3|\sqrt{\left(\frac{0.02}{1.85}\right)^2 + \left(\frac{1.06 \times 10^{-4}}{1.15 \times 10^{-3}}\right)^2} = 0.149 \times 10^{3}\,kg/m^3$ (3)

$\Rightarrow \boxed{(1.61 \pm 0.15) \times 10^{3}\,kg/m^3} \rightsquigarrow (1.6 \pm 0.2) \times 10^{3}\,kg/m^3$ (1) (2)

2. A physics book is dropped from a bridge, falling 90 m to the valley below the bridge.

   i In how much time does it pass through the last 20% of its fall?

   ii What is its speed when it begins that last 20% of its fall?

   iii What is its speed when it reaches the valley beneath the bridge?

$y - y_0 = -\frac{1}{2} g t^2$ (3)

Thus the time for full fall is found to be $t = 4.29$ s.

$y - y_0 = -90$ m (2)

because upward is chosen as (+) y direction!

The first 80% of its free-fall distance is given by $-72 = -g\tau^2/2$, which requires $\tau = 3.83$ s. (2)

a) Thus, the final 20% of its fall takes

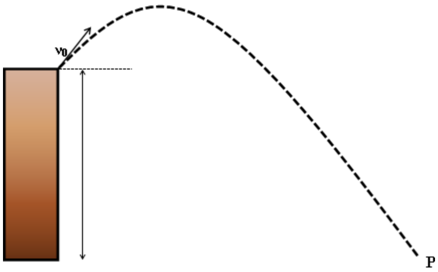
$t - \tau = 0.45$ s (3)

b) the speed $v = -g\tau$. (3)

$|v| = 38$ m/s approximately. (2)

c) Similarly, $v_{final} = -gt$ (3) $\Rightarrow |v_{final}| = 42$ m/s (2)

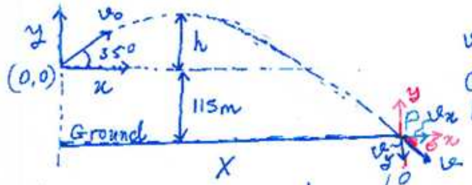
3. A projectile is shot from the edge cliff 120 m above ground level with an initial speed of 60 m/s at an angle of 30° with the horizontal.



i Determine the distance X of point P from the base of the vertical cliff.

ii What is the velocity v at point P in magnitude-angle notation and in unit-vector notation?

iii Find the maximum height reached by projectile **above ground**.



$v_0 = 65\ m/s$, $\theta = 35°$, $h_0 = 115\ m$

$v_{0x} = v_0 \cos\theta$, $v_{0y} = v_0 \sin\theta$, $\sin 35 = 0.57$, $\cos 35 = 0.82$

i) $x = x_0 + v_{0x} t$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$\Rightarrow x - x_0 = v_0 \cos\theta\, t$

$X = (65 m/s) \cos 35 (9.97 s)$

$\boxed{\cong 531 m}$

$y - y_0 = v_0 \sin\theta t - \frac{1}{2} g t^2$

$-115 m = (65 m/s) \sin 35° t - \frac{1}{2} 9.8 m/s^2 t^2$

$\Rightarrow 4.9t^2 - 37.3t - 115 = 0$

$t_{1,2} = \frac{-(-37.3) \pm \sqrt{(-37.3)^2 - 4(4.9)(-115)}}{2(4.9)}$

$t_1 = -2.36 s$ → not physical ← minus sign

$t_2 = 9.97 s = t$

ii) At point P

$v_x = v_{0x} = v_0 \cos\theta$

$v_y = v_{0y} - g t = v_0 \sin\theta - g t$

$v_x = (65 m/s) \cos 35°$

$v_y = (65 m/s) \sin 35° - (9.8 m/s^2)(9.97 s)$

$v_x = 53.2\ m/s$ & $v_y = -60.7\ m/s$

$\vec{v} = v_x \hat{i} + v_y \hat{j} = \boxed{(53.2 \hat{i} - 60.7 \hat{j}) m/s}$

→ $|\vec{v}| = \sqrt{(53.2 m/s)^2 + (-60.7 m/s)^2}$

$\boxed{= 80.7 m/s}$ $\theta = \tan^{-1} \frac{-60.7}{53.2}$

$\boxed{\cong -49°}$

iii) $h_{max} = h_0 + h$ → What is h?

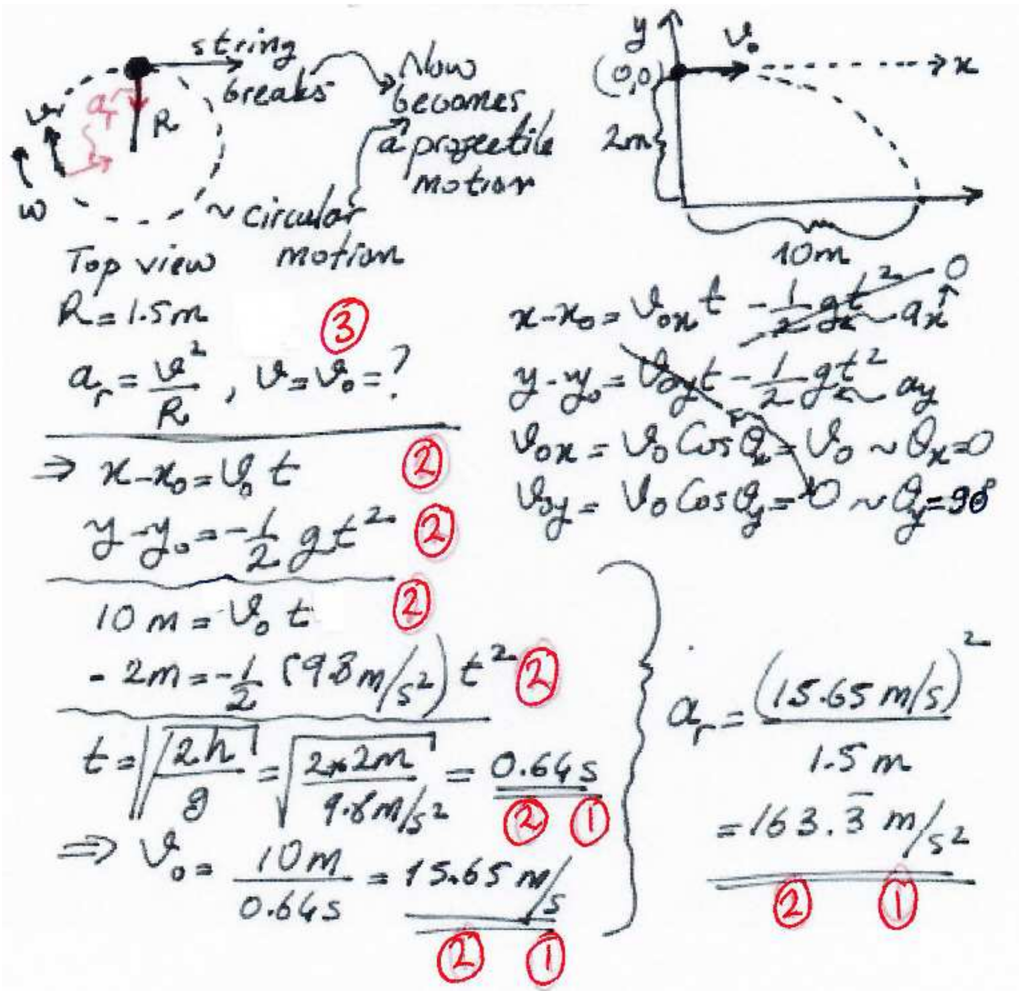
$v_y = v_{0y} - g t = 0$ at max height

$t_f = \frac{v_{0y}}{g}$ → $h = v_{0y} \frac{v_{0y}}{g} - \frac{1}{2} g \left(\frac{v_{0y}}{g}\right)^2$

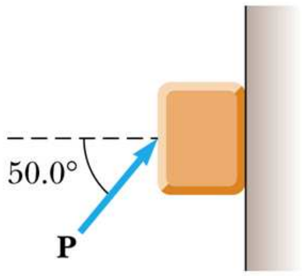
$h = \frac{1}{2} \frac{v_0^2 \sin^2\theta}{g} = \frac{1}{2} (65 m/s)^2 \frac{\sin^2 35°}{9.8 m/s^2}$

$\boxed{h \cong 71 m}$ ⇒ $h_{max} = 115 m + 71 m = \boxed{186 m}$

4. A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks, and the stone flies off horizontally and strikes the ground after traveling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone during the circular motion?



5. A block of mass 3.00 kg is pushed up against a wall by a force P that makes a 50.0° angle with the horizontal as shown in Figure below. The coefficient of static friction between the block and the wall is 0.250.



Determine minimum and maximum values for the magnitude of P that allow the block to remain stationary.