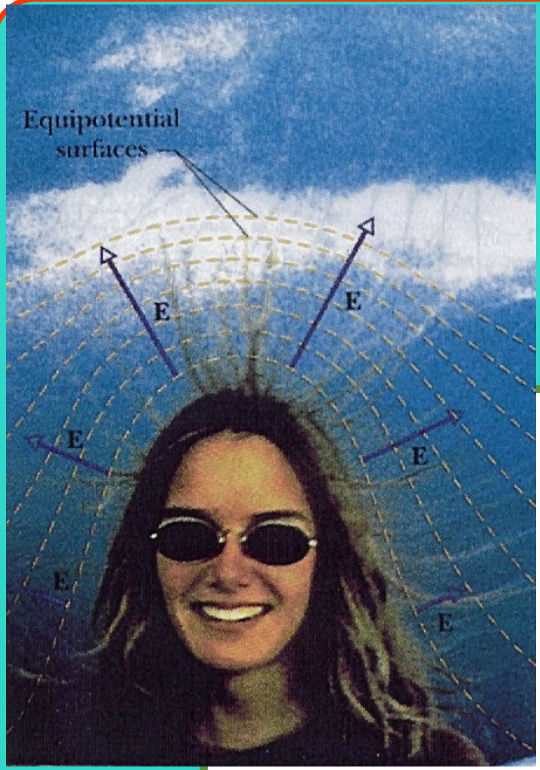
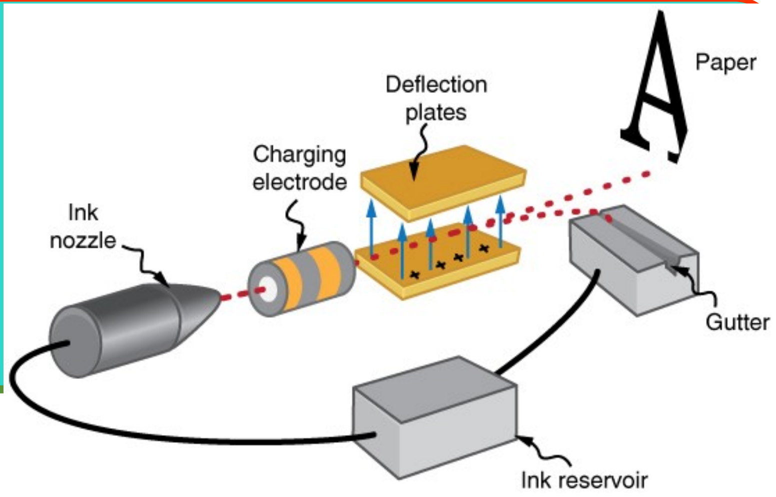
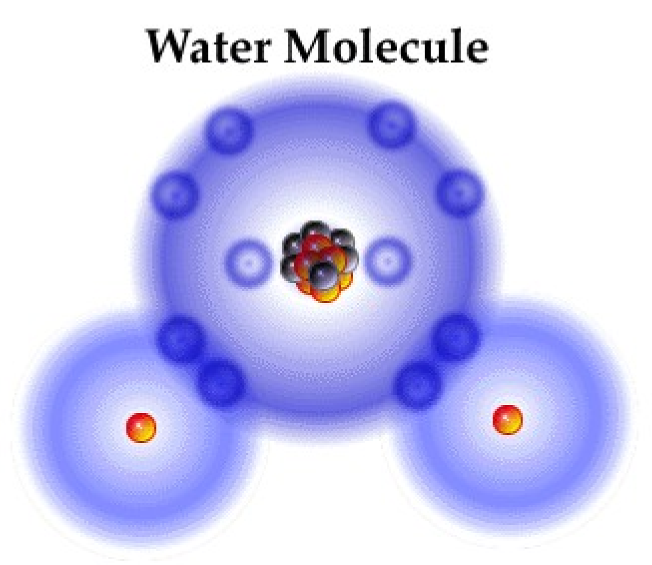




Chapter 22

Electric Fields



İZMİR KÂTİP ÇELEBİ ÜNİVERSİTESİ

22 ELECTRIC FIELDS 580

point charges (22-4, 22-5)

objects; 1D, 2D (22-6, 22-7)

How a charged particle behaves under electric field. (22-8, 22-9)

22-2 Electric Field

The particles do not touch:
How can particle 2 push on particle 1?
How can there be such an action at a distance?
How does particle 1 "know" of the presence of particle 2?

Table 22-1
Some Electric Fields

Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Within a hydrogen atom, at a radius of 5.29×10^{-11} m	5×10^{11}
Electric breakdown occurs in air	3×10^{6}
Near the charged drum of a photocopier	10^{5}
Near a charged comb	10^{3}
In the lower atmosphere	10^{2}
Inside the copper wire of household circuits	10^{-2}

- The explanation that we shall examine here is this: Particle 2 sets up an electric field at all points in the surrounding space, even if the space is a vacuum ($\rightarrow \varepsilon_0$).

distorts the E of q_2

- If we place particle 1 at any point in that space, particle 1 knows of the presence of particle 2 because it is affected by the electric field particle 2 has already set up at that point.
- Thus, particle 2 pushes on particle 1 not by touching it. Instead, particle 2 pushes by means of the electric field it has set up.

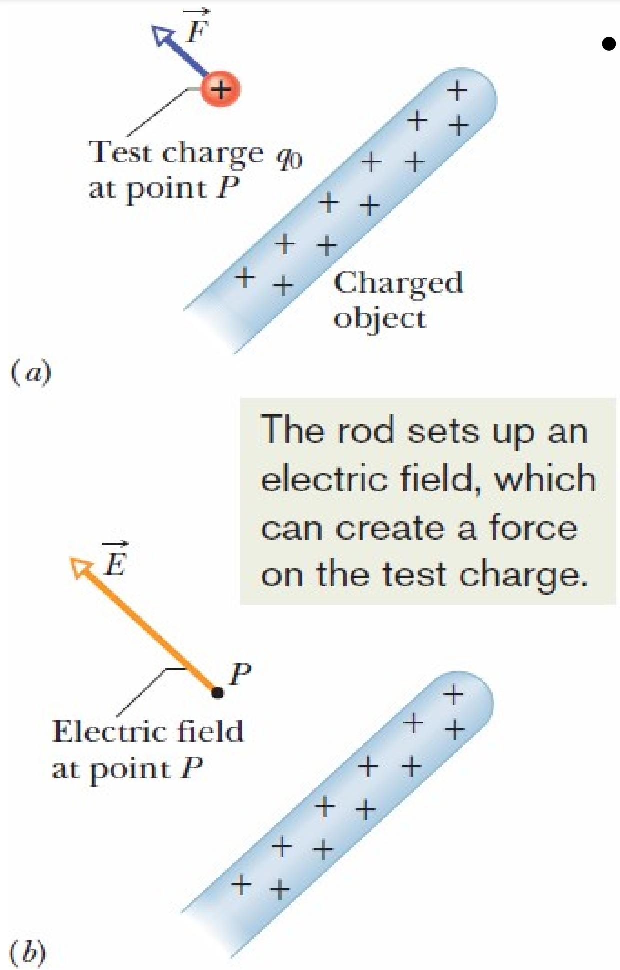


Fig. 22-1 (*a*) A positive test charge q_0 placed at point P near a charged object. An electrostatic force $\vec{F}$ acts on the test charge. (*b*) The electric field $\vec{E}$ at point P produced by the charged object.

- The electric field, E, is a *vector field*: it consists of a distribution of *vectors*, one for each point in the region around a charged object.

Vector Algebra

- We can define the electric field at some point near the charged object (Q), such as point *P* in figure, as follows:
  - A *positive* test charge q_0, placed at the point *P* will experience an electrostatic force, F.
  - The electric field at point *P* due to the charged object is defined as the electric field, E, at that point:

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{(electric field).}$$

The SI unit for the electric field is the Newton per coulomb (N/C).

$$|\vec{F}| = k\frac{|q_0||Q|}{r^2}$$

$$\vec{F} = q_0\vec{E}$$

- The electric field exists independently of the test charge: it existed both before and after the test charge was put there.
- Assumption in the defining procedure: the presence of the test charge does not affect the charge distribution on the charged object. → No disturbance on charge distribution.

Field lines: useful way to visualize electric field E

Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

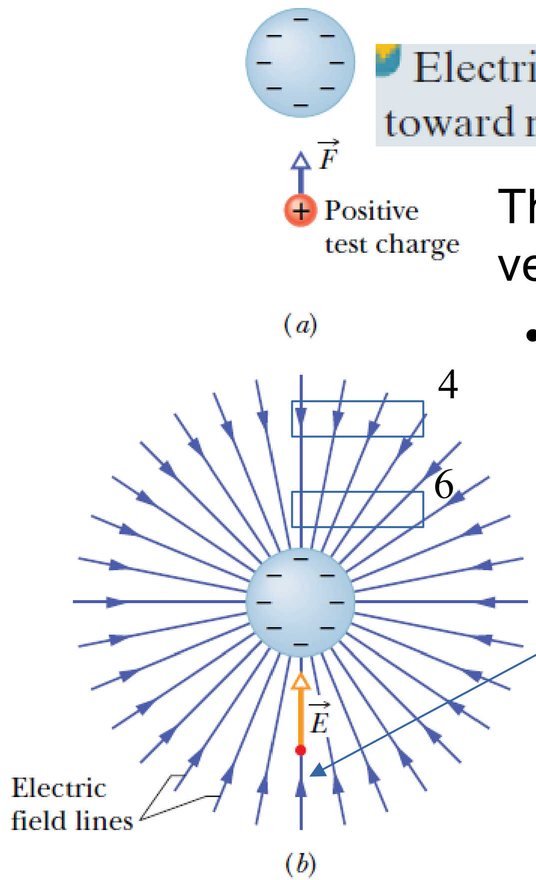


Fig. 22-2 (a) The electrostatic force $\vec{F}$ acting on a positive test charge near a sphere of uniform negative charge. (b) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend *toward* the negatively charged sphere. (They originate on distant positive charges.)

The relation between the electric field lines and electric field vectors:

- At any point, the direction of a straight field line or the direction of the tangent to a curved field line gives the direction of **E** at that point.
- The field lines are drawn so that the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the magnitude of **E**.
- Thus, **E** is large where field lines are close together and small where they are far apart.

$$F \& E \propto \frac{1}{r^2}$$

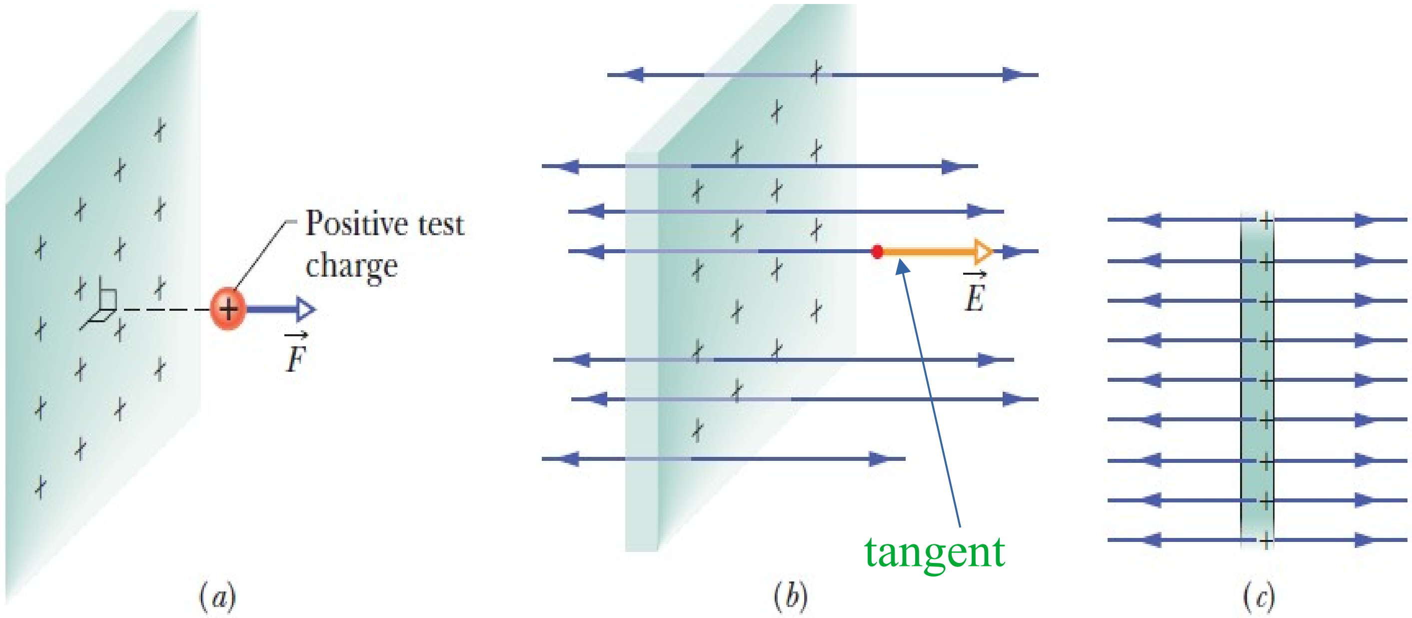


Fig. 22-3 (*a*) The electrostatic force $\vec{F}$ on a positive test charge near a very large, nonconducting sheet with uniformly distributed positive charge on one side. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sheet. The field lines extend *away from* the positively charged sheet. (*c*) Side view of (*b*).

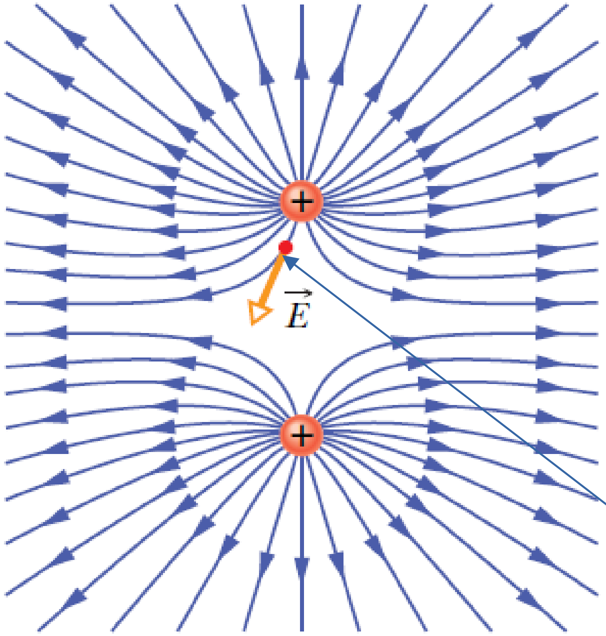
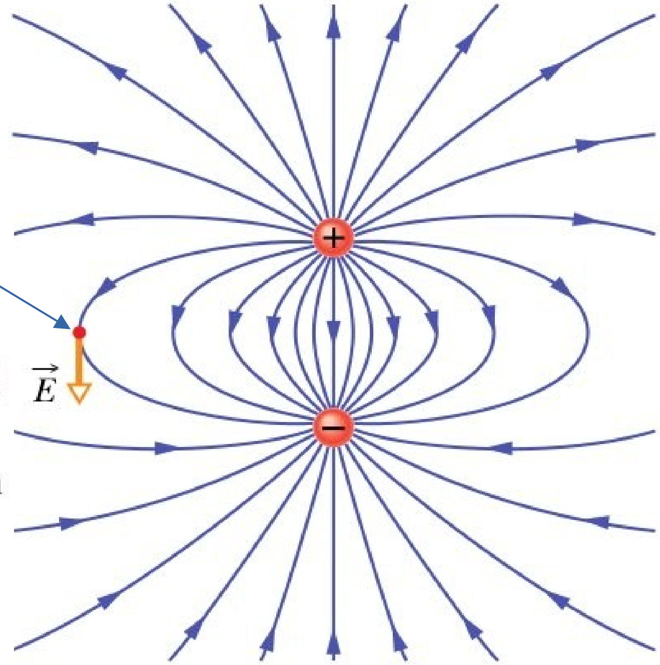


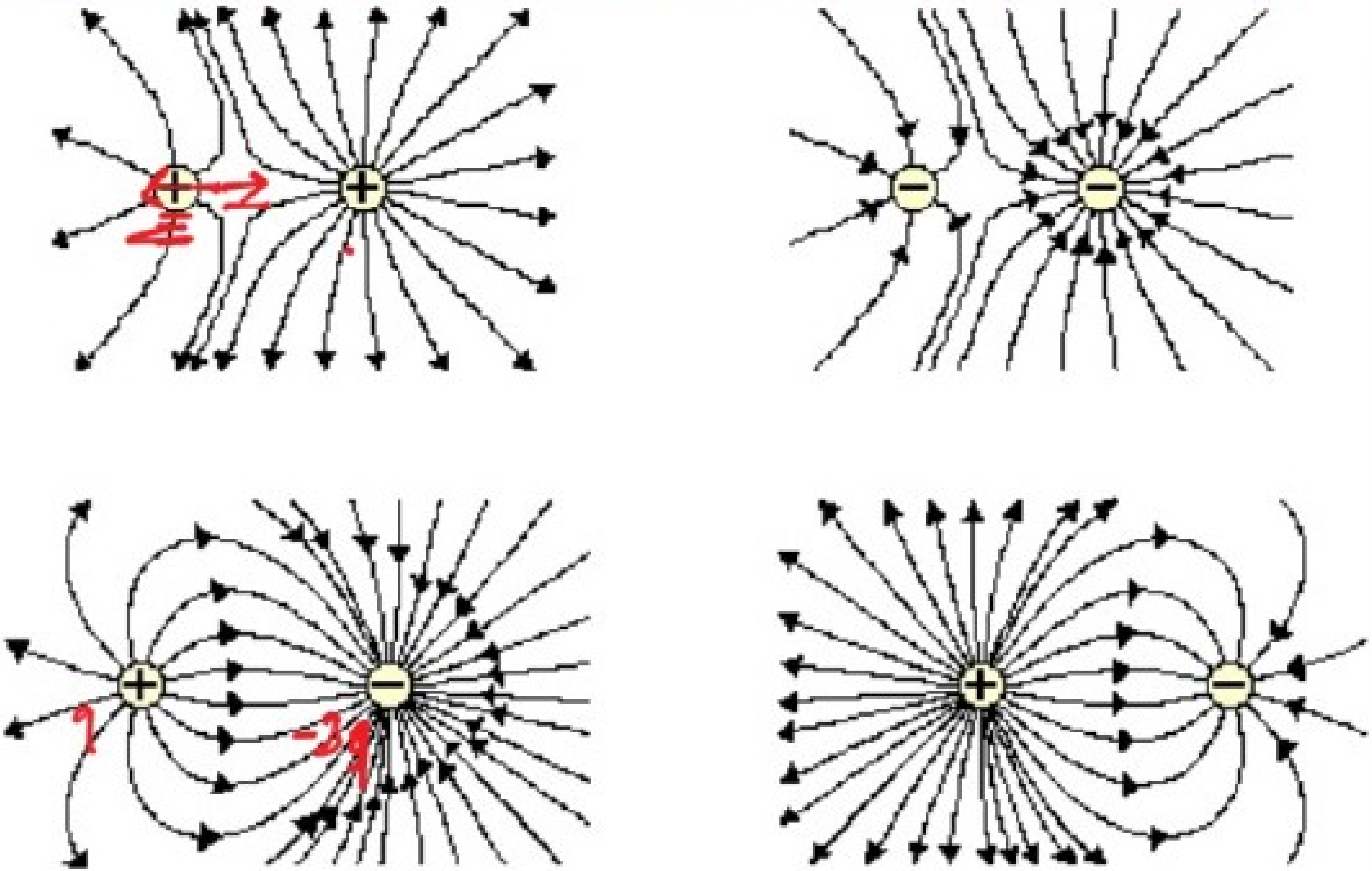
Fig. 22-4 Field lines for two equal positive point charges. The charges repel each other. (The lines terminate on distant negative charges.) To "see" the actual three-dimensional pattern of field lines, mentally rotate the pattern shown here about an axis passing through both charges in the plane of the page. The three-dimensional pattern and the electric field it represents are said to have *rotational symmetry* about that axis. The electric field vector at one point is shown; note that it is tangent to the field line through that point.

tangent

Fig. 22-5 Field lines for a positive point charge and a nearby negative point charge that are equal in magnitude. The charges attract each other. The pattern of field lines and the electric field it represents have rotational symmetry about an axis passing through both charges in the plane of the page. The electric field vector at one point is shown; the vector is tangent to the field line through the point.



Electric Field Line Patterns for Objects with Unequal Amounts of Charge

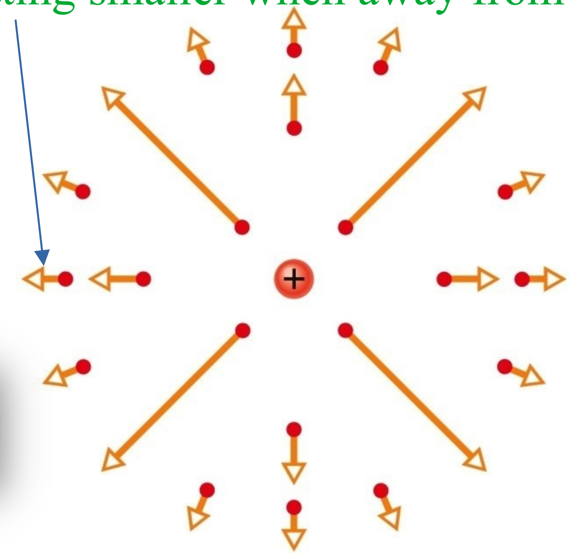


22-4 The Electric Field due to a Point Charge

- To find the electric field due to a point charge q (or charged particle) at any point at a distance r from the point charge, we put a positive test charge q_0 at that point.
- The direction of **E** is directly away from the point charge if q is positive, and directly toward the point charge if q is negative. The electric field vector is:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{(point charge).}$$

Getting smaller when away from



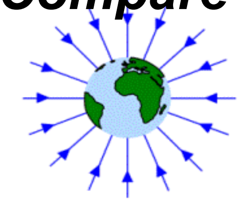
The electric field vectors at various points around a positive point charge.

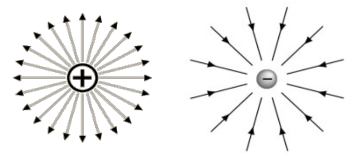
- The net, or resultant, electric field due to more than one point charge can be found by the superposition principle.
  - If we place a positive test charge q_0 near n point charges $q_1, q_2, \ldots, q_n$, then, the net force, $\mathbf{F_0}$, from the n point charges acting on the test charge is $\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$
- Therefore, the net electric field at the position of the test charge is

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0}$$
$$= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.$$

Vector Algebra

Compare to Gravitational to Electric Fields





Gravitational Force:
(Units: Newtons = N)

$$F = -\frac{GmM}{r^2}$$

Gravitational Field:
(Units: N/kg)

$$g = -\frac{GM}{r^2}$$

Given the Field, Find the Force:

$$F = mg$$

Find the Force: (Vector Form)

$$\vec{\mathbf{g}} = -GM\frac{\hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{F}} = -m\vec{\mathbf{g}} = -GmM\frac{\hat{\mathbf{r}}}{r^2}$$

Electric Force:
(Units: Newtons = N)

$$F = \frac{k|q||Q|}{r^2}$$

Electric Field:
(Units: N/C)

$$E = \frac{kQ}{r^2}$$

Given the Field, Find the Force:

$$F = qE$$

Find the Force: (Vector Form)

$$\vec{\mathbf{E}} = kQ\frac{\hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = kqQ\frac{\hat{\mathbf{r}}}{r^2}$$

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22-4 The Electric Field due to a Point Charge

Example, The net electric field due to three charges:

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

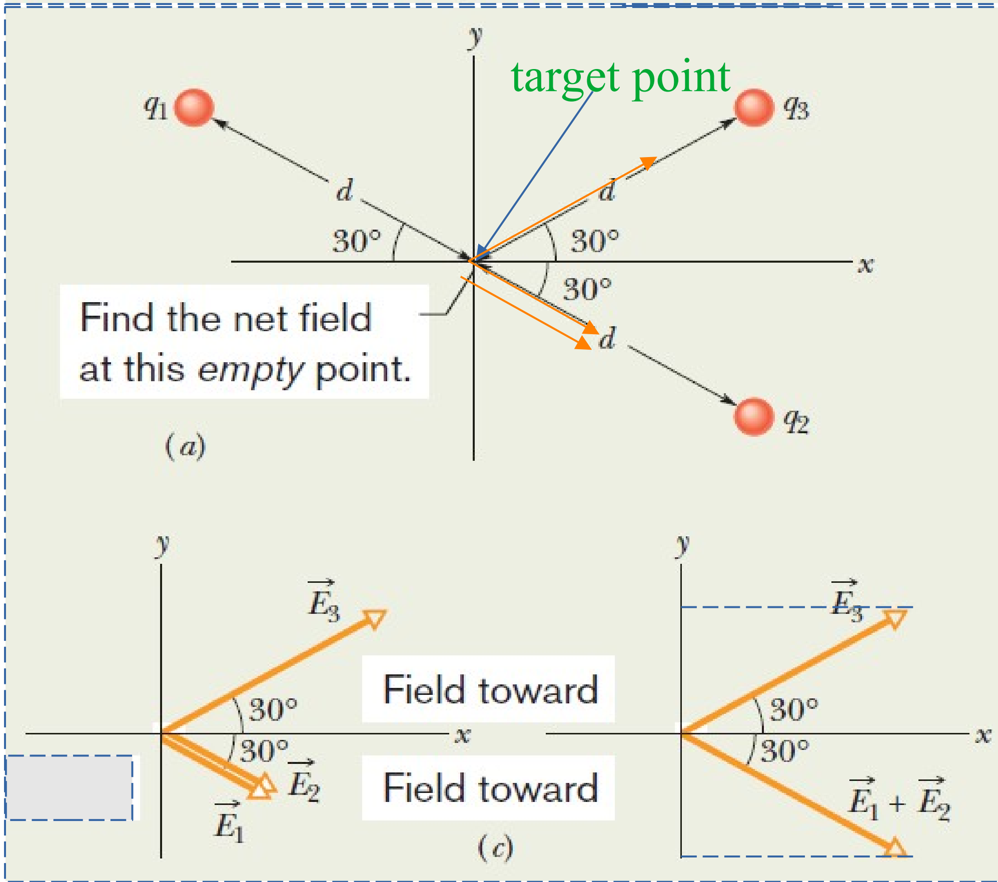


Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

From the symmetry of Fig. 22-7c, we realize that the equal *y* components of our two vectors cancel and the equal *x* components add.

Thus, the net electric field at the origin is in the positive direction of the *x* axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30°$$

$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}.$$

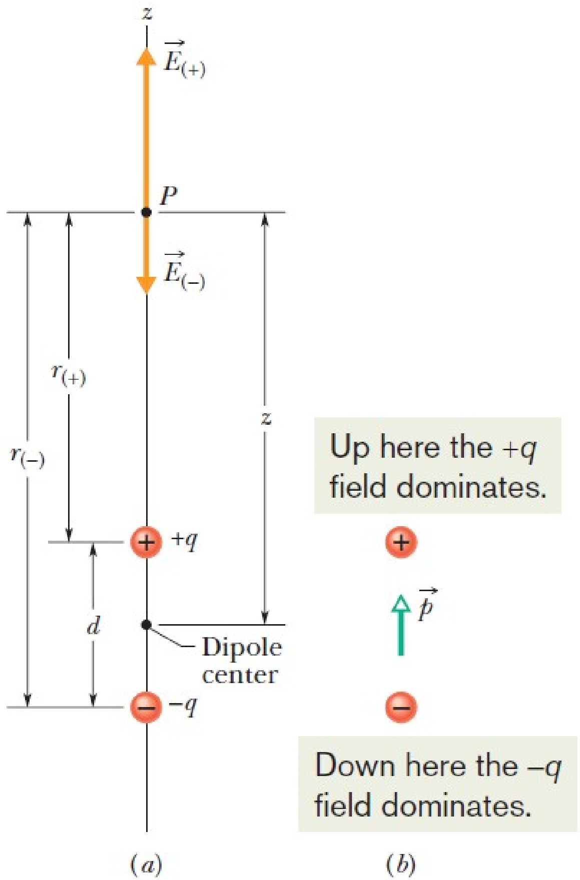


Fig. 22-8 (*a*) An electric dipole. The electric field vectors $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at point P on the dipole axis result from the dipole's two charges. Point P is at distances $r_{(+)}$ and $r_{(-)}$ from the individual charges that make up the dipole. (*b*) The dipole moment $\vec{p}$ of the dipole points from the negative charge to the positive charge.

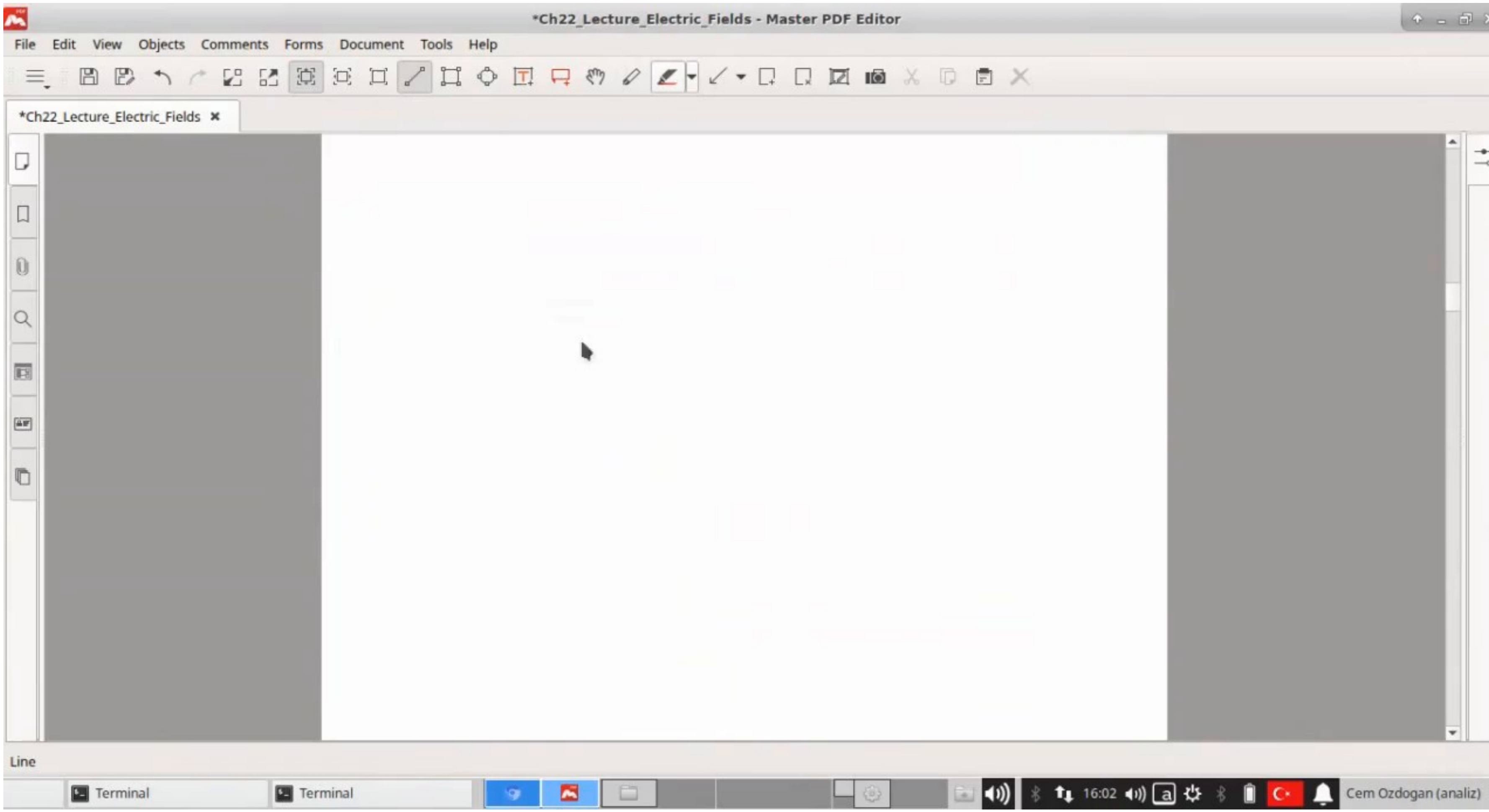
Electric Dipole

- Electric dipole: two point charges +q and –q separated by a distance d.
- Common arrangement in Nature: molecules, antennae, …
- Define "dipole moment" vector p: from –q to +q, with magnitude qd

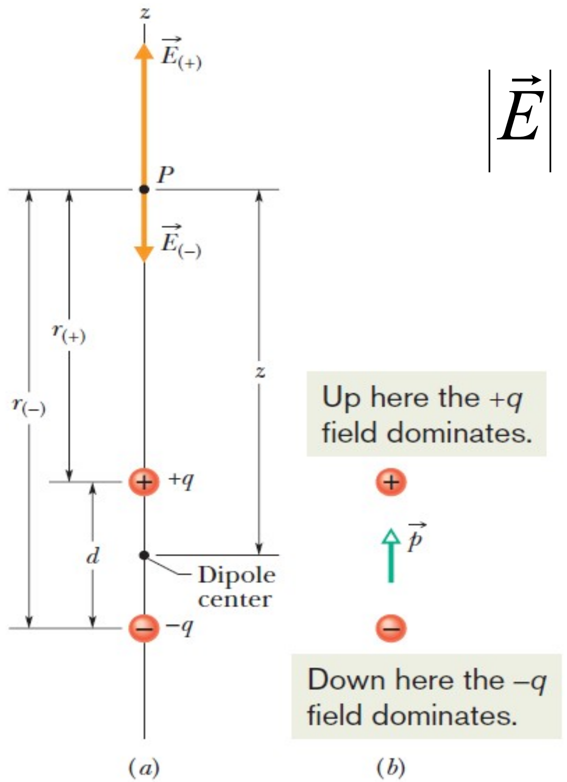
$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}, \qquad E \propto \frac{1}{z^3}$$

where z is the distance between the point and the center of the dipole.

Video: Electric Dipole



From symmetry, the electric field **E** at point P (and also the fields $\mathbf{E}_+$ and $\mathbf{E}_-$ due to the separate charges that make up the dipole) must lie along the dipole axis, which we have taken to be a *z* axis. From the superposition principle for electric fields, the magnitude E of the electric field at P is



$$|\vec{E}| \propto \frac{|\vec{p}|}{r^3}$$

$$E = E_{(+)} - E_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{q}{4\pi\varepsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z+\frac{1}{2}d)^2}.$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right).$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}.$$

$d/2z \ll 1$ → $$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$$

assumption

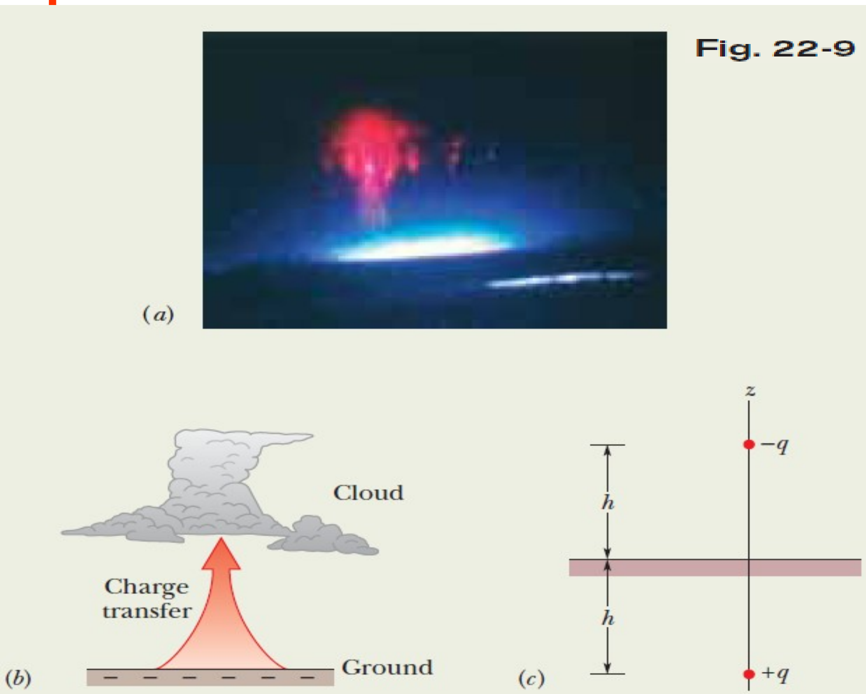
$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole).}$$

The product *qd*, which involves the two intrinsic properties *q* and *d* of the dipole, is the magnitude *p* of a vector quantity known as the ***electric dipole moment, p***, of the dipole.

polarization!

22-5 The Electric Field due to an Electric Dipole

Example, Electric Dipole and Atmospheric Sprites:



Sprites (Fig. 22-9*a)* are huge flashes that occur far above a large thunderstorm. They are still not well understood but are believed to be produced when especially powerful lightning occurs between the ground and storm clouds, particularly when the lightning transfers a huge amount of negative charge -*q* from the ground to the base of the clouds (Fig. 22-9*b).*

We can model the electric field due to the charges in the clouds and the ground by assuming a vertical electric dipole that has charge -*q* at cloud height *h* and charge +*q* at below-ground depth *h (Fig. 22-9c).* If *q =200 C* and *h =6.0* km, what is the magnitude of the dipole's electric field at altitude z_1 *=30 km* somewhat above the clouds and altitude z_2 *=60* km somewhat above the stratosphere?

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q(2h)}{z^3},$$

where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-9*c*. For the electric field at altitude $z_1 = 30$ km, we find

$$E = \frac{1}{2\pi\varepsilon_0}\frac{(200\ \text{C})(2)(6.0\times10^3\ \text{m})}{(30\times10^3\ \text{m})^3} = 1.6\times10^3\ \text{N/C}. \quad \text{(Answer)}$$

Similarly, for altitude $z_2 = 60$ km, we find

$$E = 2.0\times10^2\ \text{N/C}. \quad \text{(Answer)}$$

Left as exercise

22-6 The Electric Field due to a Continuous Charge

When we deal with continuous charge distributions, it is most convenient to express the charge on an object as a ***charge density*** (λ,σ,ρ) rather than as a total charge.

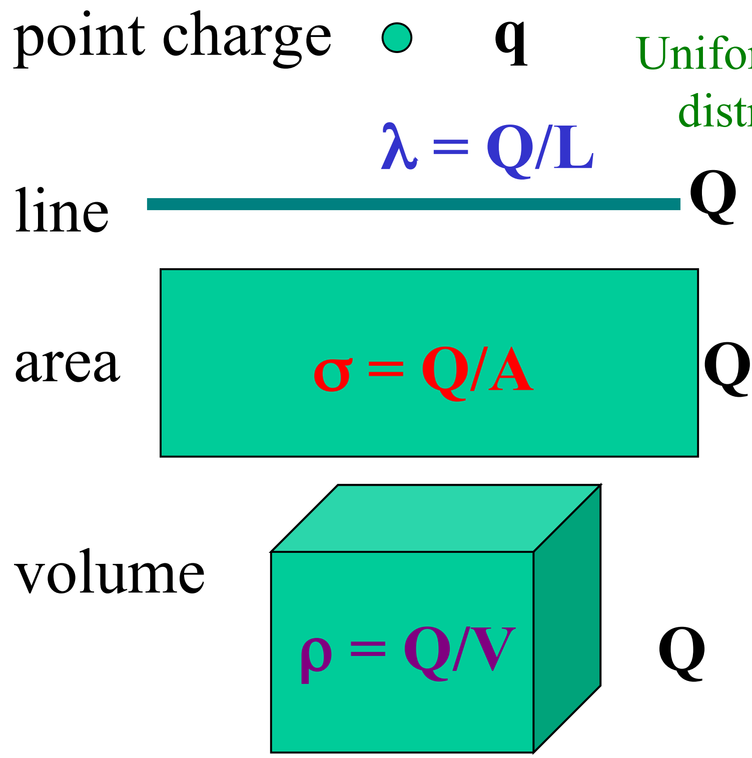
- For a line of charge, for example, we would report the *linear charge density* (or charge per unit length) λ, whose SI unit is the coulomb per meter.
- Table 22-2 shows the other charge densities we shall be using.

$$\lambda = \frac{Q}{L} = \frac{\Delta Q}{\Delta L} = \frac{dq}{dx}$$

Table 22-2

Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3



Uniform charge distribution

regular shaped objects

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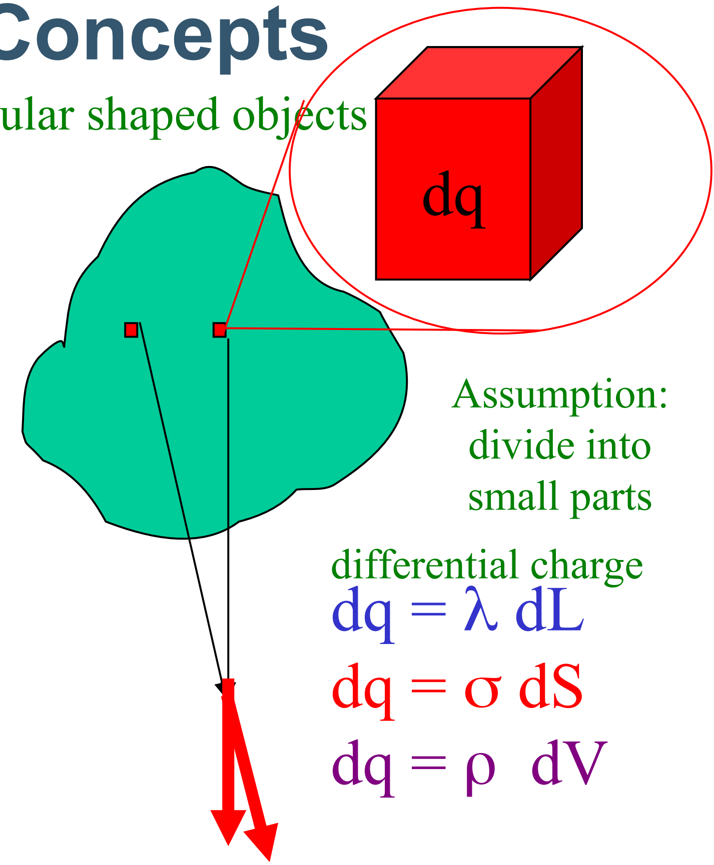
$$Q \mapsto dq \mapsto dE \mapsto \sum \Delta E \mapsto \int dE$$

Key Concepts

- The equation for the electric field set up by a particle *does not apply* to an **extended object** with charge (said to have a continuous charge distribution).
- To find the electric field of an extended object at a point, we first consider the electric field set up by a charge element **dq** in the object, where the element is *small enough* for us to apply the equation for a particle.
- Then we sum, via integration, components of the electric fields ***dE*** from all the charge elements.
- Because the individual electric fields *dE* have different magnitudes and point in different directions, *we first see if symmetry allows us to cancel out any of the fields, to simplify the integration.*



$$dq = \lambda\, dL$$

$$dq = \sigma\, dS$$

$$dq = \rho\, dV$$

$$\sum \Delta E = \int dE$$

$$|d\vec{E}| = \frac{k|dq|}{r^2}$$

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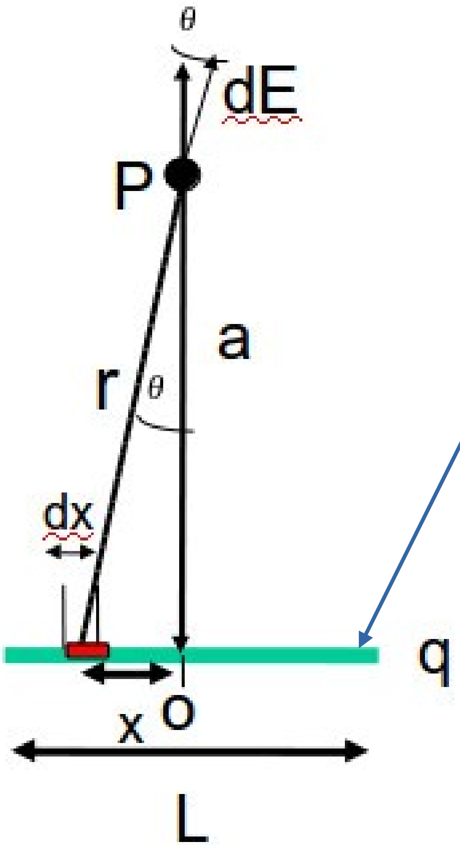
E Due to a Line of Charge: Field on bisector

point P at the middle



Distance hypotenuse: $r = \left(a^2 + x^2\right)^{1/2}$

Charge per unit length: $\lambda = \dfrac{q}{L}$ [C/m]

$$dE = \frac{k(dq)}{r^2}$$

$$dE_y = dE\ \cos\theta = \frac{k(\lambda\ dx)a}{\left(a^2 + x^2\right)^{3/2}}$$

$$\cos\theta = \frac{a}{r} = \frac{a}{(a^2 + x^2)^{1/2}}$$

Adjacent Over Hypotenuse

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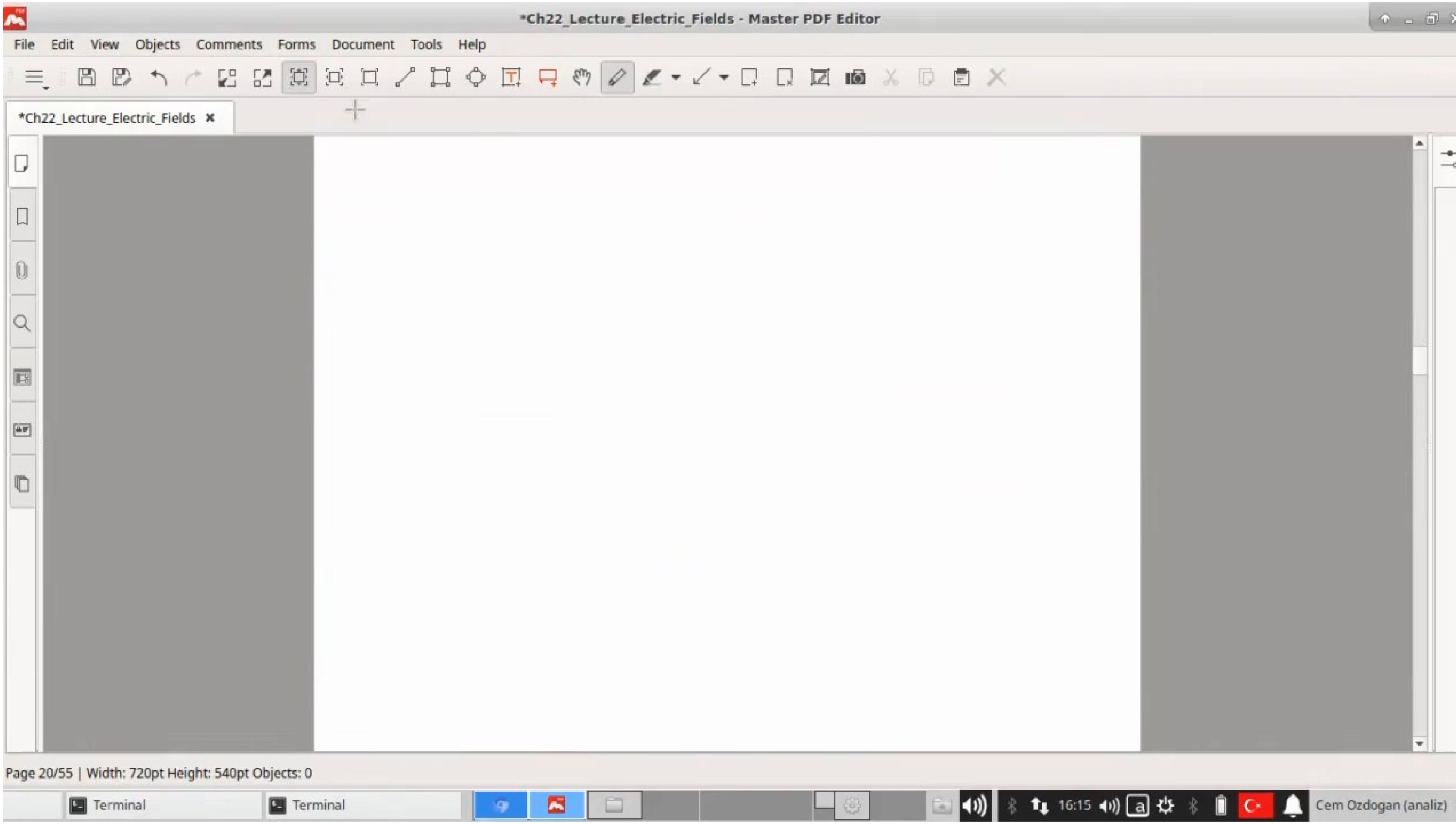
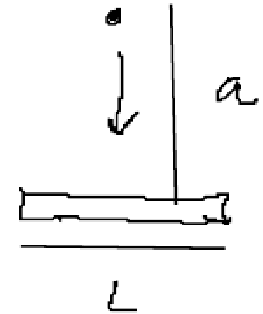
Video: Line of Charge



E Due to a Line of Charge: Field on bisector

$$E_y = k\lambda\, a \int_{-L/2}^{L/2} \frac{dx}{\left(a^2+x^2\right)^{3/2}} = k\lambda\, a \left[\frac{x}{a^2\sqrt{x^2+a^2}} \right]_{-L/2}^{L/2}$$



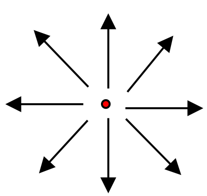
Integrate: Trig Substitution!

$$= \frac{2k\lambda L}{a\sqrt{4a^2+L^2}}$$

away from the rod: point charge

Point Charge Limit: L << a

$$E_y = \frac{2k\lambda L}{a\sqrt{4a^2+L^2}} \approx \frac{k\lambda L}{a^2} = \frac{kq}{a^2}$$

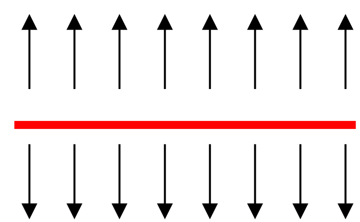


Coulomb's Law!

near by the rod: infinite rod

Line Charge Limit: L >> a

$$E_y = \frac{2k\lambda L}{a\sqrt{4a^2+L^2}} \approx \frac{2k\lambda}{a}$$



Units Check!

$$\left[\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\frac{1}{\mathrm{m}}\frac{\mathrm{C}}{\mathrm{m}}\right] = \left[\frac{\mathrm{N}}{\mathrm{C}}\right]$$

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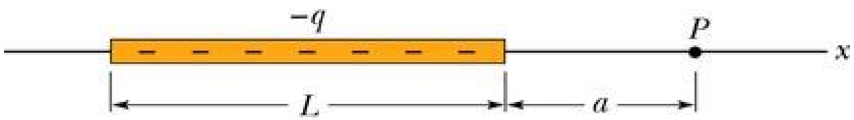
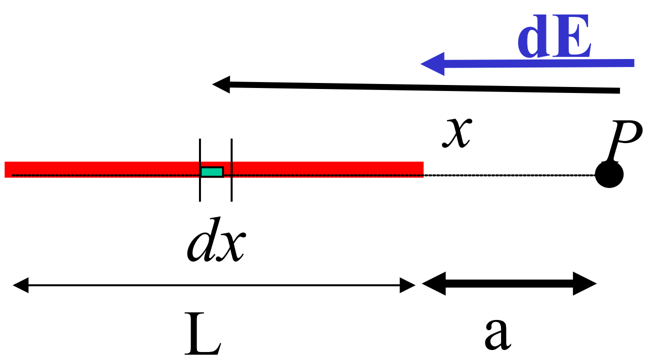
E Due to a Line of Charge: Field on x-axis direction



Calculate the <u>magnitude</u> of the electric field at point *P*.



$$dE = \frac{k(dq)}{r^2} = \frac{k(dq)}{x^2} = \frac{k(\lambda dx)}{x^2}$$

$$E = \int_a^{a+L} dE = \int_a^{a+L} \frac{k(\lambda dx)}{x^2}$$

$$= k\lambda \int_a^{a+L} x^{-2}\, dx = k\lambda \left[-x^{-1}\right]_a^{a+L}$$

$$= k\lambda \left[\frac{1}{a} - \frac{1}{a+L}\right] = k\frac{q}{L}\left[\frac{1}{a} - \frac{1}{a+L}\right]$$

E_x=-kq/(a(-L+a))

Now, if a>>L

E_x=-kq/a^2 (point charge)

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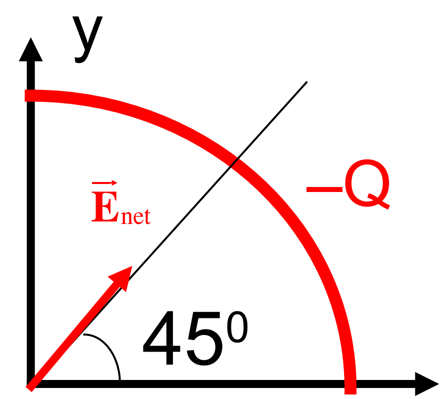
E Due to Arc of Charge

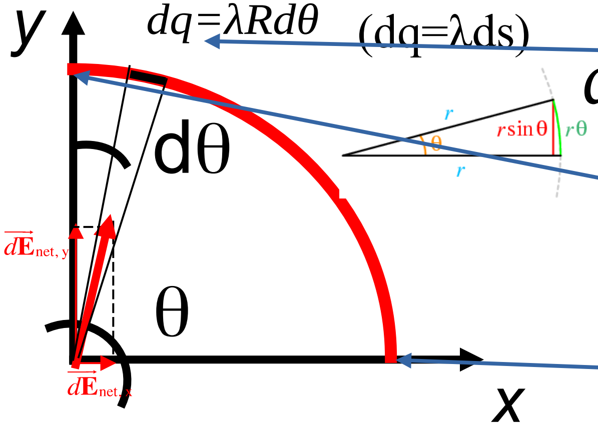


- Figure shows a uniformly charged rod of charge **-Q** bent into a circular arc of radius *R*, centered at (0,0).
- Which way does net E-field point?
- Compute the direction & magnitude of E at the origin.

$$\lambda = \frac{Q}{L} = \frac{Q}{2\pi R/4} = \frac{2Q}{\pi R}$$

circumference/4

$dq = \lambda R d\theta$ (dq=λds)



$$dE_x = dE\cos\theta = \frac{kdq}{R}\cos\theta$$

$$E_x = \int_0^{\pi/2} \frac{k\lambda R\cos\theta}{R^2} d\theta = \frac{k\lambda}{R}\int_0^{\pi/2}\cos\theta d\theta$$

$$E_x = \frac{k\lambda}{R} \qquad E_y = \frac{k\lambda}{R}$$

$$E = \sqrt{E_x^2 + E_y^2} \qquad E_{net} = \sqrt{2}\frac{k\lambda}{R}$$

E of a Charged Circular Rod

Figure 22-11*a* shows a plastic rod having a uniformly distributed charge $-Q$. The rod has been bent in a 120° circular arc of radius r. We place coordinate axes such that the axis of symmetry of the rod lies along the x axis and the origin is at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}$$

$ds = R\,d\theta$

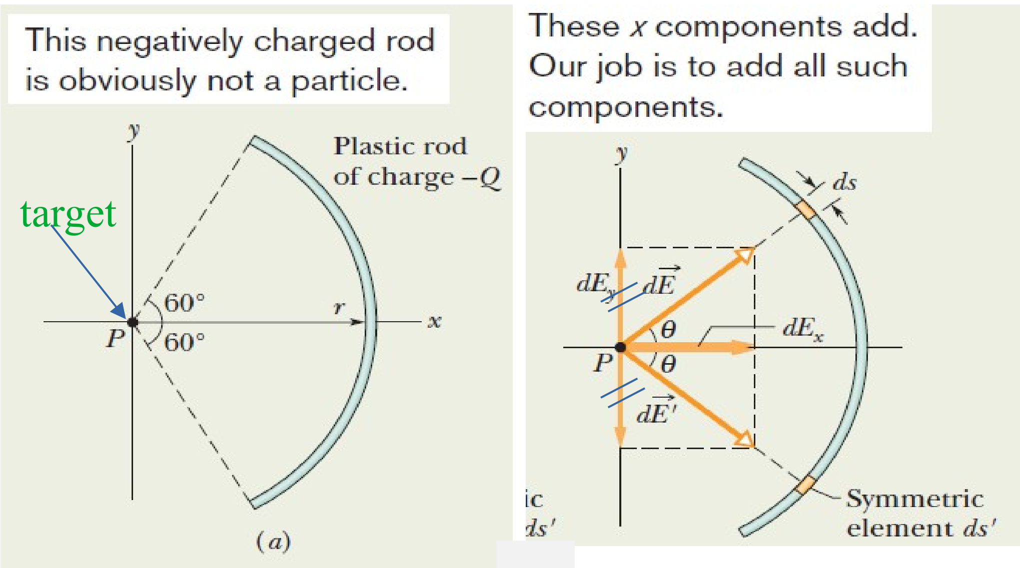


Fig. 22-11 (a) A plastic rod of charge Q is a circular section of radius r and central angle 120°; point *P* is the center of curvature of the rod. (b) The field components from symmetric elements from the rod.

Our element has a symmetrically located (mirror image) element ds' in the bottom half of the rod.

If we resolve the electric field vectors of ds and ds' into x and y components as shown in we see that their y components cancel (because they have equal magnitudes and are in opposite directions). We also see that their x components have equal magnitudes and are in the same direction.

$$E = \int dE_x = \int_{-60^\circ}^{60^\circ} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\, d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-60^\circ}^{60^\circ}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-60^\circ}^{60^\circ}$$

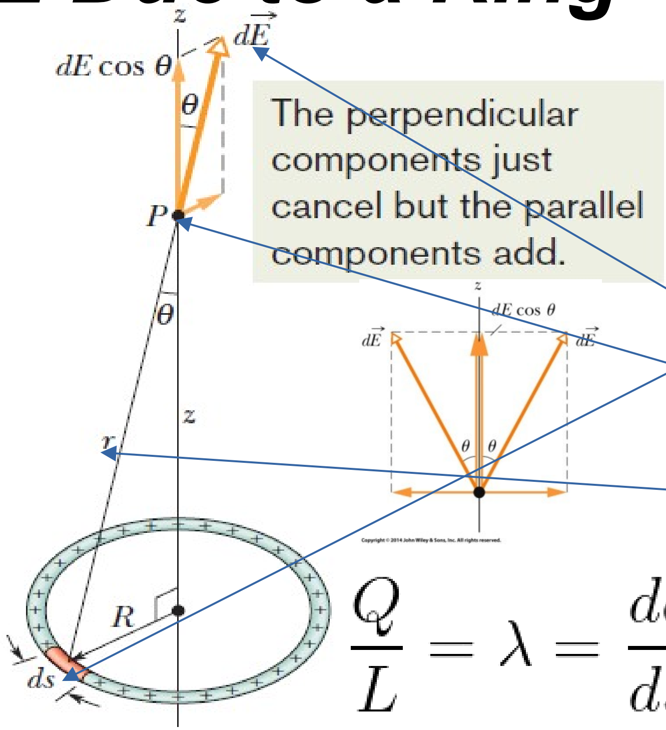
$$= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin 60^\circ - \sin(-60^\circ)]$$

$$= \frac{1.73\lambda}{4\pi\varepsilon_0 r} = \frac{0.83Q}{4\pi\varepsilon_0 r^2}.$$

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}.$$

$$dq = \lambda\, ds.$$

E Due to a Ring



$$\frac{Q}{L} = \lambda = \frac{dq}{ds}$$

Fig. 22-10 A ring of uniform positive charge. A differential element of charge occupies a length ds (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point P. The component of $d\vec{E}$ along the central axis of the ring is $dE\cos\theta$.

1) **Define:** We can mentally divide the ring into *differential elements* of charge that are so small that they are like point charges, and then we can apply

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

to each of them.

- Let *ds* be the (arc) length of any differential element of the ring.
- Since λ is the charge per unit (arc) length, the element has a charge of magnitude $dq = \lambda\, ds.$
- This differential charge sets up a differential electric field ***dE*** at point *P*, a distance *r* from the element.

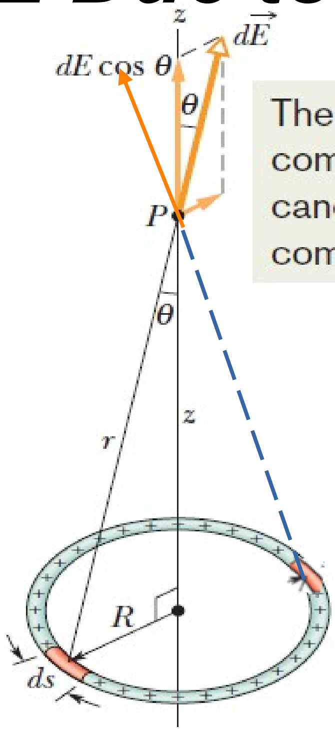
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}.$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}.$$

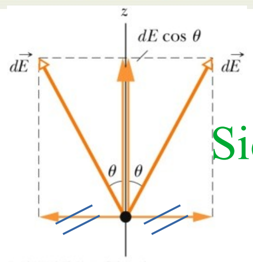
2) **Adding:** Next, we can add the electric fields set up at *P by all the differential elements.*

- The vector sum (Σ→ ∫) of the fields gives us the field set up at *P by the ring.*

E Due to a Ring



The perpendicular components just cancel but the parallel components add.



Side view

Top view

Fig. 22-10 A ring of uniform positive charge. A differential element of charge occupies a length *ds* (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point *P*. The component of $d\vec{E}$ along the central axis of the ring is $dE\cos\theta$.

All the ***dE*** vectors have components parallel and perpendicular to the central axis:

- The perpendicular components (to z-axis) are identical in magnitude but point in different directions, so they sum up to zero.
- The parallel components are

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\,ds.$$

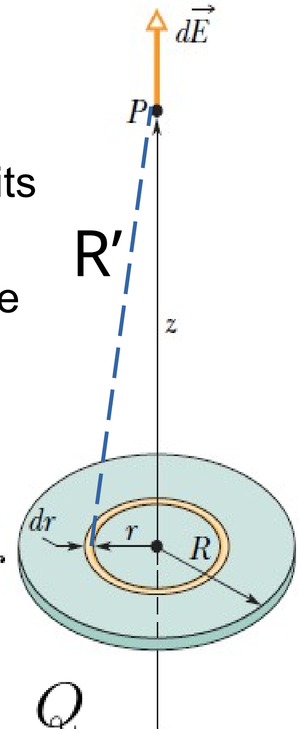
3)**Integrating:** Finally, for the entire ring,

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}.$$

E Due to Charged Disk

- We need to find the electric field at point *P*, a distance *z* from the disk along its central axis.
- **Define & Adding:** Divide the disk into concentric flat rings and then calculate the electric field at point *P* by adding up (that is, by integrating) the contributions of all the rings.
  - The figure shows one such ring, with radius *r* and radial width *dr*. If σ is the charge per unit area, the charge on the ring is



$$A = \pi r^2 \; ; \; dA = 2\pi r\,dr$$

$$dq = \sigma\,dA = \sigma\,(2\pi r\,dr),$$

$$dE = \frac{z\sigma 2\pi r\,dr}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}} = \frac{\sigma z}{4\varepsilon_0}\frac{2r\,dr}{(z^2 + r^2)^{3/2}}$$

$$\frac{Q}{A} = \sigma = \frac{dq}{dA}$$

- **Integrating:** We can now find *E* by integrating *dE* over the surface of the disk— that is, by integrating with respect to the variable *r* from *r =0 to r =R.*

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2 + r^2)^{-3/2}(2r)\,dr. = \frac{\sigma z}{4\varepsilon_0}\left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R.$$

$$\Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \quad \text{(charged disk)}$$

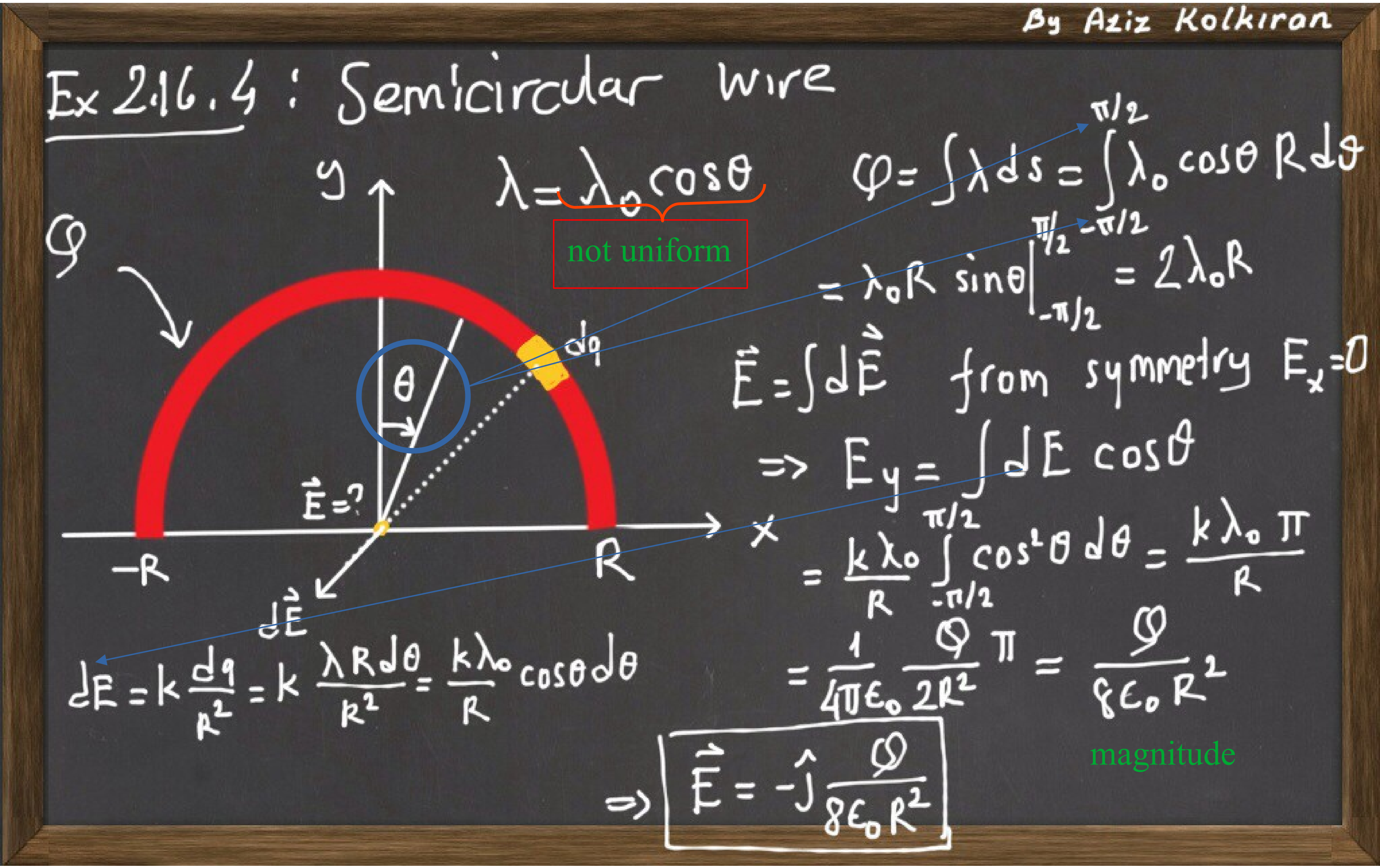
$1/R \rightarrow 0$

If we let R →∞, while keeping z finite, the second term in the parentheses in the above equation approaches zero, and this equation reduces to

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(infinite sheet).}$$

This is the electric field produced by an infinite sheet of uniform charge.

22-7 The Electric Field due to a Continuous Charge

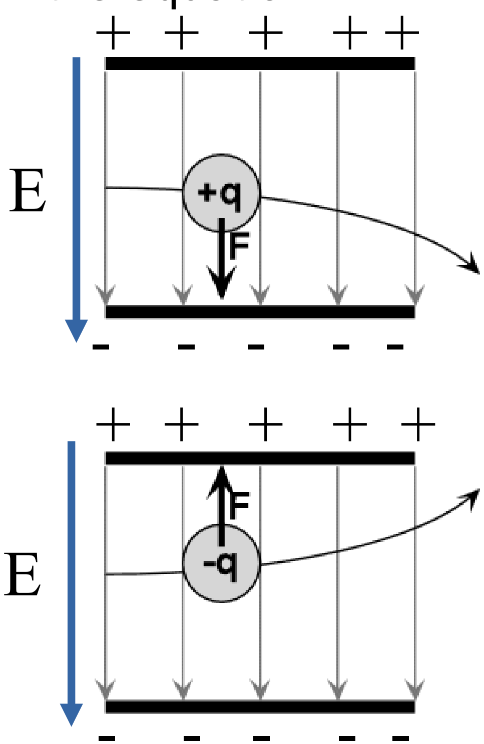


The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.

When a charged particle, of charge *q*, is in an electric field, **E,** *set up by other stationary or slowly moving charges*, an electrostatic force, **F,** acts on the charged particle as given by the equation:

$$\vec{F} = q\vec{E},$$

Force on target particle — target — External Field



Example: Ink-Jet Printing

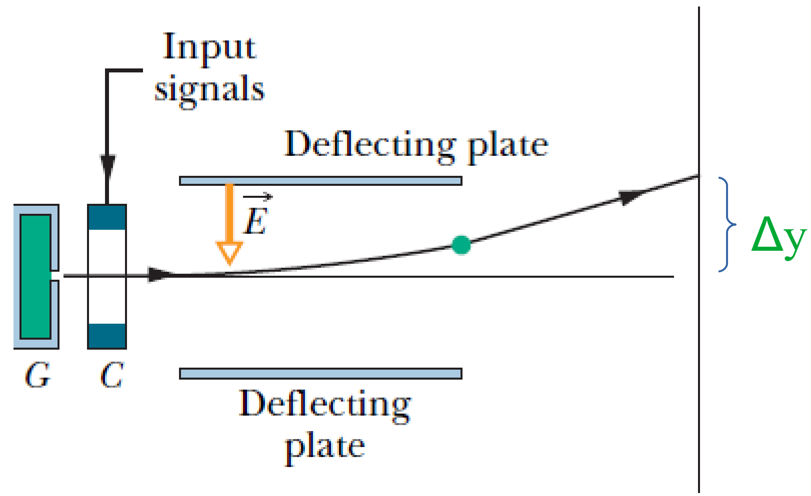


Fig. 22-15 Ink-jet printer. Drops shot from generator G receive a charge in charging unit C. An input signal from a computer controls the charge and thus the effect of field $\vec{E}$ where the drop lands on the paper.

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

Example, Motion of a Charged Particle in an Electric Field

Figure 22-17 shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x = 18$ m/s. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)

$$y - y_0 = \Delta y = h = \frac{1}{2} a_y t^2$$

$$x - x_0 = L = v_{0x} t$$

X$_g$ & F_E : constant E → constant acceleration

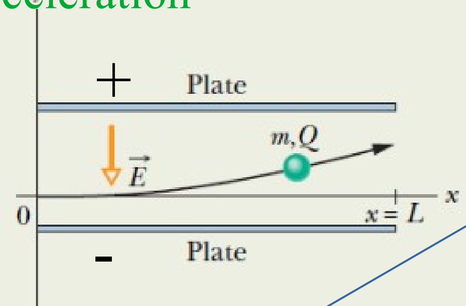


Fig. 22-17 An ink drop of mass m and charge magnitude Q is deflected in the electric field of an ink-jet printer.

$v_{0x} = v_x$ & $v_{0y} = 0$ $\quad x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$F_E = ma_y = QE \quad y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

KEY IDEA

The drop is negatively charged and the electric field is directed *downward*. From Eq. 22-28, a constant electrostatic force of magnitude QE acts *upward* on the charged drop. Thus, as the drop travels parallel to the x axis at constant speed v_x, it accelerates upward with some constant acceleration a_y.

Calculations: Applying Newton's second law ($F = ma$) for components along the y axis, we find that

$$a_y = \frac{F}{m} = \frac{QE}{m}. \tag{22-30}$$

Let t represent the time required for the drop to pass through the region between the plates. During t the vertical and horizontal displacements of the drop are

$$y = \tfrac{1}{2} a_y t^2 \quad \text{and} \quad L = v_x t, \tag{22-31}$$

respectively. Eliminating t between these two equations and substituting Eq. 22-30 for a_y, we find

$$y = \frac{QEL^2}{2mv_x^2}$$

$$= \frac{(1.5 \times 10^{-13}\ \text{C})(1.4 \times 10^6\ \text{N/C})(1.6 \times 10^{-2}\ \text{m})^2}{(2)(1.3 \times 10^{-10}\ \text{kg})(18\ \text{m/s})^2}$$

$$= 6.4 \times 10^{-4}\ \text{m}$$

$$= 0.64\ \text{mm}. \quad \text{(Answer)}$$

22-9 A Dipole in an Electric Field

- When an **electric dipole** is placed in a region where there is an **external electric field**, **E**, electrostatic forces act on the charged ends of the dipole.
- If the electric field is uniform, those forces act in opposite directions and with the same magnitude **F** =*q***E**.
- Although the net force on the dipole from the field is zero, and the center of mass of the dipole does not move, the forces on the charged ends do produce a net torque $\boldsymbol{\tau}$ on the dipole about its center of mass.
- The center of mass lies on the line connecting the charged ends, at some distance *x* from one end and a distance *d*-*x* from the other end.

The net torque is:

$$\tau = Fx \sin\theta + F(d - x)\sin\theta = Fd\sin\theta. = pE\sin\theta.$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}$$

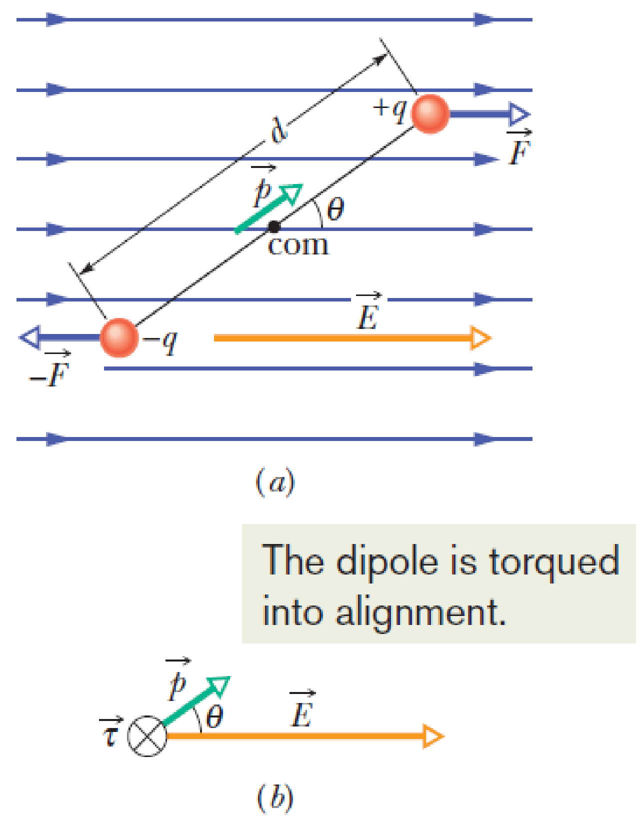


Fig. 22-19 (*a*) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance *d*. The line between them represents their rigid connection. (*b*) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

Potential Energy

- Potential energy can be associated with the orientation of an electric dipole in an electric field.
- The dipole has its *least potential energy* when it is in its *equilibrium orientation*, which is when its moment **p** is lined up with the field **E.**
- The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ (Fig.22-19) is 90°.
- **The potential energy *U* of the dipole** at any other value of θ can be found by calculating the work *W* done by the field on the dipole when the dipole is rotated to that value of θ from 90°.

$$U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta = -pE\cos\theta.$$

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}$$

$U_f - U_i = \Delta U = -W$

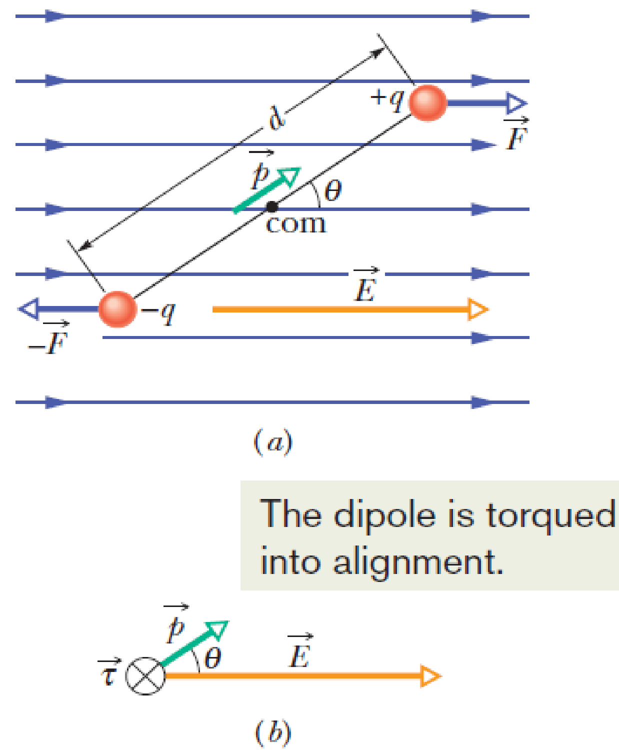
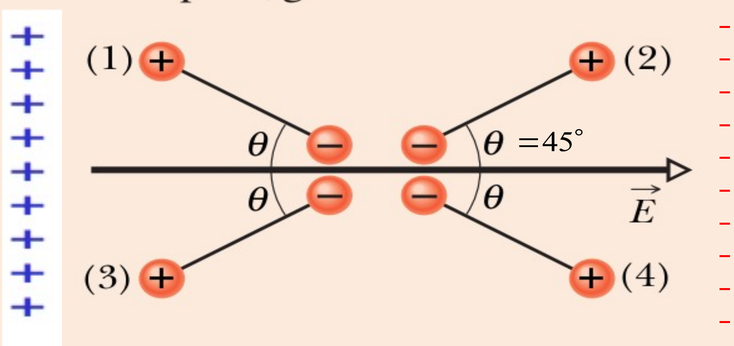


Fig. 22-19 (*a*) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (*b*) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

22-9 A Dipole in an Electric Field

CHECKPOINT 4

The figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.



1 and 3 are "uphill".
2 and 4 are "downhill".
U1 = U3 > U2 = U4

$$U_1 = -pE\cos(135^\circ) = +0.71\,pE$$
$$U_2 = -pE\cos(+45^\circ) = -0.71\,pE$$
$$U_3 = -pE\cos(-135^\circ) = +0.71\,pE$$
$$U_4 = -pE\cos(-45^\circ) = -0.71\,pE$$

$$|\tau_1| = pE\,|\sin(45^\circ + 45^\circ + 45^\circ)| = pE\,|\sin(135^\circ)| = 0.71\,pE$$
$$|\tau_2| = pE\,|\sin(45^\circ)| = 0.71\,pE$$
$$|\tau_3| = pE\,|\sin(-135^\circ)| = 0.71\,pE$$
$$|\tau_4| = pE\,|\sin(-45^\circ)| = 0.71\,pE$$
$$|\tau_1| = |\tau_2| = |\tau_3| = |\tau_4|$$

(a) all tie;
(b) 1 and 3 tie, then 2 and 4 tie

Left as exercise

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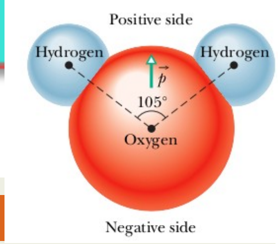
22-9 A Dipole in an Electric Field

Example, Torque, Energy of an Electric Dipole in an Electric Field



A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude 6.2×10^{-30} C·m.

(a) How far apart are the molecule's centers of positive and negative charge?

KEY IDEA

A molecule's dipole moment depends on the magnitude q of the molecule's positive or negative charge and the charge separation d.

Calculations: There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which d is the separation we are seeking and e is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \text{C}\cdot\text{m}}{(10)(1.60 \times 10^{-19}\ \text{C})}$$

$$= 3.9 \times 10^{-12}\ \text{m} = 3.9\ \text{pm}. \qquad \text{(Answer)}$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

(b) If the molecule is placed in an electric field of 1.5×10^4 N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

KEY IDEA

The torque on a dipole is maximum when the angle θ between $\vec{p}$ and $\vec{E}$ is 90°.

Calculation: Substituting $\theta = 90°$ in Eq. 22-33 yields

$$\tau = pE \sin\theta$$

$$= (6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})(\sin 90°)$$

$$= 9.3 \times 10^{-26}\ \text{N}\cdot\text{m}. \qquad \text{(Answer)}$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

initially 0; **p||E**

KEY IDEA

The work done by an external agent (by means of a torque applied to the molecule) is equal to the change in the molecule's potential energy due to the change in orientation.

Calculation: From Eq. 22-40, we find

$$W_a = U_{180°} - U_0 \qquad U_f - U_i$$

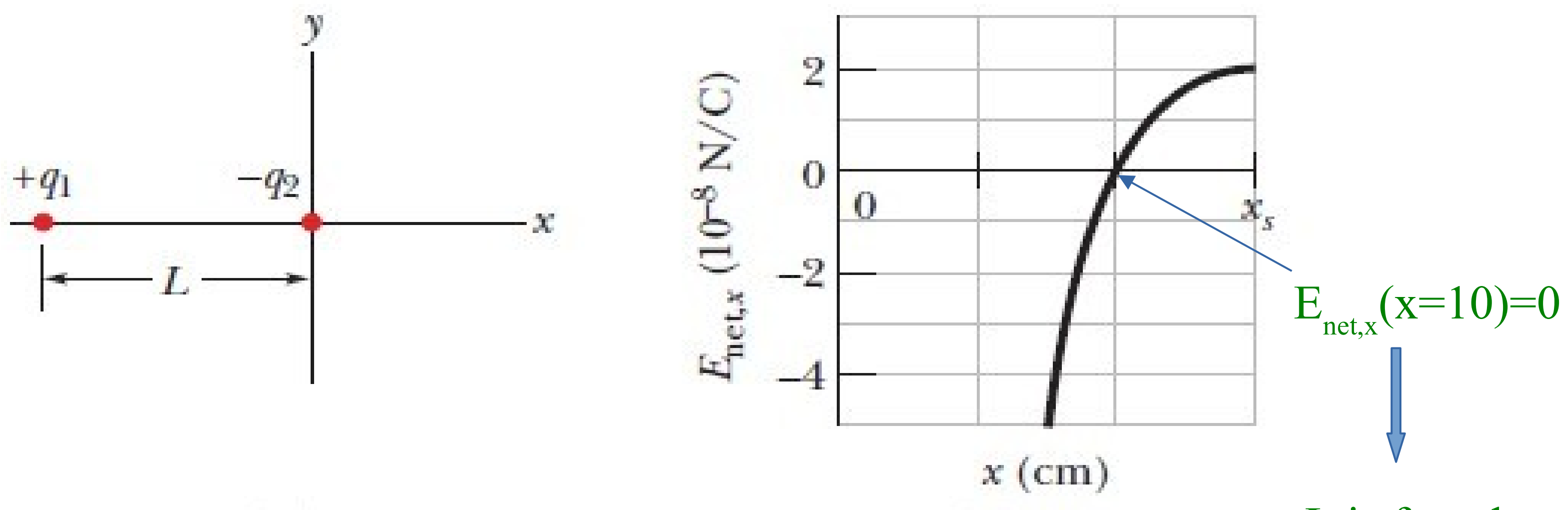
$$= (-pE\cos 180°) - (-pE\cos 0)$$

$$= 2pE = (2)(6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})$$

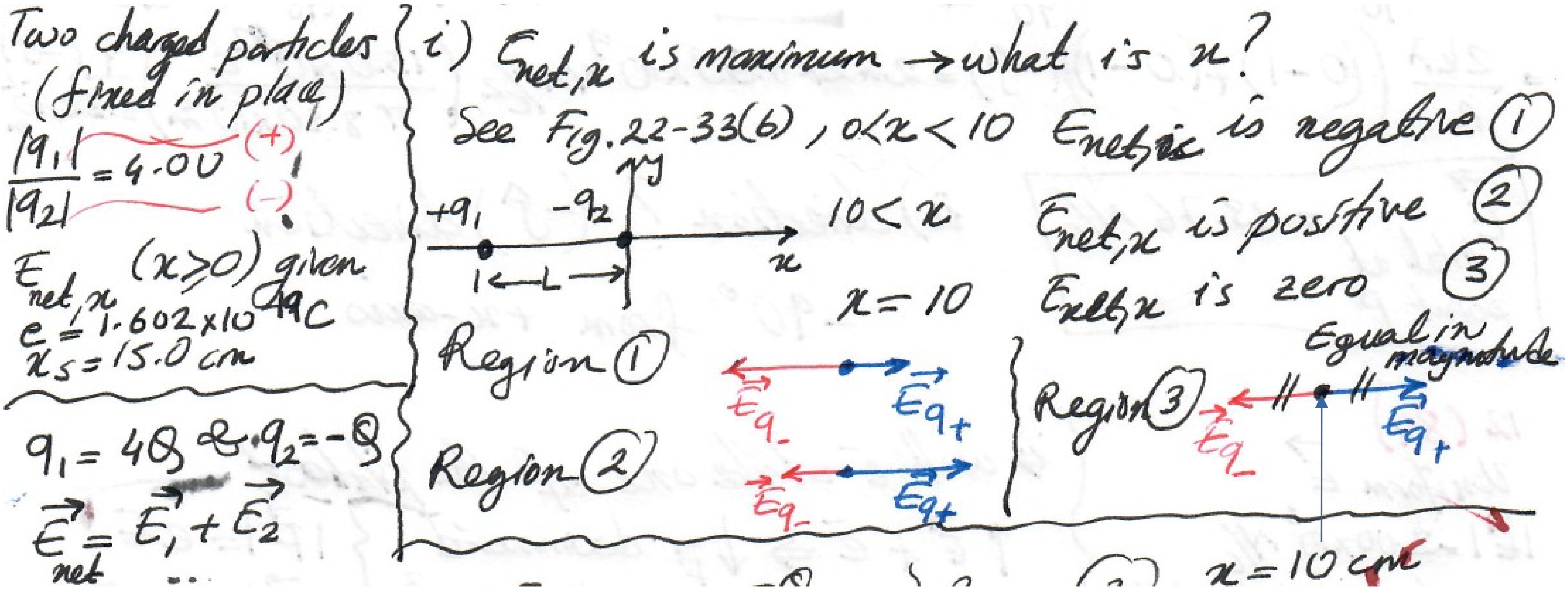
$$= 1.9 \times 10^{-25}\ \text{J}. \qquad \text{(Answer)}$$

(+) positive value→ work done on the system

1. (10) Figure (a) shows two charged particles fixed in place on an x-axis with separation L. The ratio q_1/q_2 of their charge magnitudes is 4.00. Figure (b) shows the x component $E_{net,x}$ of their net electric field along the x axis just to the right of particle 2. The x axis scale is set by x_s= 15.0 cm. (a) At what value of x >0 is $E_{net,x}$ maximum? (b) If particle 2 has charge $-q_2$ = -3e, what is the value of that maximum?



(a) (b)



$$\vec{E}_{net,x} = 0 = \vec{E}_{q+} + \vec{E}_{q-}$$

$$|\vec{E}_{q+}| = |\vec{E}_{q-}|$$

Next: first derivative of E with respect to x should be equal to 0

E_{net} :

$E_{net,x} = E_{1,x} + E_{2,x} = \frac{4Q}{4\pi\varepsilon_0 (L+x)^2} + \frac{-Q}{4\pi\varepsilon_0 x^2}$ } Region (3) $x = 10$ cm, $E_{net,x} = 0$

$0 = \frac{4Q}{4\pi\varepsilon_0 (L+0.1m)^2} + \frac{-Q}{4\pi\varepsilon_0 (0.1m)^2}$ { $4 \times (0.1m)^2 = (L+0.1m)^2$

$0.2m = L + 0.1m$

$\Rightarrow L = 0.1m = 10cm$

For being maximum, derivative should be equal to zero.

$\frac{dE_{net,x}}{dx} = 0$ | $\frac{d}{dx}\left(\frac{Q}{4\pi\varepsilon_0}\right)\left(\frac{4}{(0.1m+x)^2} - \frac{1}{x^2}\right) = 0$ | $\frac{4(-2)(1)}{(0.1m+x)^3} = \frac{(-2)1}{x^3}$

$4 = \frac{(0.1m+x)^3}{x^3}$ | $4^{1/3} x = 0.1m + x$ | $x = \frac{0.1m}{(4^{1/3}-1)} = 0.17m = 17.0$ cm

ii) if $-q_2 = -3e$, what $E_{net,x,max} = ?$

$E_{net,x,max} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4 \times 3e}{(0.1m+0.17m)^2} + \frac{-3e}{(0.17m)^2}\right) = 8.76 \times 10^{-8}$ N/C

$k = 8.99 \times 10^9$ Nm^2/C^2

magnitude

Left Right

2. (16) Figure shows a plastic ring of radius R = 50.0 cm. Two small charged beads are on the ring: Bead 1 of charge +2.00 μC is fixed in place at the left side; bead 2 of charge +6.00 μC can be moved along the ring. The two beads produce a net electric field of magnitude E at the center of the ring. At what (a) positive and (b) negative value of angle θ should bead 2 be positioned such that E = 2.00 x 105 N/C?

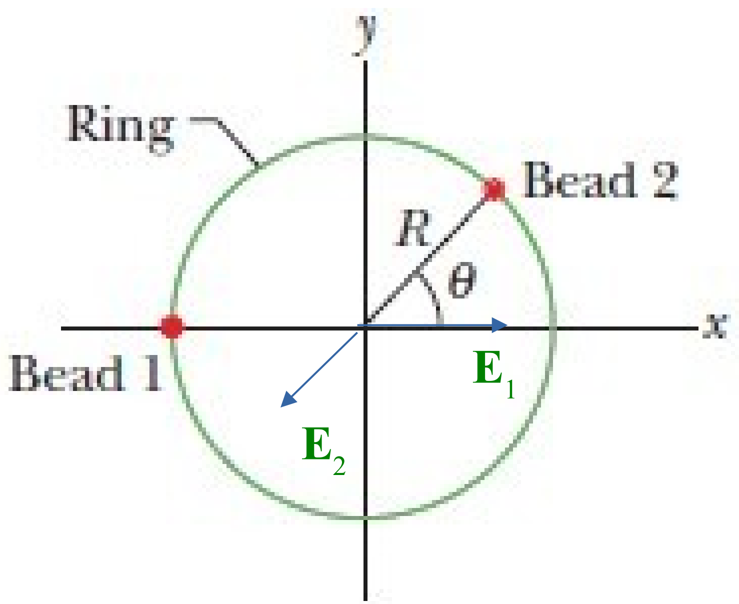
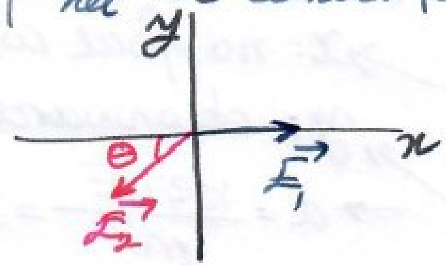


Fig. 22-38 Problem 16.

plastic ring
Radius $R = 50\times10^{-2}\,m$
two charged objects
$q_1 = 2\,\mu C$: fixed
$q_2 = 6\,\mu C$: moving

$|\vec{E}_{net}$ at center$| = |\vec{E}_1| + \vec{E}_2|$



$x:\ E_{1,x} + E_{2,x} = |\vec{E}_1| + |\vec{E}_2|\cos\theta$

$$E_{net,x} = k\frac{q_1}{R^2} - k\frac{q_2}{R^2}\cos\theta$$

$$y:\ E_{net,y} = -k\frac{q_2}{R^2}\sin\theta$$

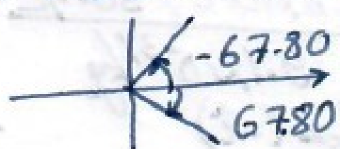
$$E_{net,center} = \sqrt{|E_{net,x}|^2 + |E_{net,y}|^2} = \frac{k}{R^2}\left((q_1 - q_2\cos\theta)^2 + (-q_2\sin\theta)^2\right)^{1/2}$$

$$= \frac{k}{R^2}\left((q_1^2 + q_2^2\cos^2\theta - 2q_1q_2\cos\theta + q_2^2\sin^2\theta)\right)^{1/2} = \frac{k}{R^2}\left(q_1^2 + q_2^2 - 2q_1q_2\cos\theta\right)^{1/2}$$

$$2\times10^5\,N/C = \frac{1}{4\pi\epsilon_0 R^2}\left(q_1^2 + q_2^2 - 2q_1q_2\cos\theta\right)^{1/2} = 8.99\times10^9\,\frac{Nm^2}{C^2}\left(\frac{1}{50\times10^{-2}m}\right)^2\left[(2\times10^{-6}C)^2 + (6\times10^{-6}C)^2 - 2(2\times10^{-6}C)(6\times10^{-6}C)\cos\theta\right]^{1/2}$$

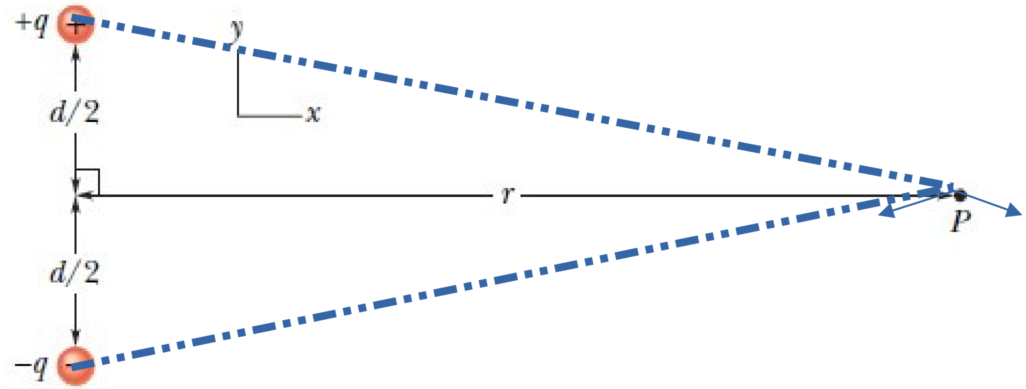
$$2\times10^5\,\frac{N}{C}\,\frac{(50\times10^{-2}m)^2\,C^2}{8.99\times10^9\,Nm^2} = \left(40\times10^{-12}C^2 - 24\times10^{-12}C^2\cos\theta\right)^{1/2}$$

$$\frac{(5.56\times10^{-6}C)^2 - 40\times10^{-12}C^2}{-24\times10^{-12}C^2} = \cos\theta = 0.377 \rightarrow \theta = \cos^{-1}(0.377) = \underline{67.80^\circ}$$

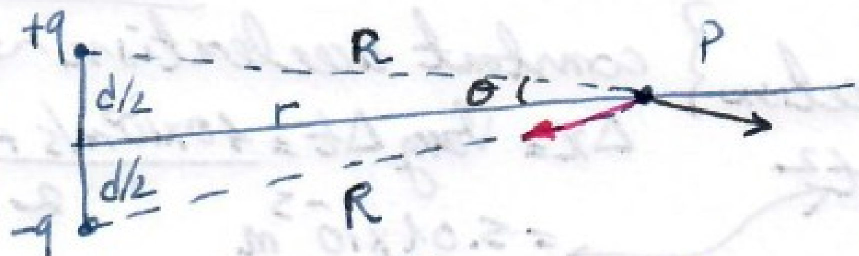


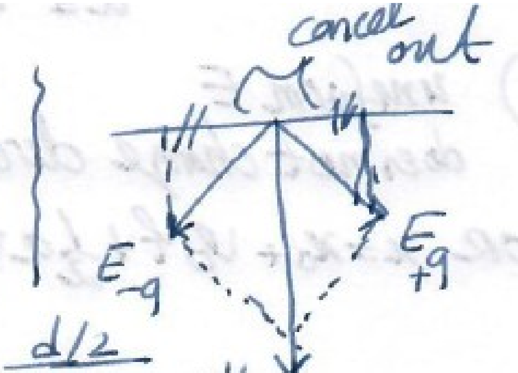
→ two angles

3. (19) Figure shows an electric dipole. What are the (a) magnitude and (b) direction of the dipole's electric field at point *P*, located at a distance r>>d.



Electric dipole. magnitude and direction of E_p





$$|\vec{E}_p| = 2E\sin\theta = 2\left(k\frac{q}{R^2}\right)\frac{d/2}{R} = 2k\frac{q}{((d/2)^2+r^2)}\frac{d/2}{((d/2)^2+r^2)^{1/2}}$$

(E: E_{+q} OR E_{-q})

$$= 2k\frac{q\,d/2}{((d/2)^2+r^2)^{3/2}}$$ → Electric field at point P

Now, assumption $r >> d$ → $((\frac{d}{2})^2+r^2)^{3/2} \to r^3$ ($\frac{d}{2} \to 0$) → $E = k\frac{qd}{r^3} = k\frac{p}{r^3}$

magnitude & $(-\hat{j})$

4. (27) In Figure, two curved plastic rods, one of charge $+q$ and the other of charge $-q$, form a circle of radius $R = 8.50$ cm in an xy-plane. The x axis passes through both of the connecting points, and the charge is distributed uniformly on both rods. If $q = 15.0$ pC, what are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field ***E*** produced at P, the center of the circle?

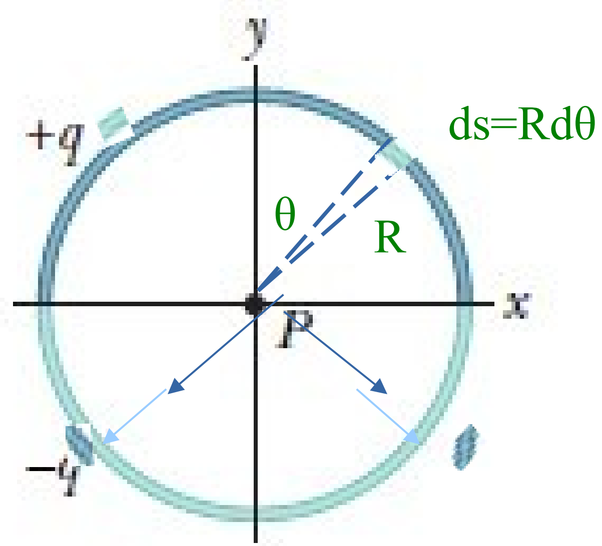


Fig. 22-46 Problem 27.

$net, x, max \quad (4\pi\varepsilon_0)\,(0.1_m + 0.17m)^2 \quad (0.17m)$

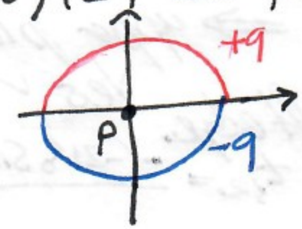
$k = 8.99\times10^{9}\ Nm^2/C^2$

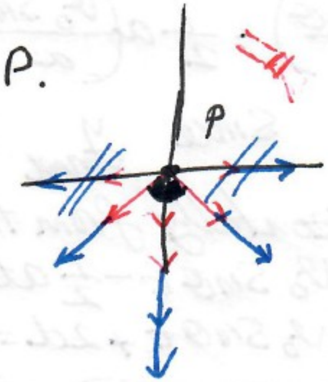
4 (27)

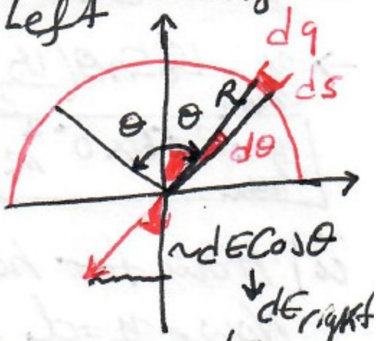
Two curved plastic rods $+q$ & $-q$ } $q = 15.0\,pC$

Radius, $R = 8.50\,cm$

i) $|\vec{E}|$ ~ magnitude at point P.







$$\frac{dq}{ds} = \lambda = \frac{dq}{R\,d\theta}$$

$$dE = k\frac{dq}{R^2} = k\frac{\lambda R\,d\theta}{R^2} \quad \text{; 1st step}$$

$$\vec{E}_{total} = \vec{E} + \vec{E} = 2\vec{E} = 2\left(\vec{E}_{right} + \vec{E}_{left}\right) = 2\left(\int_0^{90} dE\cos\theta + \int_0^{90} dE\cos\theta\right)(-\hat{j})$$

$$= 2\left(\int_0^{90}\frac{k\lambda}{R}d\theta\cos\theta + \int_0^{90}\frac{k\lambda}{R}d\theta\cos\theta\right)(-\hat{j}) = \frac{2k\lambda}{R}\left(\sin\theta\Big|_0^{90} + \sin\theta\Big|_0^{90}\right)(-\hat{j})$$

$$= \frac{2k\lambda}{R}\left((1-0)+(1-0)\right)(-\hat{j}) = 2\times2\times8.99\times10^{9}\,Nm^2/C^2\left(\frac{15.0\times10^{-12}\,C}{\pi\,8.50\times10^{-2}m}\right)\frac{1}{8.50\times10^{-2}m}(-\hat{j})$$

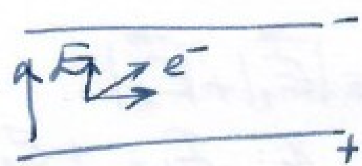
$$\vec{E}_{total\ at\ point\ P} = 23.76\ N/C$$

ii) direction? $(-\hat{j})$ direction

$-90°$ from $+x$-axis

5. (50) At some instant the velocity components of an electron moving between two charged parallel plates are v_x=1.5x10^5 m/s and v_y=3.0x10^3 m/s. Suppose the electric field between the plates is given by ***E***=(120 N/C)**j**. In unit-vector notation, what are (a) the electron's acceleration in that field and (b) the electron's velocity when its x coordinate has changed by 2.0 cm?

an electron
two charged plates
$v_{0x}=2.5\times10^{5}$ m/s
$v_{0y}=5\times10^{3}$ m/s
$\vec{E}=120\,\frac{N}{C}\,\hat{j}$: constant



a) $\vec{F}_e=q\vec{E}=m\vec{a}$
x: no force components
y: downward force since it is electron

$$a=\frac{(-e)E}{m}=\frac{(1.6\times10^{-19}\,C)\,120\,N/C}{9.1\times10^{-31}\,kg}=2.1\times10^{+13}\,m/s^2$$

$$\Rightarrow \vec{a}=2.1\times10^{+13}\,m/s^2\,(-\hat{j})$$

b) $\Delta x=2\times10^{-2}$ m, v_x does not change $\rightarrow \Delta t=\frac{\Delta x}{v_x}=\frac{2\times10^{-2}\,m}{2.5\times10^{5}\,m/s}=8\times10^{-8}\,s$

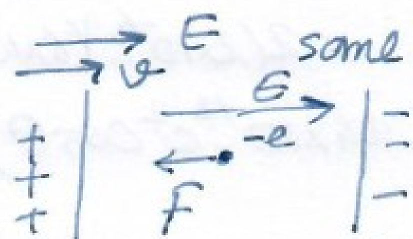
$$\rightarrow v_y=v_{0y}+a_y t=5\times10^{3}\,m/s-(2.1\times10^{13}\,m/s^2)(8\times10^{-8}\,s)$$

$$=-1.675\times10^{6}\,m/s \rightarrow \vec{v}=2.5\times10^{5}\,m/s\,\hat{i}+1.67\times10^{6}\,m/s\,(-\hat{j})$$

$v_{x0}=v_x$ $\qquad$ v_y

6. (52) An electron enters a region of uniform electric field with an initial velocity of 40 km/s in the same direction as the electric field, which has magnitude $E = 50$ N/C. (a) What is the speed of the electron 1.5 ns after entering this region? (b) How far does the electron travel during the 1.5 ns interval?

an electron
uniform $E = 50\,N/C$
$v_0 = 40\,000\,m/s$
$\Delta t = 1.5\,ns$



same direction → e^- will not change direction as deflection but deceleration

a) $qE = ma \rightsquigarrow a = \frac{(-e)E}{m} = \frac{(1.6\times10^{-19})\,50\,N/C}{9.1\times10^{-31}\,kg}$

$\vec{a} = 8.79\times10^{12}\,m/s^2\,(-\hat{i})$

$v = v_{0x} - 8.79\times10^{12}\,m/s^2 \cdot 1.5\times10^{-9}\,s$, with $v_{0x} = 40\times10^{3}\,m/s$

$v = 2.7\times10^{4}\,m/s$

b) uniform E does not change direction } constant acceleration $\rightsquigarrow v_{avg} = \frac{v + v_0}{2}$

$\Delta x = v_{avg}\,\Delta t = \frac{40\times10^{3}\,m/s + 27\times10^{3}\,m/s}{2}(1.5\times10^{-9}\,s)$

OR $x = x_0 + v_0 t + \frac{1}{2}at^2$

$= 5.01\times10^{-5}\,m$

7. (84) In Fig. 22-63, a uniform, upward electric field ***E*** of magnitude 2.00 x103 N/C has been set up between two horizontal plates by charging the lower plate positively and the upper plate negatively. The plates have length L = 10.0 cm and separation d = 2.00 cm. An electron is then shot between the plates from the left edge of the lower plate. The initial velocity $\boldsymbol{V_0}$ of the electron makes an angle θ = 45.0° with the lower plate and has a magnitude of 6.0×10^6 m/s. (a) Will the electron strike one of the plates? (b) If so, which plate and how far horizontally from the left edge will the electron strike?

maximum height → $v_y = 0$

projectile

PATH?

range equation

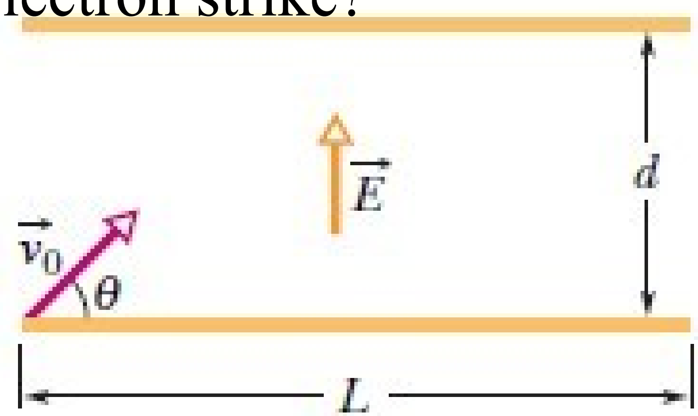


Fig. 22-63 Problem 84.

Uniform $\vec{E}$

$|\vec{E}| = 2.00 \times 10^3$ N/C

Two horizontal plates

$L = 10.0$ cm

$d = 2.00$ cm

$\bar{e}$ shoot (negative charge)

$|\vec{v}_0| = 6.00 \times 10^6$ m/s

$\theta = 45.0°$ with lower plate

$m_e = 9.11 \times 10^{-31}$ kg

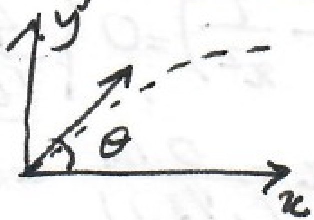
i) will $\bar{e}$ strike one of the plates?

$\uparrow \vec{E} + \bar{e} \Rightarrow \downarrow \vec{F}$ downward $\left\{ \begin{array}{l} |\vec{F}| = eE \\ |\vec{F}| = m_e a \end{array} \right.$

$\Rightarrow a = \dfrac{eE}{m_e} = \dfrac{1.602 \times 10^{-19} C \; 2.00 \times 10^3 N/C}{9.11 \times 10^{-31} kg}$

$a = 3.52 \times 10^{14} m/s^2$

Now a projectile under the influence of a force with acceleration of $3.51 \times 10^{14} m/s^2$



$\downarrow F \quad \downarrow a$

$x = v_{0x} t = v_0 \cos\theta \, t$

$y = v_{0y} t - \frac{1}{2} a t^2$

$v_y = v_{0y} - at$ & $v_x = v_{0x}$

Find y_{max} and compare with d

- less than $d \Rightarrow$ lower plate (if Range < L)
- more than $d \Rightarrow$ upper plate (if Range < L)

$y_{max} \rightarrow v_y = 0$

$v_0 \sin\theta = at$

$t = \dfrac{v_0 \sin\theta}{a}$

$$\rightarrow y_{max} = v_0 \sin\theta \left(\frac{v_0 \sin\theta}{a}\right) - \frac{1}{2} a \left(\frac{v_0 \sin\theta}{a}\right)^2 = \frac{1}{2}\frac{v_0^2 \sin^2\theta}{a} = \frac{1}{2}\frac{(6\times10^6 \, m/s)^2 \sin^2 45}{3.52\times10^{14} \, m/s^2}$$

$$\boxed{y_{max} = 2.56\times10^{-2} \, m}$$

Since $y_{max} > d \Rightarrow$ upper plate Yes ✓

iii) How far horizontally from the left edge.

Now, $y = d = v_0 \sin\theta \, t - \frac{1}{2} a t^2$; $t_{1,2} = \frac{-(-2v_0 \sin\theta) \pm \sqrt{(-2v_0 \sin\theta)^2 - 4a2d}}{2a}$

$\Rightarrow a t^2 - 2 v_0 \sin\theta \, t + 2d = 0$

$$= \frac{v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta - 2ad}}{a}$$

$$= \frac{(6\times10^6 \, m/s) \sin 45 \pm \sqrt{(6\times10^6 \, m/s \, \sin 45)^2 - 2\times 3.52\times10^{14} \, m/s^2 \times 0.02 \, m}}{3.52\times10^{14} \, m/s^2}$$

$= 6.43\times10^{-9} \, s$ & $1.76\times10^{-8} \, s$

$6.43\times10^{-9} < 1.76\times10^{-8}$ (t_1, t_2)

take the first hit time

$x = v_{0x} t = v_0 \cos\theta \, t_1 =$

$= 6\times10^6 \, m/s \cos 45 \times 6.43\times10^{-9} \, s = 2.72\times10^{-2} \, m$ $\boxed{= 2.72 \, cm}$

Definition of Electric Field

- The electric field at any point

$$\vec{E} = \frac{\vec{F}}{q_0}.$$ **Eq. 22-1**

Electric Field Lines

- provide a means for visualizing the directions and the magnitudes of electric fields

Field due to a Point Charge

- The magnitude of the electric field ***E*** set up by a point charge *q* at a distance *r* from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}.$$ **Eq. 22-3**

Field due to an Electric Dipole

- The magnitude of the electric field set up by the dipole at a distant point on the dipole axis is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$ **Eq. 22-9**

Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$ **Eq. 22-26**

Force on a Point Charge in an Electric Field

- When a point charge *q* is placed in an external electric field ***E***

$$\vec{F} = q\vec{E}.$$ **Eq. 22-28**

Dipole in an Electric Field

- The electric field exerts a torque on a dipole

$$\vec{\tau} = \vec{p} \times \vec{E}.$$ **Eq. 22-34**

- The dipole has a potential energy *U* associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}.$$ **Eq. 22-38**