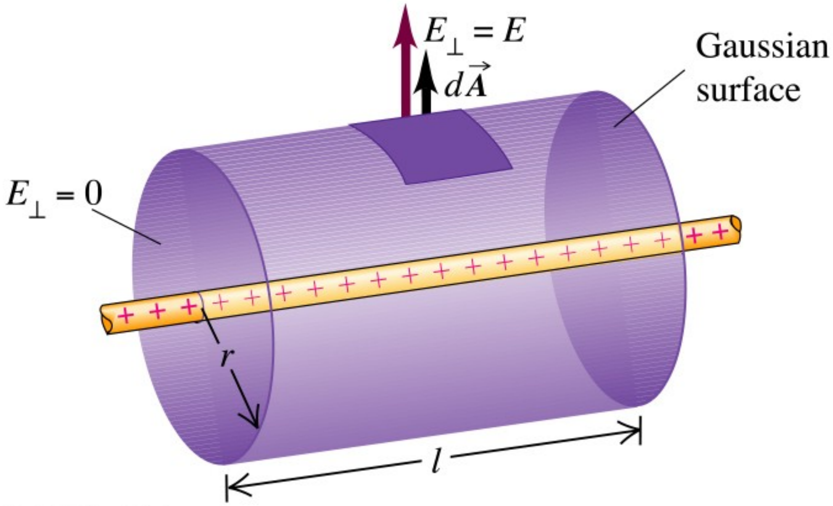
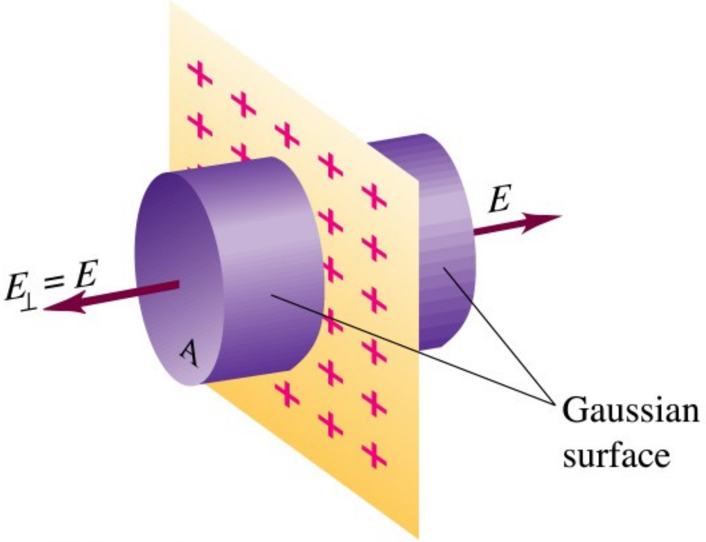
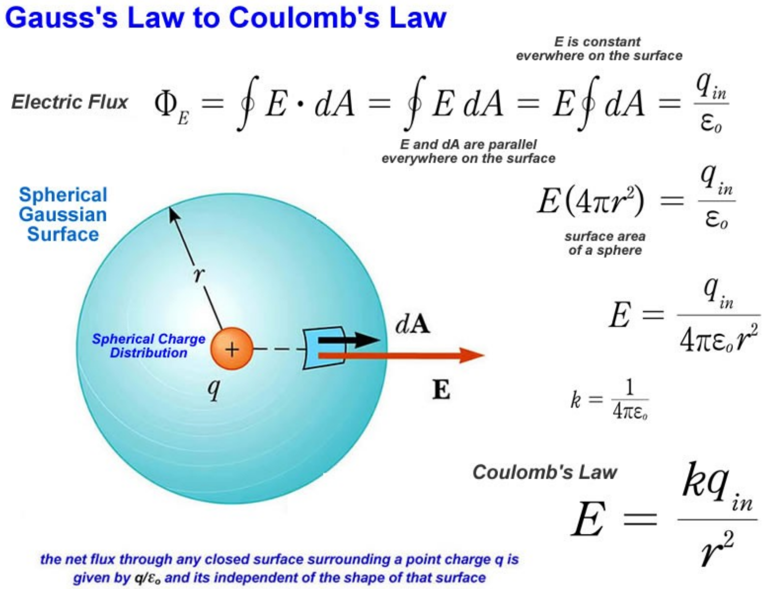




Chapter 23

Gauss' Law



Gauss's Law to Coulomb's Law

E is constant everwhere on the surface

Electric Flux $\Phi_E = \oint E \cdot dA = \oint E\,dA = E\oint dA = \frac{q_{in}}{\varepsilon_o}$

E and dA are parallel everywhere on the surface

$$E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

surface area of a sphere

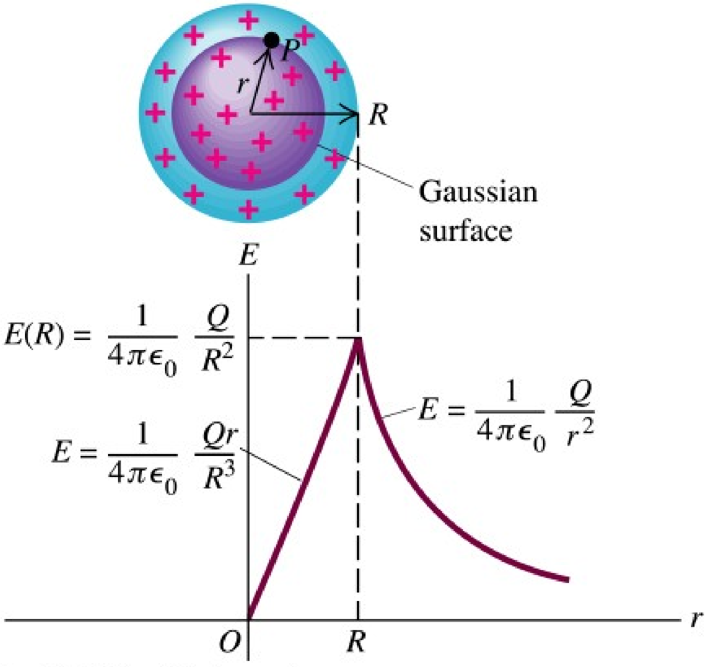
$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2}$$

$$k = \frac{1}{4\pi\varepsilon_o}$$

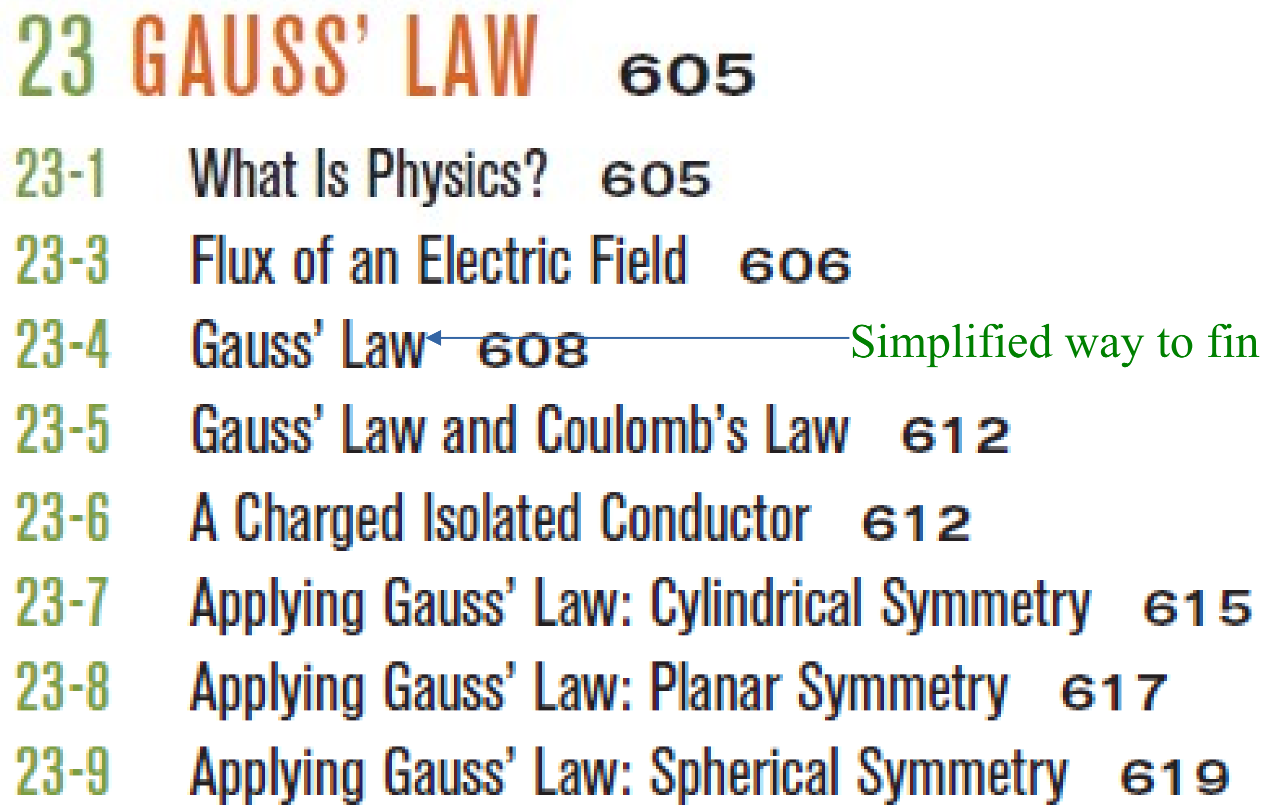
Coulomb's Law

$$E = \frac{kq_{in}}{r^2}$$

the net flux through any closed surface surrounding a point charge q is given by q/ε₀ and its independent of the shape of that surface



23 Gauss’Law



23 GAUSS' LAW 605

Simplified way to find the electric field

23-1 What is Physics?

- Gauss' law relates the electric fields at points on a (**closed**) *Gaussian surface* to the net charge enclosed by that surface.

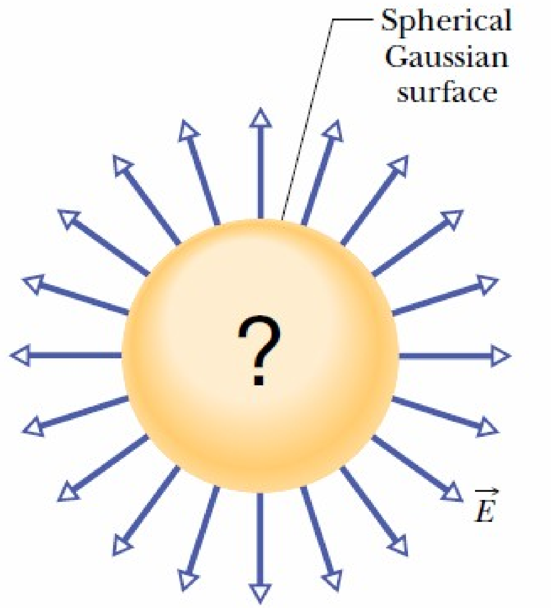


Fig. 23-1 A spherical Gaussian surface. If the electric field vectors are of uniform magnitude and point radially outward at all surface points, you can conclude that a net positive distribution of charge must lie within the surface and have spherical symmetry.

- Gauss' law considers a hypothetical (imaginary) closed surface enclosing the charge distribution.
- This **Gaussian surface,** as it is called, can have any shape, but the shape that minimizes our calculations of the electric field is one that mimics the ***symmetry*** of the charge distribution.
- Electric Field & Force Law Depends on Geometry

23-2 Flux

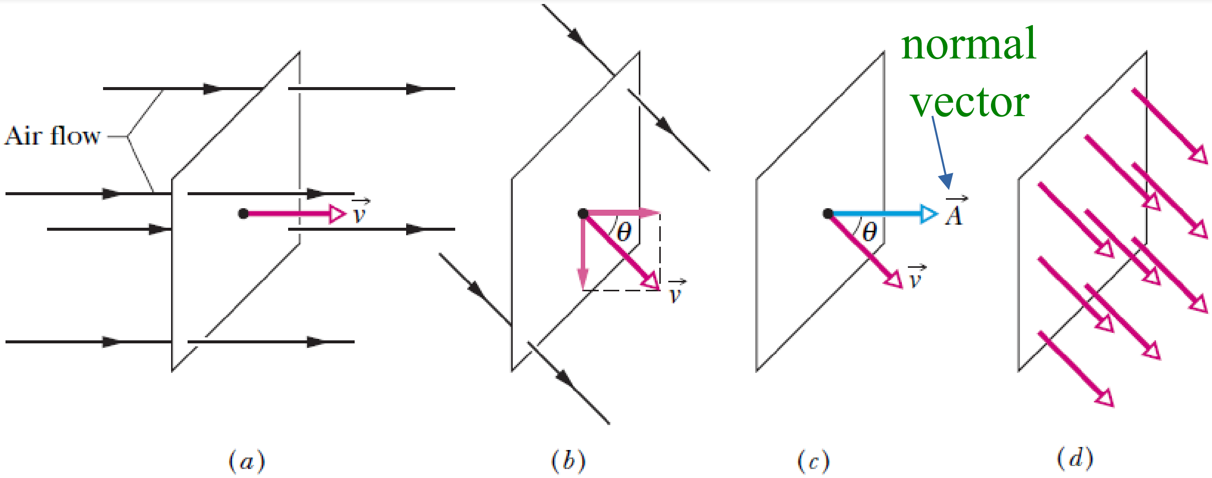


Fig. 23-2 (a) A uniform airstream of velocity ***v*** is perpendicular to the plane of a square loop of area *A*.
(b) The component of ***v*** perpendicular to the plane of the loop is $v\cos\theta$, where θ, is the angle between ***v***
and a normal to the plane.
(c) The area vector ***A*** is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop; that is *A*. Here, ***A*** makes an angle θ, with ***v***.
(d) The velocity field intercepted by the area of the loop.

- The *volume flow rate* (volume per unit time) at which air flows through the loop is $\Phi = (v \cos \theta)A$.
- This rate of flow through an area is an example of a flux- a *volume flux in this situation*- which can be rewritten in terms of vectors as

scalar

$$\Phi = vA\cos\theta = \vec{v}\cdot\vec{A},$$

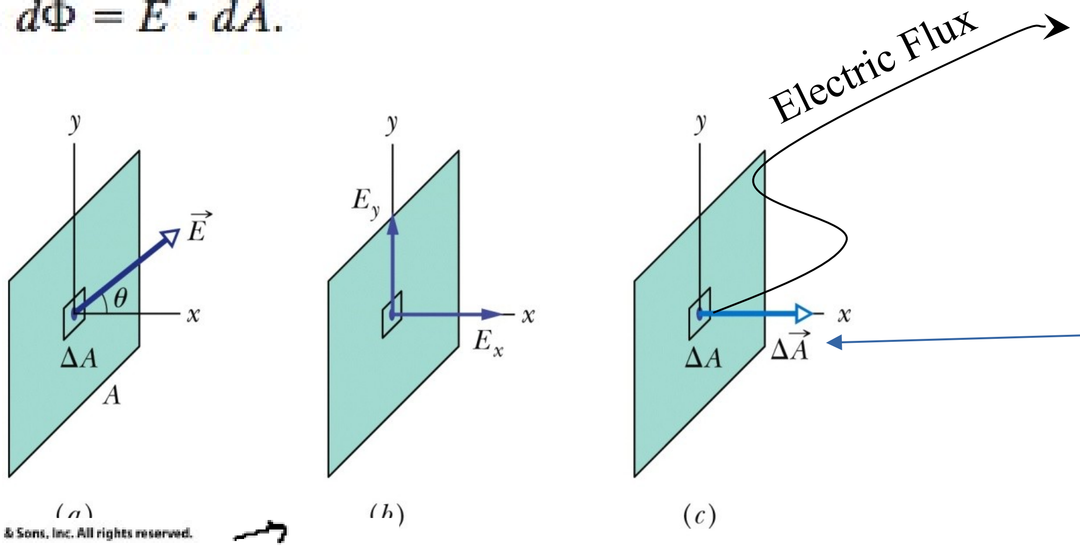
Air Flow Analogy

- This equation can also be interpreted as the *flux of the velocity field through the loop*. With this interpretation, flux is no longer means the actual flow of something through an area - rather it means the **product of an area and the field across that area**.

23-3 Electric Flux

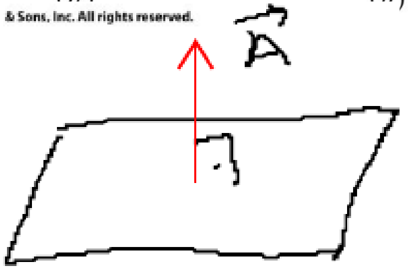
- The **area vector** *dA* for an area element (patch element) on a surface is a vector that is perpendicular to the element and has a magnitude equal to the area *dA* of the element.
- The **electric flux** *d*ϕ through a patch element with area vector ***dA*** is given by a dot product:

$$d\Phi = \vec{E} \cdot d\vec{A}.$$



$$\Delta\Phi = (E \cos\theta)\, \Delta A.$$

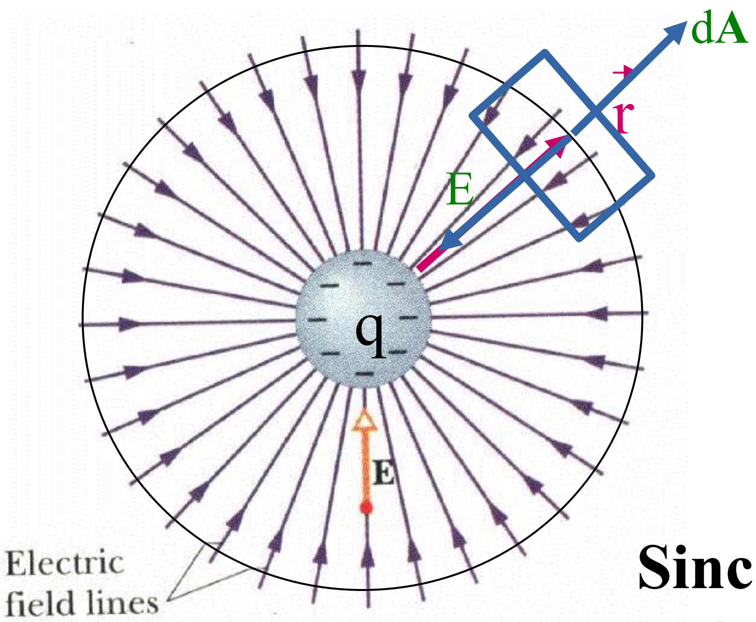
(a) An electric field vector pierces a small square patch on a flat surface.
(b) Only the *x* component actually pierces the patch; the *y* component skims across it.
(c) The area vector of the patch is perpendicular to the patch, with a magnitude equal to the patch's area.



$$\Phi = \vec{E} \cdot \vec{A}$$

what is the source of the **E**?

23-3 Electric Flux



$$d\vec{A} = +(dA)\hat{r}$$

(dA: outward!)

$$\vec{\mathbf{E}} = -\frac{kq}{r^2}\hat{\mathbf{r}}$$

(E: inward!)

$$\vec{E}\cdot d\vec{A} = EdAcos(180^\circ) = -EdA$$

antiparallel

Since r is Constant on the Sphere - Remove E Outside the Integral!

closed surface

$$\Phi = \oint \vec{E}\cdot d\vec{A} = - E\underbrace{\oint dA}_{\text{total area}} = \left(-\frac{kq}{\cancel{r}^2}\right)\underline{(4\cancel{\pi r}^2)}$$

Sphere Surface Area

$$= -\frac{q}{\cancel{4\pi}\epsilon_0}(\cancel{4}\pi) = -q/\epsilon_0$$

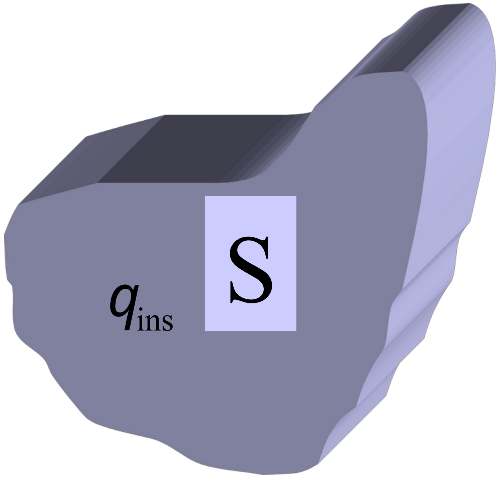
Gauss' Law: Special Case!

$$\Phi = \frac{q_{enc}}{\epsilon_0} = \oint \vec{E}\cdot d\vec{A}$$

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

Gauss' Law: General Case

- Consider any ARBITRARY CLOSED surface S -- NOTE: this does NOT have to be a "real" physical object!
- The TOTAL ELECTRIC FLUX through S is proportional to the TOTAL CHARGE ENCLOSED!
- The results of a complicated integral is a very simple formula: it avoids long calculations!



simplifications

$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{ins}}}{\varepsilon_0}$$

(One of Maxwell's 4 equations!)

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

23-3 Electric Flux

Electric flux through a Gaussian surface is proportional to the *net number of electric field lines* passing through that surface.

- For small ΔA, ***E*** can be taken as constant over ΔA. Thus, the flux of electric field for the Gaussian surface of Fig. 23-3 is

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}.$$

- Now we can find the **total flux** by integrating the dot product over the full surface. The **total flux** through a surface is given by

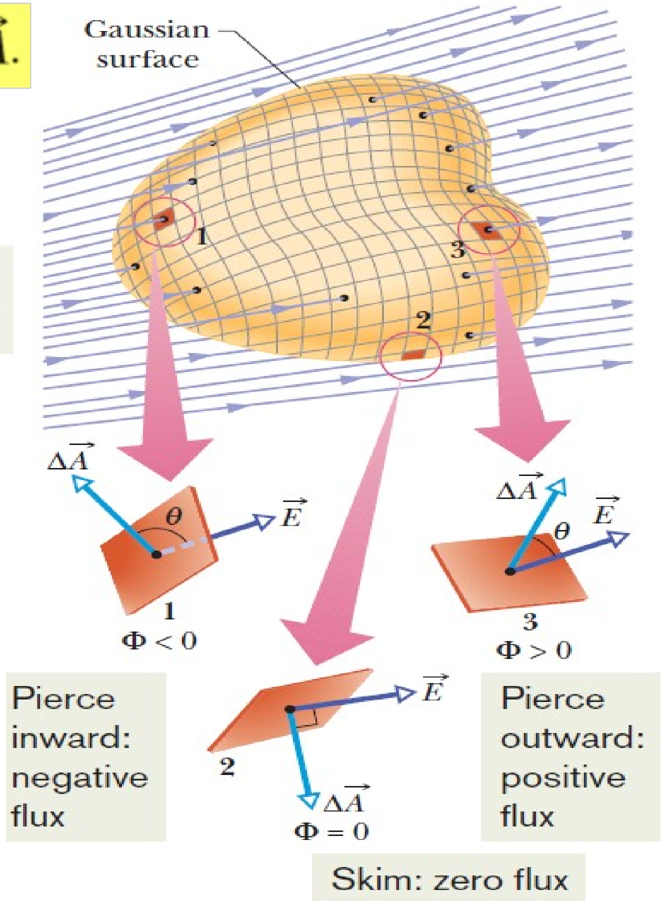
$$\Phi = \int \vec{E} \cdot d\vec{A} \qquad \text{(total flux).}$$

- The **net flux** through a closed surface (which is used in Gauss' law) is given by

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface).}$$

The loop sign indicates that the integration is to be taken over the entire (closed) surface. The SI unit for Φ is (N.m^2/C).

Fig. 23-3 A Gaussian surface of arbitrary shape immersed in an electric field. The surface is divided into small squares of area Δ *A*. The electric field vectors ***E*** and the area vectors Δ***A*** for three representative squares, marked 1, 2, and 3, are shown.



Pierce inward: negative flux

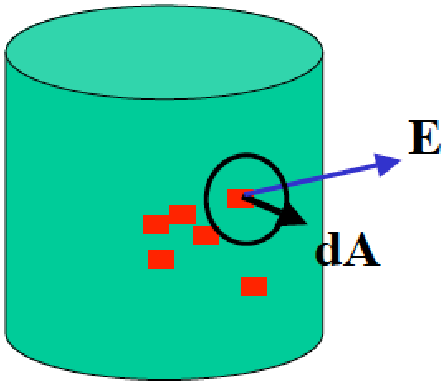
Pierce outward: positive flux

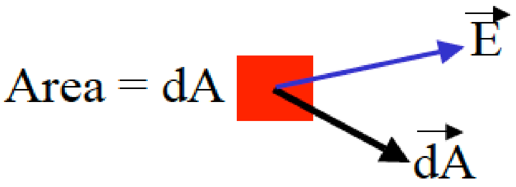
Skim: zero flux

Electric Flux: General Surface

- For any general surface: break up into infinitesimal planar patches
- Electric Flux $\Phi = \int \vec{E} \bullet d\vec{A}$
- Surface integral
- $d\vec{A}$ is a vector normal to each patch and has a magnitude = $|d\mathbf{A}| = dA$
- CLOSED surfaces:
  - define the vector dA as pointing OUTWARDS
  - Inward E gives negative flux Φ
  - Outward E gives positive flux Φ

as the sign of enclosed charge





http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

23-3 Electric Flux

Example, Flux through a closed cylinder, uniform field:

Figure 23-4 shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field $\vec{E}$, with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?

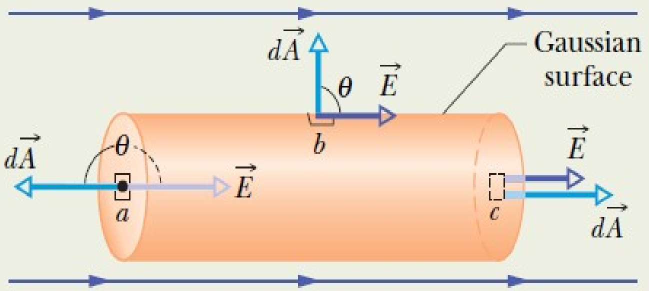


Fig. 23-4 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

KEY IDEA

We can find the flux Φ through the Gaussian surface by integrating the scalar product $\vec{E}\cdot d\vec{A}$ over that surface.

Calculations: We can do the integration by writing the flux as the sum of three terms: integrals over the left cylinder cap a, the cylindrical surface b, and the right cap c. Thus, from Eq. 23-4,

$$\Phi = \oint \vec{E}\cdot d\vec{A}$$

$$= \int_a \vec{E}\cdot d\vec{A} + \int_b \vec{E}\cdot d\vec{A} + \int_c \vec{E}\cdot d\vec{A}. \qquad (23\text{-}5)$$

For all points on the left cap, the angle θ between $\vec{E}$ and $d\vec{A}$ is 180° and the magnitude E of the field is uniform. Thus,

$$\int_a \vec{E}\cdot d\vec{A} = \int E(\cos 180°)\, dA = -E \int dA = -EA,$$

left cap

where $\int dA$ gives the cap's area $A\ (= \pi R^2)$. Similarly, for the right cap, where $\theta = 0$ for all points,

right cap

$$\int_c \vec{E}\cdot d\vec{A} = \int E(\cos 0)\, dA = EA.$$

Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E}\cdot d\vec{A} = \int E(\cos 90°)\, dA = 0.$$

in=out

Net flux

Substituting these results into Eq. 23-5 leads us to

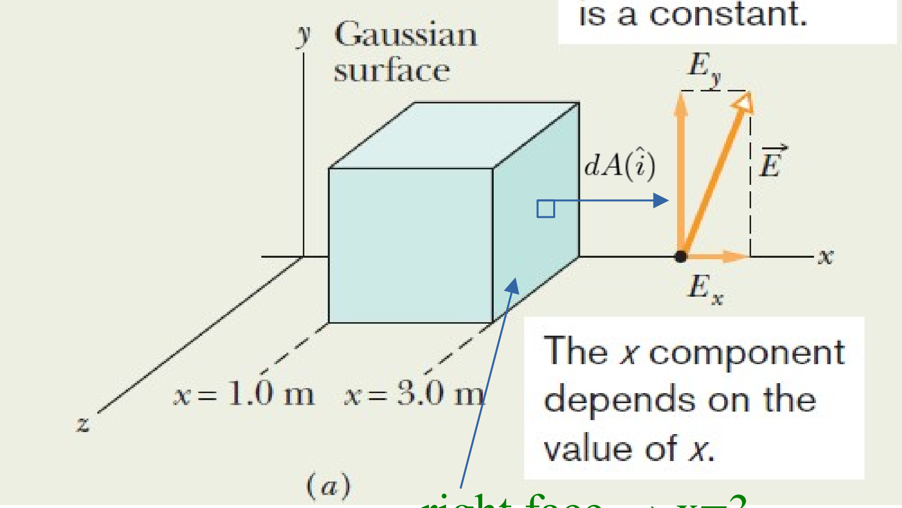
$$\Phi = -EA + 0 + EA = 0. \qquad \text{(Answer)}$$

The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

23-3 Electric Flux

Example, Flux through a closed cube, nonuniform field: x dependence

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}}$ pierces the Gaussian cube shown in Fig. 23-5*a*. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)



The y component is a constant.

The x component depends on the value of x.

right face → x=3

Right face: An area vector ***A*** is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector ***dA*** for any area element on the right face of the cube must point in the positive direction of the x axis. The most convenient way to express the vector is in unit-vector notation,

$$d\vec{A} = dA\hat{\mathrm{i}}.$$

$$\Phi_r = \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}}) \cdot (dA\hat{\mathrm{i}})$$

$$= \int [(3.0x)(dA)\hat{\mathrm{i}} \cdot \hat{\mathrm{i}} + (4.0)(dA)\hat{\mathrm{j}} \cdot \hat{\mathrm{i}}]$$

$$= \int (3.0x\, dA + 0) = 3.0 \int x\, \underbrace{dA}_{dydz}.$$

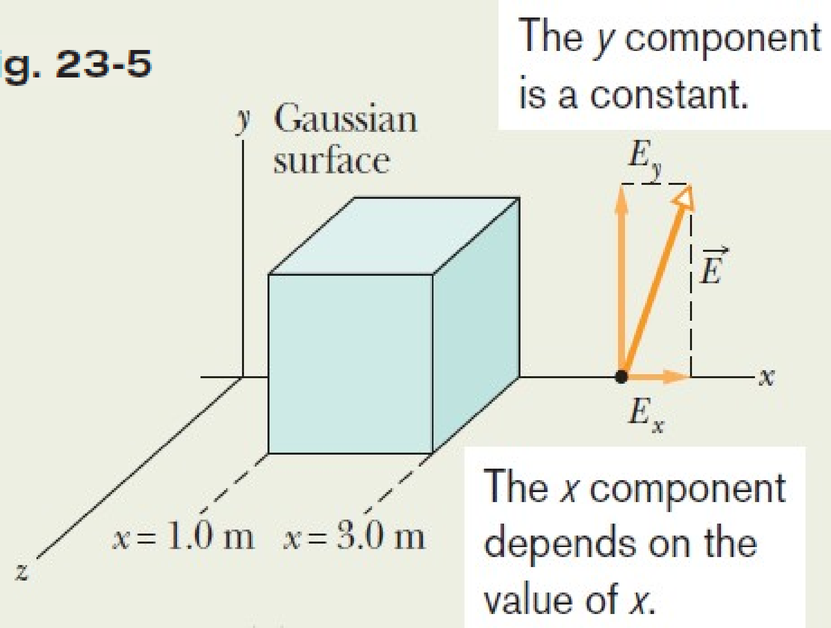
Although x is certainly a variable as we move left to right across the figure, because the right face is perpendicular to the x axis, every point on the face has the same x coordinate. (The y and z coordinates do not matter in our integral.) Thus, we have

$$\Phi_r = 3.0 \int (3.0)\, dA = 9.0 \underbrace{\int dA.}_{\text{total area}} = (9.0\ \mathrm{N/C})(4.0\ \mathrm{m^2}) = 36\ \mathrm{N \cdot m^2/C}. \qquad \text{(Answer)}$$

23-3 Electric Flux

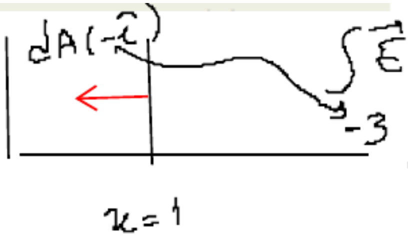
A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-5*a*. (*E* is in newtons per coulomb and *x* is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)

Fig. 23-5



The *y* component is a constant.

The *x* component depends on the value of *x*.



$$\int \vec{E}\cdot\vec{dA} = -3\int x\, dA$$

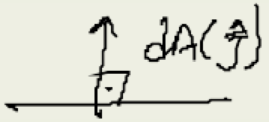
$$\int_1 (-3)(4) = \Phi_{left}$$

area

Left face: The procedure for finding the flux through the left face is the same as that for the right face. However, two factors change. (1) The differential area vector $d\vec{A}$ points in the negative direction of the *x* axis, and thus $d\vec{A} = -dA\hat{i}$ (Fig. 23-5*d*). (2) The term *x* again appears in our integration, and it is again constant over the face being considered. However, on the left face, $x = 1.0$ m. With these two changes, we find that the flux Φ_l through the left face is

$$\Phi_l = -12\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

Top face: The differential area vector $d\vec{A}$ points in the positive direction of the *y* axis, and thus $d\vec{A} = dA\hat{j}$ (Fig. 23-5*e*). The flux Φ_t through the top face is then



$$\Phi_t = \int (3.0x\hat{i} + 4.0\hat{j})\cdot(dA\hat{j})$$

$$= \int [(3.0x)(dA)\hat{i}\underset{0}{\cancel{\cdot}}\hat{j} + (4.0)(dA)\hat{j}\underset{1}{\cancel{\cdot}}\hat{j}]$$

$$= \int (0 + 4.0\, dA) = 4.0\int dA \xrightarrow{} \int_1^3 dx \int_0^2 dz$$

$$= 16\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

$$|\Phi_{top}| = |\Phi_{bottom}|$$

23-4 Gauss' Law

Gauss's law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the *net* charge q_{enc} that is *enclosed* by that surface.

$$\varepsilon_0 \Phi = q_{\text{enc}} \qquad \text{(Gauss' law).}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \qquad \text{(Gauss' law).}$$

- The net charge q_{enc} is the algebraic sum of all the *enclosed* positive and negative charges, and it can be positive, negative, or zero.
- If q_{enc} is positive, the net flux is *outward*; if q_{enc} is negative, the net flux is *inward*.

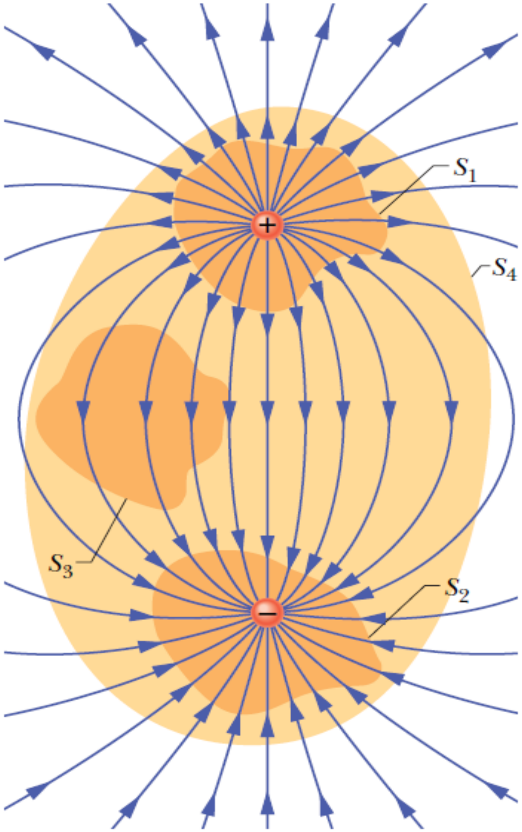


Fig. 23-6 Two point charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section. Surface S_1 encloses the positive charge. Surface S_2 encloses the negative charge. Surface S_3 encloses no charge. Surface S_4 encloses both charges and thus no net charge.

23-4 Gauss' Law

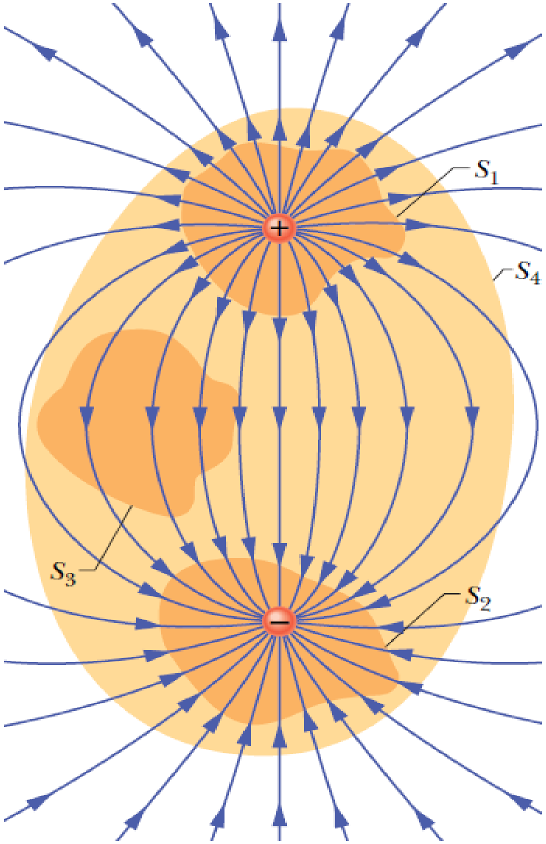
Surface S1.The electric field is outward for all points on this surface. Thus, the flux of the electric field through this surface is **positive**, and so is the net charge within the surface, as Gauss' law requires.

Surface S2.The electric field is inward for all points on this surface. Thus, the flux of the electric field through this surface is **negative** and so is the enclosed charge, as Gauss' law requires.

Surface S3.This surface encloses no charge, and thus q_{enc} = 0. Gauss' law requires that the net flux of the electric field through this surface be **zero**.

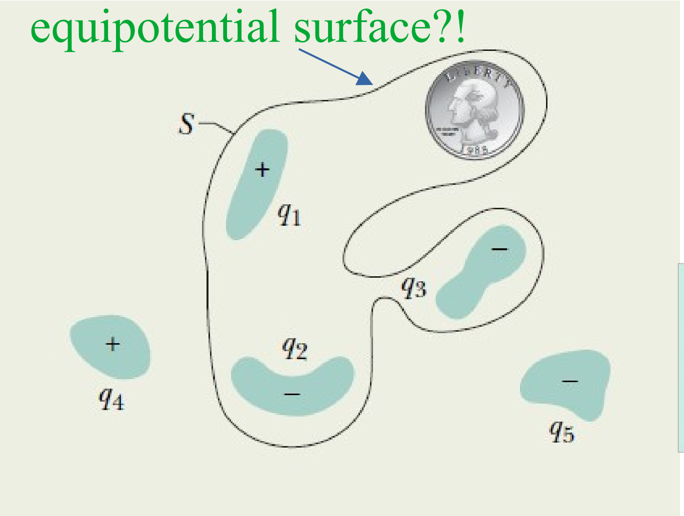
Surface S4.This surface encloses no net charge, because the enclosed positive and negative charges have equal magnitudes. Gauss' law requires that the net flux of the electric field through this surface be **zero**. That is reasonable because *there are as many field lines leaving surface S4 as entering it*.

In = Out



Two charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section.

23-4 Gauss' Law



Example, Relating the net enclosed charge and the net flux:

$$\Phi = \frac{q_{enc}}{\epsilon_0}$$

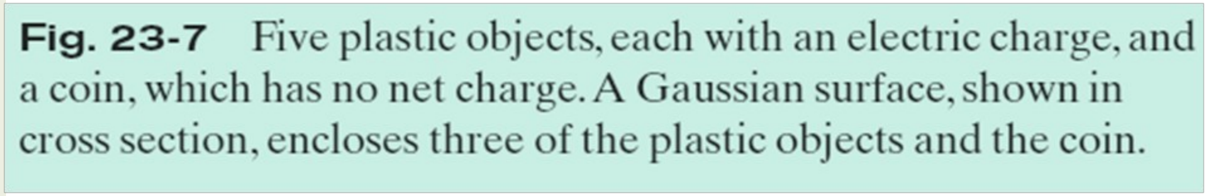


Fig. 23-7 Five plastic objects, each with an electric charge, and a coin, which has no net charge. A Gaussian surface, shown in cross section, encloses three of the plastic objects and the coin.

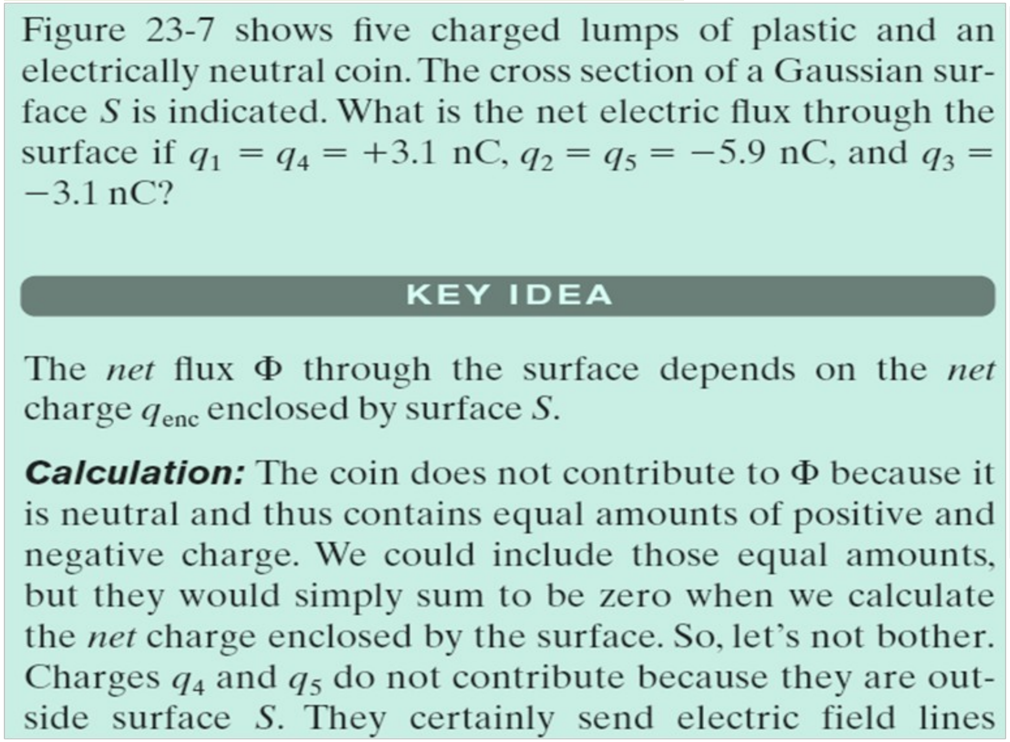
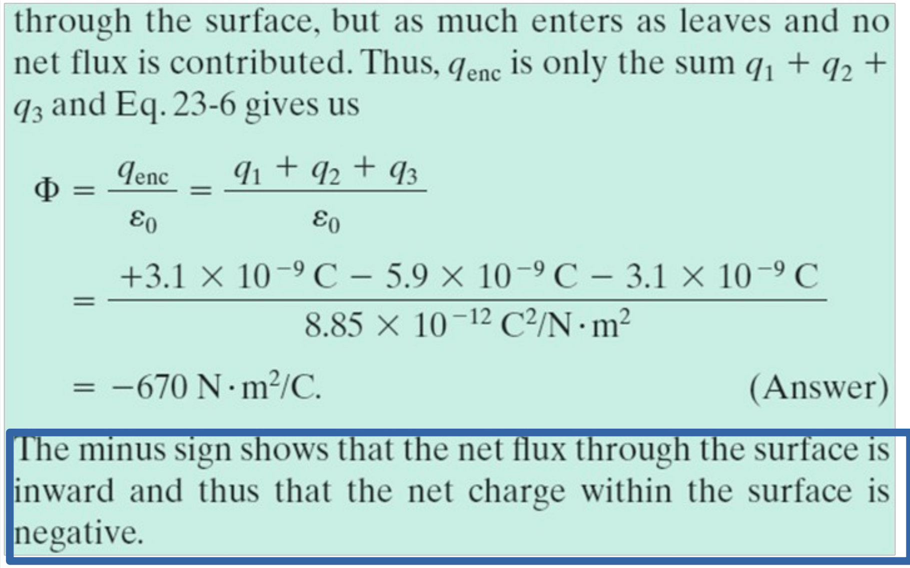


Figure 23-7 shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1$ nC, $q_2 = q_5 = -5.9$ nC, and $q_3 = -3.1$ nC?

KEY IDEA

The *net* flux Φ through the surface depends on the *net* charge q_{enc} enclosed by surface S.

Calculation: The coin does not contribute to Φ because it is neutral and thus contains equal amounts of positive and negative charge. We could include those equal amounts, but they would simply sum to be zero when we calculate the *net* charge enclosed by the surface. So, let's not bother. Charges q_4 and q_5 do not contribute because they are outside surface S. They certainly send electric field lines through the surface, but as much enters as leaves and no net flux is contributed. Thus, q_{enc} is only the sum $q_1 + q_2 + q_3$ and Eq. 23-6 gives us



$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{+3.1 \times 10^{-9}\,\text{C} - 5.9 \times 10^{-9}\,\text{C} - 3.1 \times 10^{-9}\,\text{C}}{8.85 \times 10^{-12}\,\text{C}^2/\text{N} \cdot \text{m}^2}$$

$$= -670\ \text{N} \cdot \text{m}^2/\text{C}. \qquad \text{(Answer)}$$

The minus sign shows that the net flux through the surface is inward and thus that the net charge within the surface is negative.

negative flux

23-4 Gauss' Law

Example, Enclosed charge in a nonuniform field:

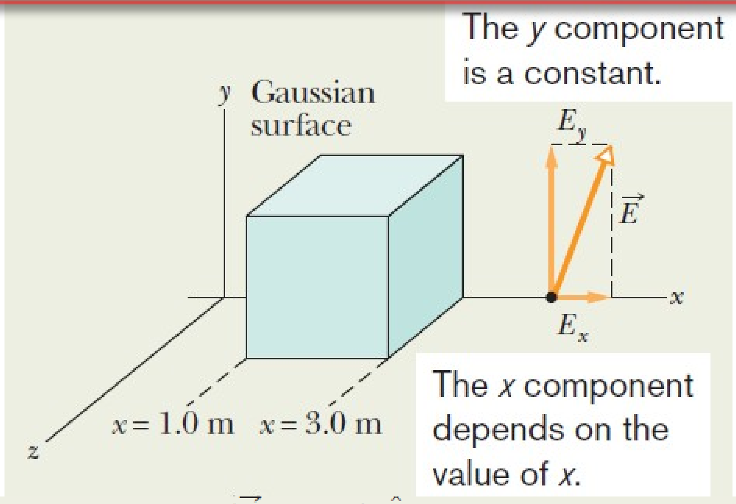
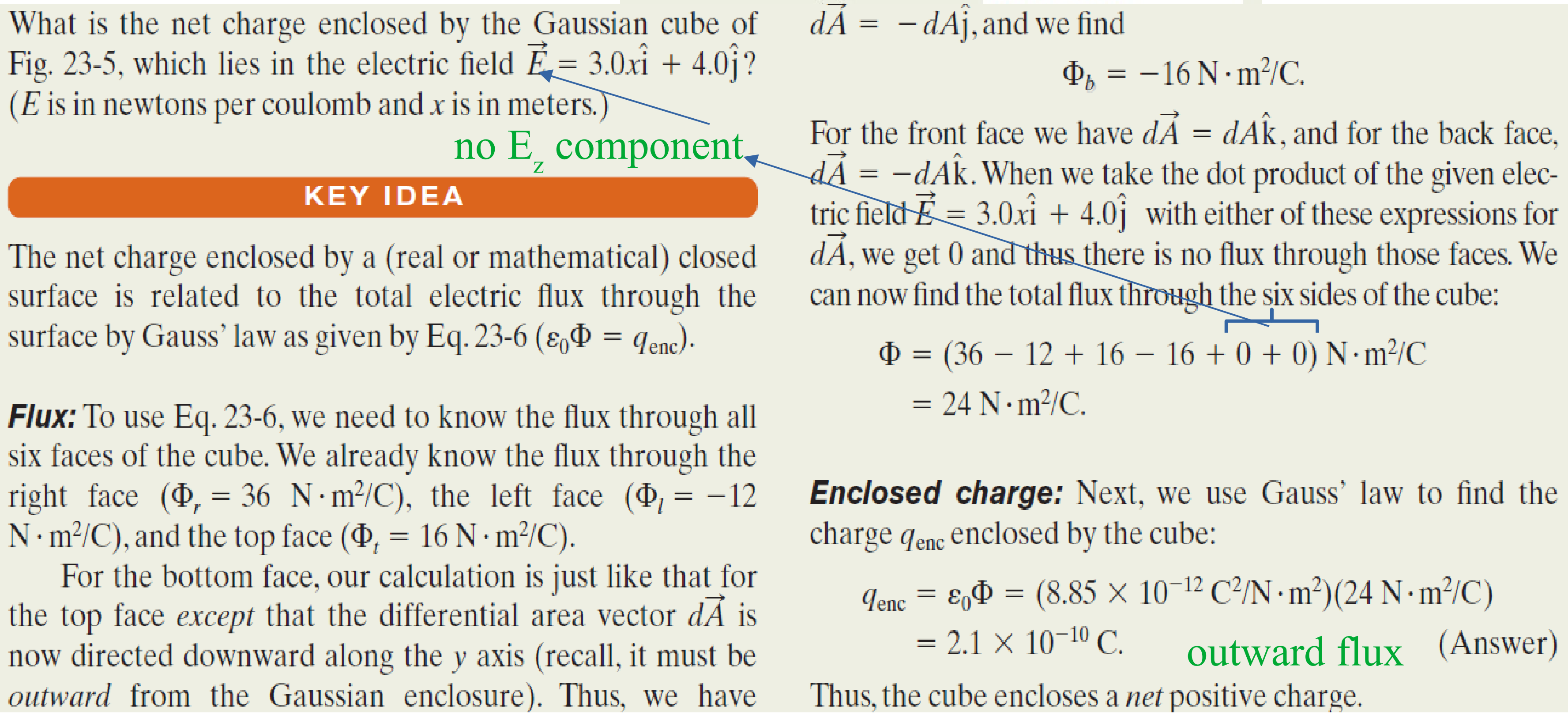


Fig. 23-5



What is the net charge enclosed by the Gaussian cube of Fig. 23-5, which lies in the electric field $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$? ($E$ is in newtons per coulomb and x is in meters.)

no E_z component

KEY IDEA

The net charge enclosed by a (real or mathematical) closed surface is related to the total electric flux through the surface by Gauss' law as given by Eq. 23-6 ($\varepsilon_0\Phi = q_{enc}$).

Flux: To use Eq. 23-6, we need to know the flux through all six faces of the cube. We already know the flux through the right face ($\Phi_r = 36\ \text{N}\cdot\text{m}^2/\text{C}$), the left face ($\Phi_l = -12\ \text{N}\cdot\text{m}^2/\text{C}$), and the top face ($\Phi_t = 16\ \text{N}\cdot\text{m}^2/\text{C}$).

For the bottom face, our calculation is just like that for the top face *except* that the differential area vector $d\vec{A}$ is now directed downward along the y axis (recall, it must be *outward* from the Gaussian enclosure). Thus, we have $d\vec{A} = -dA\hat{j}$, and we find

$$\Phi_b = -16\ \text{N}\cdot\text{m}^2/\text{C}.$$

For the front face we have $d\vec{A} = dA\hat{k}$, and for the back face, $d\vec{A} = -dA\hat{k}$. When we take the dot product of the given electric field $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ with either of these expressions for $d\vec{A}$, we get 0 and thus there is no flux through those faces. We can now find the total flux through the six sides of the cube:

$$\Phi = (36 - 12 + 16 - 16 + \underbrace{0 + 0}_{})\ \text{N}\cdot\text{m}^2/\text{C}$$
$$= 24\ \text{N}\cdot\text{m}^2/\text{C}.$$

Enclosed charge: Next, we use Gauss' law to find the charge q_{enc} enclosed by the cube:

$$q_{enc} = \varepsilon_0\Phi = (8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(24\ \text{N}\cdot\text{m}^2/\text{C})$$
$$= 2.1 \times 10^{-10}\ \text{C}. \qquad \text{(Answer)}$$

outward flux

Thus, the cube encloses a *net* positive charge.

23-5 Gauss' Law and Coulomb's Law

- Figure 23-8 shows a positive point charge *q*, around which a concentric spherical Gaussian surface of radius *r* is drawn.

1. Divide this surface into differential areas *dA.*
2. The area vector ***dA*** at any point is perpendicular to the surface and directed outward from the interior.
3. From the *symmetry* of the situation, at any point the electric field, ***E***, is also perpendicular to the surface and directed outward from the interior.
4. Thus, since the angle θ between ***E*** and ***dA*** is zero, we can rewrite Gauss's law as

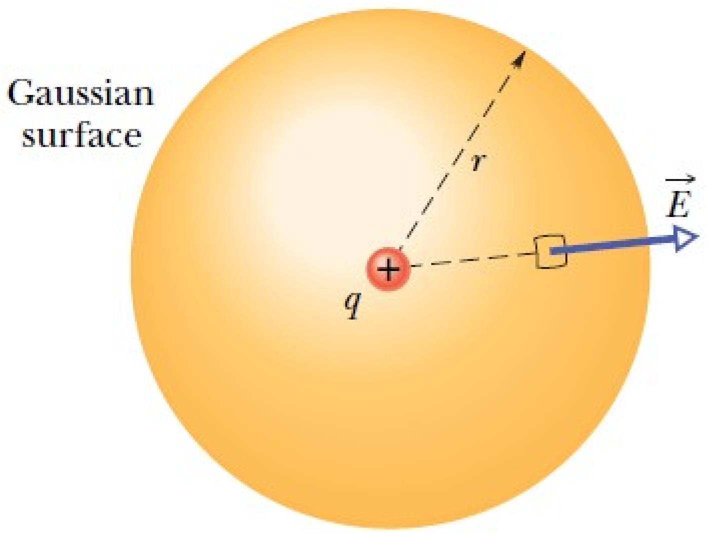


Fig. 23-8 A spherical Gaussian surface centered on a point charge *q*.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}.$$

$$\varepsilon_0 E \underbrace{\oint dA}_{\text{total area of sphere surface}} = q. \qquad \varepsilon_0 E(4\pi r^2) = q$$

radius of Gaussian surface

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

This is exactly what Coulomb's law yielded.

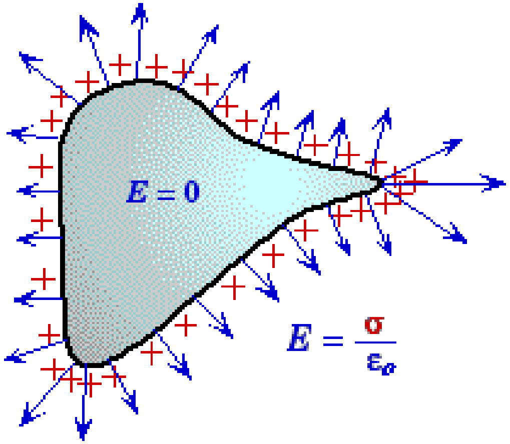
23-6 A Charged Isolated Conductor

- If an **excess charge** is placed on an *isolated* conductor, that amount of charge will move entirely to the surface of the conductor.

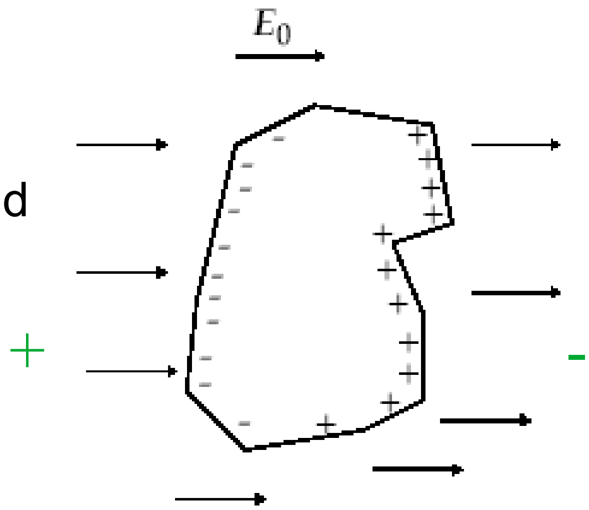
as much as far away

1. **Inside a Conductor** in electrostatic equilibrium, the electric field Is ZERO. Why?
    - Because if the field is not zero, then charges inside the conductor would be moving.
    - Charges in a conductor redistribute themselves wherever they are needed to make the field inside the conductor ZERO. $E_{inside}=0$ (metals)
2. **On the surface of conductors** in electrostatic equilibrium, the electric field is always perpendicular to the surface. Why?

$$\vec{E}||\vec{A}$$



- Because if not, charges on the surface of the conductors would move with the electric field.



http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

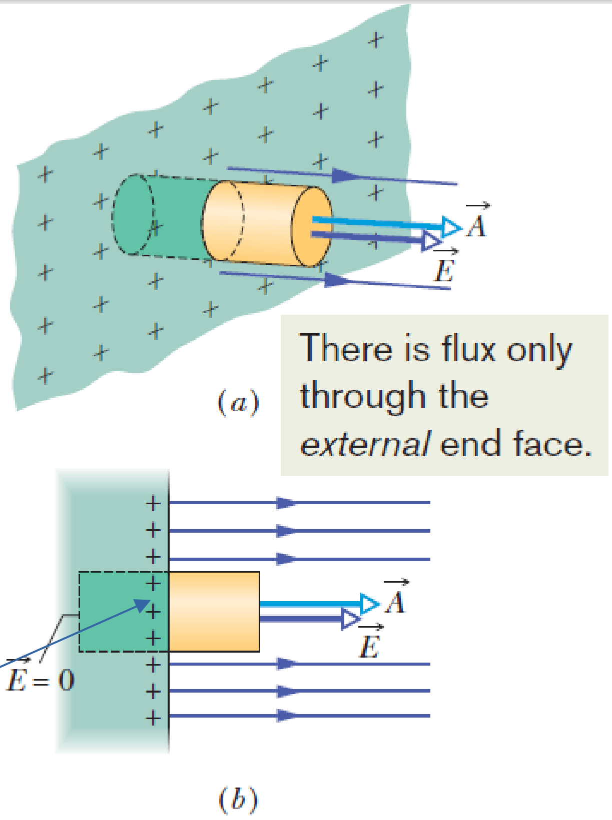
23-6 A Charged Isolated Conductor

3. **<u>Just outside the surface of a conductor</u>**, the electric field is easy to determine using Gauss's law.
   - A tiny cylindrical Gaussian surface is embedded in the section as in Fig. 23-10.
   - We assume that the cap area *A* is small enough that the field magnitude *E* is constant over the cap. Then, the flux through the cap is *EA*, and that is the net flux Φ through the Gaussian surface.
   - The charge q_{enc} enclosed by the Gaussian surface lies on the conductor's surface in an area *A*.
   - If σ is the charge per unit area, then q_{enc} is equal to σA. Then, Gauss's law becomes

$$\varepsilon_0 E\cancel{A} = \sigma\cancel{A},$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface).}$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad \text{(nonconducting surface)}$$



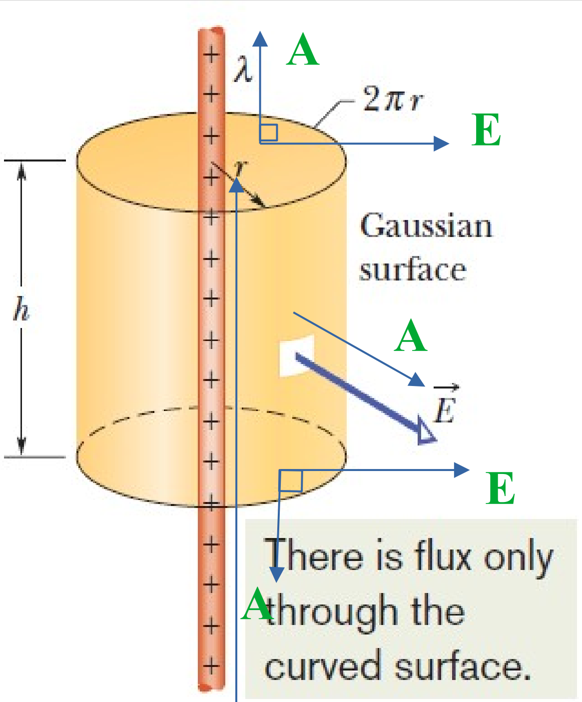
There is flux only through the *external* end face.

Fig. 23-10 (*a*) Perspective view and (*b*) side view of a tiny portion of a large, isolated conductor with excess positive charge on its surface. A (closed) cylindrical Gaussian surface, embedded perpendicularly in the conductor, encloses some of the charge. Electric field lines pierce the external end cap of the cylinder, but not the internal end cap. The external end cap has area *A* and area vector $\vec{A}$.

23-7 Applying Gauss'Law: Cylindrical Symmetry

- Figure 23-12 shows a section of an infinitely long cylindrical plastic rod with a uniform positive linear charge density λ.
  - Let us find an expression for the magnitude of the electric field ***E*** at a distance *r* from the axis of the rod.
  - Gaussian surface should match the *symmetry* of the problem, which is **cylindrical**.
  - Choose a cylinder of radius *r* and length *h*, coaxial with the rod. Because the Gaussian surface must be **closed**, include two end caps as part of the surface.
  - At every point on the cylindrical part of the Gaussian surface, ***E*** must have the same magnitude *E* and Φ must be directed radially outward (for a positively charged rod).
  - The flux of ***E*** through this cylindrical surface is

$$\Phi = EA\cos\theta = E(2\pi rh)\cos 0 = E(2\pi rh).$$



There is flux only through the curved surface.

Fig. 23-12 A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

$$\lambda = \frac{Q}{L} = \frac{q_{enc}}{h}$$

- Then, applying the Gauss's law:

$$\varepsilon_0\Phi = q_{\text{enc}},$$

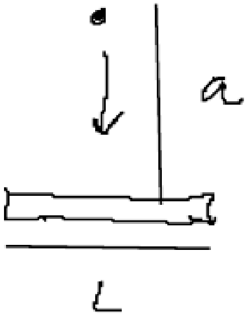
$$\varepsilon_0 E(2\pi r\cancel{h}) = \lambda\cancel{h},$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(line of charge).}$$

- Compare with the solution at Chapter 22!!

E Due to a Line of Charge: Field on bisector

$$E_y = k\lambda\, a \int_{-L/2}^{L/2} \frac{dx}{\left(a^2+x^2\right)^{3/2}} \qquad = k\lambda\, a\left[\frac{x}{a^2\sqrt{x^2+a^2}}\right]_{-L/2}^{L/2}$$



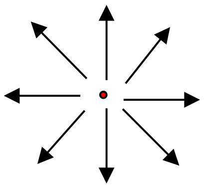
Integrate: Trig Substitution!

$$= \frac{2k\lambda L}{a\sqrt{4a^2+L^2}}$$

away from the rod: point charge

Point Charge Limit: L << a

$$E_y = \frac{2k\lambda L}{a\sqrt{4a^2+\cancel{L^2}}} \approx \frac{k\boxed{\lambda L}}{a^2} = \frac{k\boxed{q}}{a^2}$$

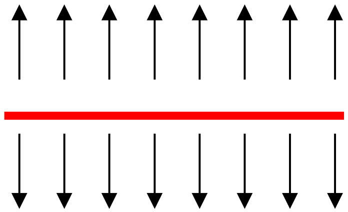


Coulomb's Law!

near by the rod: infinite rod

Line Charge Limit: L >> a

$$E_y = \frac{2k\lambda L}{a\sqrt{\cancel{4a^2}+L^2}} \approx \frac{2k\lambda}{a}$$



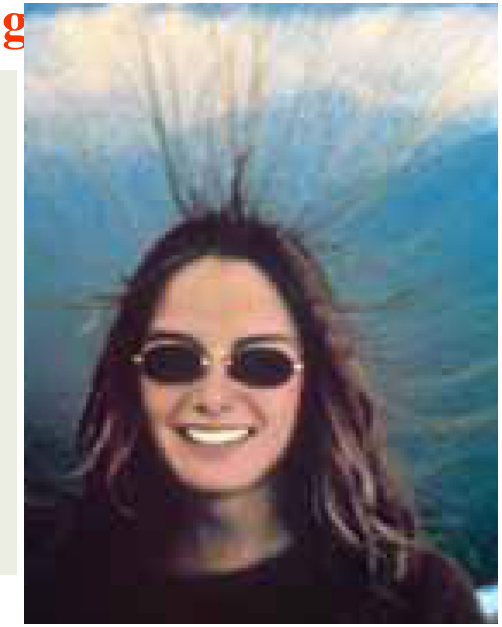
Units Check!

$$\left[\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\,\frac{1}{\mathrm{m}}\,\frac{\mathrm{C}}{\mathrm{m}}\right] = \left[\frac{\mathrm{N}}{\mathrm{C}}\right]$$

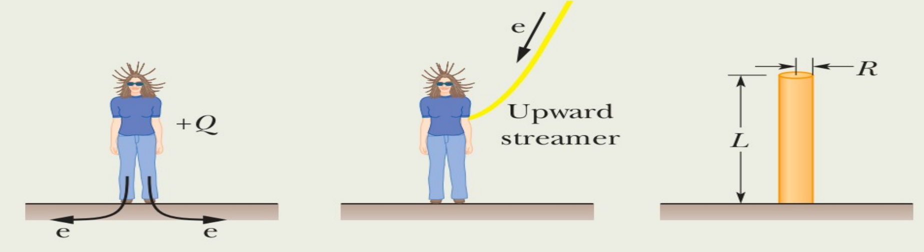
http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

23-7 Applying Gauss'Law: Cylindrical Symmetry

Example, Gauss's Law and an upward streamer in a lightning



Lightning did not strike the woman, but she was in extreme danger because that electric field was on the verge of causing electrical breakdown in the surrounding air. Such a breakdown would have occurred along a path extending away from her in what is called an *upward streamer.* An upward streamer is dangerous because the resulting ionization of molecules in the air suddenly frees a tremendous number of electrons from those molecules.



Let's model her body as a narrow vertical cylinder of height $L = 1.8$ m and radius $R = 0.10$ m (Fig. 23-14*c*). Assume that charge Q was uniformly distributed along the cylinder and that electrical breakdown would have occurred if the electric field magnitude along her body had exceeded the critical value $E_c = 2.4$ MN/C. What value of Q would have put the air along her body on the verge of breakdown?

Calculations: Substituting the critical value E_c for E, the cylinder radius R for radial distance r, and the ratio Q/L for linear charge density λ, we have

$$E_c = \frac{Q/L}{2\pi\varepsilon_0 R},$$

or

$$Q = 2\pi\varepsilon_0 RLE_c.$$

Substituting given data then gives us

$$\begin{aligned} Q &= (2\pi)(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.10\ \mathrm{m}) \\ &\quad \times (1.8\ \mathrm{m})(2.4 \times 10^6\ \mathrm{N/C}) \\ &= 2.402 \times 10^{-5}\ \mathrm{C} \approx 24\ \mu\mathrm{C}. \qquad \text{(Answer)} \end{aligned}$$

Left as exercise

23-8 Applying Gauss'Law: Planar Symmetry

Nonconducting Sheet:

- Figure 23-15 shows a portion of a thin, infinite, nonconducting sheet with a uniform (positive) surface charge density σ.
- We need to find the electric field ***E*** at a distance *r* in front of the sheet.
    - A useful Gaussian surface is a closed cylinder with end caps of area *A*, arranged to pierce the sheet perpendicularly as shown.
    - From *symmetry*, ***E*** must be perpendicular to the sheet and hence to the end caps.
    - Since the charge is positive, ***E*** is directed away from the sheet.

only caps contribute

    - Because the field lines do not pierce the curved surface, there is no flux through this portion of the Gaussian surface.

no surface contribution

- Thus ***E.dA*** is simply *EdA*, then the Gauss's law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}},$$

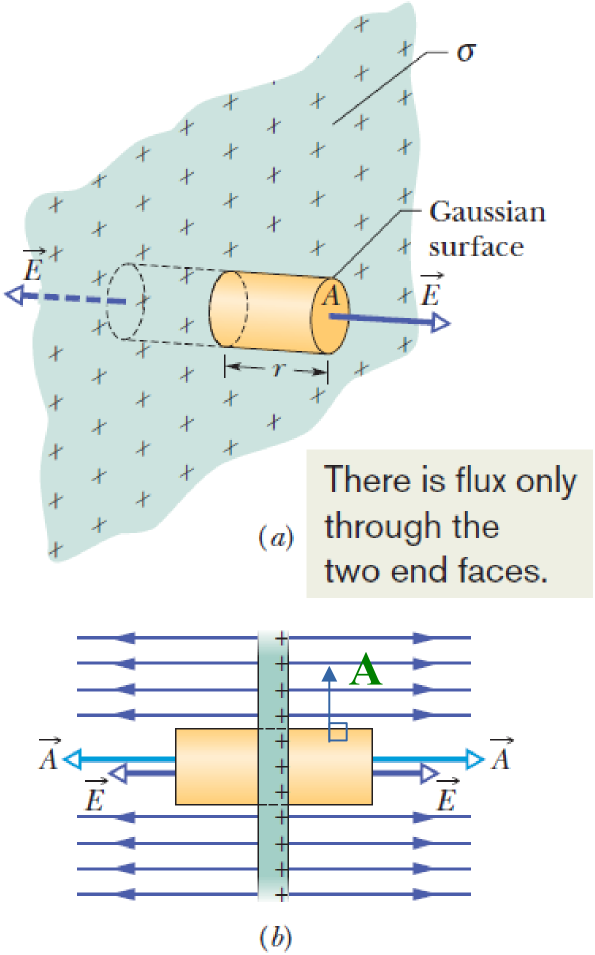
uniform charge distribution

$$\varepsilon_0(EA + EA) = \sigma A,$$

Here σ*A* is the charge enclosed by the Gaussian surface. Therefore,

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(sheet of charge).}$$

Compare with the solution at Chapter 22!!



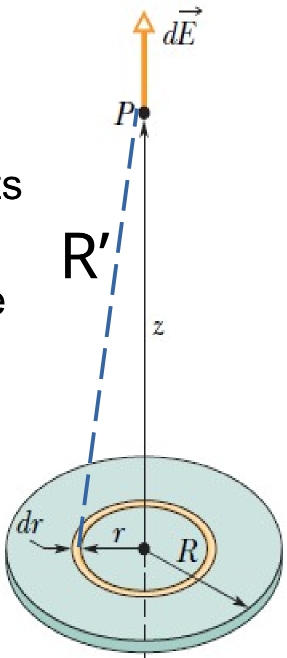
There is flux only through the two end faces.

Fig. 23-15 (*a*) Perspective view and (*b*) side view of a portion of a very large, thin plastic sheet, uniformly charged on one side to surface charge density σ. A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

22-7 The Electric Field due to a Continuous Charge

E Due to Charged Disk

- We need to find the electric field at point *P*, a distance *z* from the disk along its central axis.
- **Define & Adding:** Divide the disk into concentric flat rings and then calculate the electric field at point *P* by adding up (that is, by integrating) the contributions of all the rings.
  - The figure shows one such ring, with radius *r* and radial width *dr*. If σ is the charge per unit area, the charge on the ring is



$$dq = \sigma\, dA = \sigma\,(2\pi r\, dr),$$

$$A = \pi r^2 \;;\; dA = 2\pi r\, dr$$

$$dE = \frac{z\sigma 2\pi r\, dr}{4\pi\varepsilon_0 (z^2 + r^2)^{3/2}} = \frac{\sigma z}{4\varepsilon_0} \frac{2r\, dr}{(z^2 + r^2)^{3/2}}$$

$$\frac{Q}{A} = \sigma = \frac{dq}{dA}$$

- **Integrating:** We can now find *E* by integrating *dE* over the surface of the disk— that is, by integrating with respect to the variable *r* from *r =0 to r =R.*

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr. = \frac{\sigma z}{4\varepsilon_0} \left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R.$$

$$\Rightarrow\quad E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\cancel{2\ z^2 + R^2}}\right) \qquad \text{(charged disk)}$$

1/R→ 0

If we let R →∞, while keeping z finite, the second term in the parentheses in the above equation approaches zero, and this equation reduces to

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(infinite sheet).}$$

This is the electric field produced by an infinite sheet of uniform charge.

23-8 Applying Gauss'Law: Planar Symmetry

Two Conducting Plates:

- Figure 23-16*a* shows a cross section of a thin, infinite conducting plate **with excess positive charge**.
  - The plate is thin and very large, and all the excess charge is on the two large **faces of the plate**.
  - It will spread out on the two faces with a uniform surface charge density of magnitude σ_1.
  - Just outside the plate this charge sets up an electric field of magnitude $E = \sigma_1/\varepsilon_0$.
- Figure 23-16*b* shows an identical plate **with excess negative charge** having the same magnitude of surface charge density σ_1.
  - Now the electric field is directed toward the plate.
- If we arrange for the plates to be close to each other and parallel (Fig. 23-16*c*), the excess charge on one plate attracts the excess charge on the other plate, and *all the excess charge moves onto the inner faces of the plates* as in Fig. 23-16*c*.
- The new surface charge density, σ, on each inner face is twice σ_1.

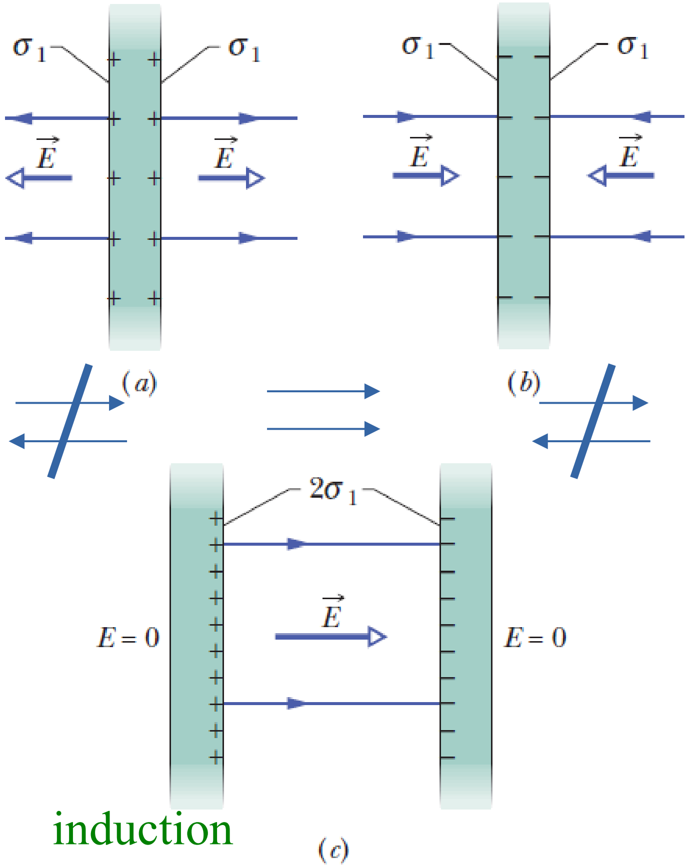


Fig. 23-16 (*a*) A thin, very large conducting plate with excess positive charge. (*b*) An identical plate with excess negative charge. (*c*) The two plates arranged so they are parallel and close.

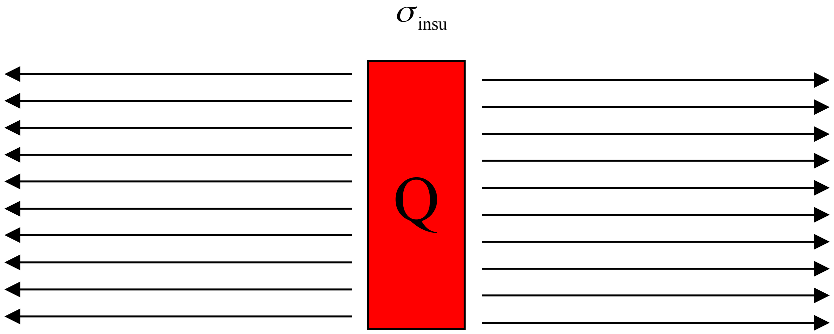
Thus, the electric field at any point between the plates has the magnitude:

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

23-8 Applying Gauss'Law: Planar Symmetry

Insulating and Conducting Planes

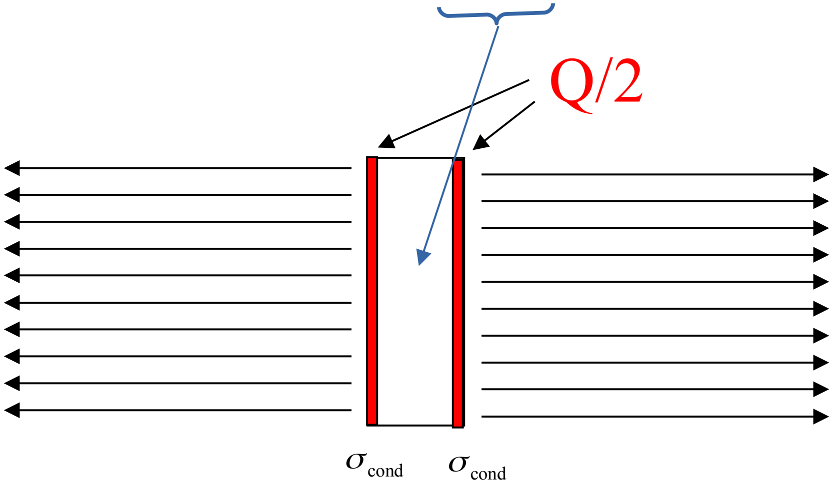
Insulating Plate: Charge Distributed Homogeneously.



$$E_{\text{insu}} = \frac{\sigma_{\text{insu}}}{2\varepsilon_0}$$

$$\sigma_{\text{insu}} = Q/A$$

Conducting Plate: Charge Distributed on the Outer Surfaces.
Electric Field Inside a Conductor is ZERO!



$$E_{\text{cond}} = \frac{\sigma_{\text{cond}}}{\varepsilon_0} = 2E_{\text{insu}}$$

$$\sigma_{\text{cond}} = Q/(2A)$$

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

23-8 Applying Gauss'Law: Planar Symmetry

Example, Electric Field:

Figure 23-17*a* shows portions of two large, parallel, nonconducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8\ \mu\text{C/m}^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3\ \mu\text{C/m}^2$ for the negatively charged sheet.

Find the electric field $\vec{E}$ (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.

KEY IDEA

With the charges fixed in place (they are on nonconductors), we can find the electric field of the sheets in Fig. 23-17*a* by (1) finding the field of each sheet as if that sheet were isolated and (2) algebraically adding the fields of the isolated sheets via the superposition principle. (We can add the fields algebraically because they are parallel to each other.)

Calculations: At any point, the electric field $\vec{E}_{(+)}$ due to the positive sheet is directed *away* from the sheet and, from Eq. 23-13, has the magnitude

$$E_{(+)} = \frac{\sigma_{(+)}}{2\varepsilon_0} = \frac{6.8 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.84 \times 10^5\ \text{N/C}.$$

Similarly, at any point, the electric field $\vec{E}_{(-)}$ due to the negative sheet is directed *toward* that sheet and has the magnitude

$$E_{(-)} = \frac{\sigma_{(-)}}{2\varepsilon_0} = \frac{4.3 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.43 \times 10^5\ \text{N/C}.$$

Figure 23-17*b* shows the fields set up by the sheets to the left of the sheets (L), between them (B), and to their right (R).

The resultant fields in these three regions follow from the superposition principle. To the left, the field magnitude is

$$\begin{aligned} E_L &= E_{(+)} - E_{(-)} \\ &= 3.84 \times 10^5\ \text{N/C} - 2.43 \times 10^5\ \text{N/C} \\ &= 1.4 \times 10^5\ \text{N/C}. \qquad \text{(Answer)} \end{aligned}$$

Because $E_{(+)}$ is larger than $E_{(-)}$, the net electric field $\vec{E}_L$ in this region is directed to the left, as Fig. 23-17*c* shows. To the right of the sheets, the electric field has the same magnitude but is directed to the right, as Fig. 23-17*c* shows.

Between the sheets, the two fields add and we have

$$\begin{aligned} E_B &= E_{(+)} + E_{(-)} \\ &= 3.84 \times 10^5\ \text{N/C} + 2.43 \times 10^5\ \text{N/C} \\ &= 6.3 \times 10^5\ \text{N/C}. \qquad \text{(Answer)} \end{aligned}$$

The electric field $\vec{E}_B$ is directed to the right.

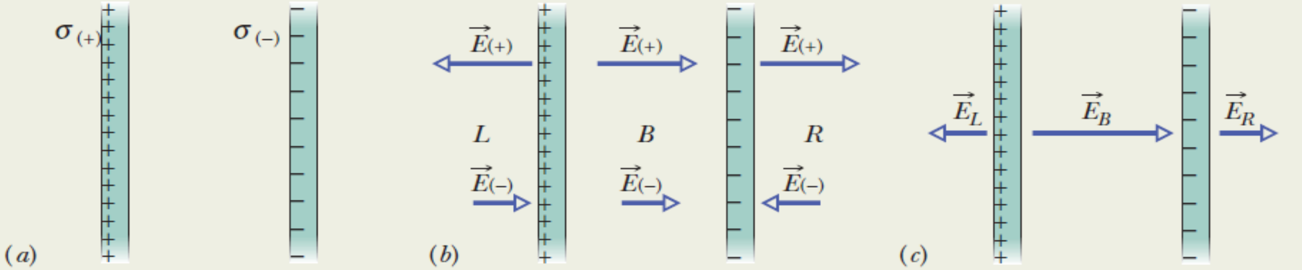


Fig. 23-17 (*a*) Two large, parallel sheets, uniformly charged on one side. (*b*) The individual electric fields resulting from the two charged sheets. (*c*) The net field due to both charged sheets, found by superposition.

23-9 Applying Gauss'Law: Spherical Symmetry

1. A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were **concentrated at the center of the shell (ST1)**.
2. If a charged particle is located inside a shell of uniform charge, there is **no net electrostatic force** on the particle from the shell **(ST2)**.

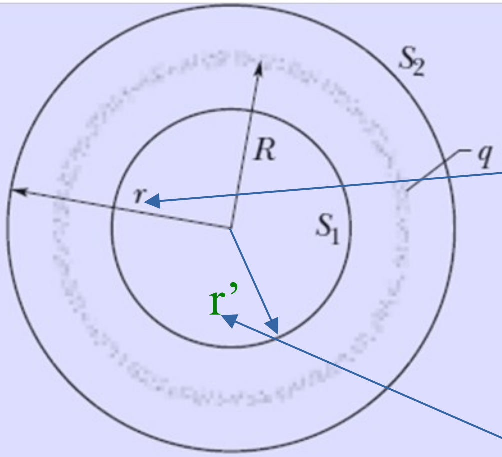


Fig. 23-18 A thin, uniformly charged, spherical shell with total charge q, in cross section. Two Gaussian surfaces S_1 and S_2 are also shown in cross section. Surface S_2 encloses the shell, and S_1 encloses only the empty interior of the shell.

Using Gauss' law, it is easy to prove these shell theorems:

$$\oint \vec{E}\cdot\vec{A} = E\int dA = E(4\pi r^2) = \frac{q_{enc}}{\epsilon_0}$$

1- Applying Gauss' law for spherical Gaussian surface S_2 yields $(r \geq R)$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{(spherical shell, field at } r \geq R).$$

This field is the same as one set up by a point charge *q* at the center of the shell of charge. Thus, the force produced by a shell of charge *q* on a charged particle placed outside the shell is the same as the force produced by a point charge *q* **located at the center of the shell**.

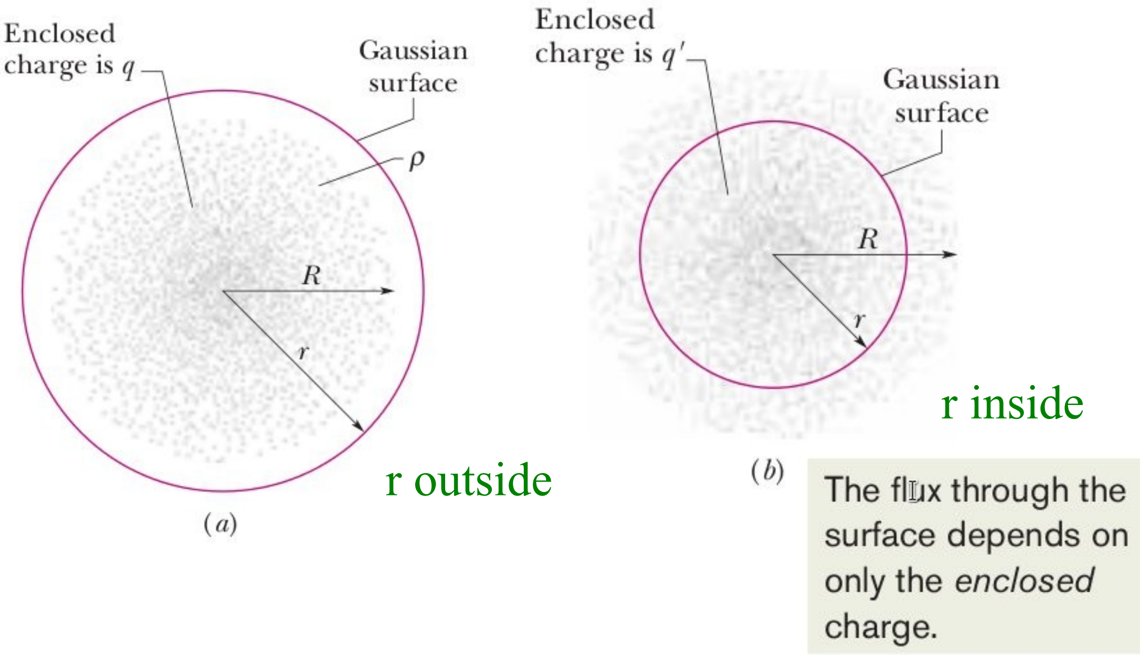
2- Applying Gauss's law for spherical Gaussian surface S_1 yields $(r' < R)$

$$E = 0 \qquad \text{(spherical shell, field at } r < R),$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = 0$$

because this Gaussian surface encloses no charge. Thus, if a charged particle were enclosed by the shell, the shell would exert **no net electrostatic force** on the particle.

23-9 Applying Gauss’Law: Spherical Symmetry



The flux through the surface depends on only the *enclosed* charge.

Fig. 23-19 The dots represent a spherically symmetric distribution of charge of radius *R*, whose volume charge density ρ is a function only of distance from the center.
The charged object is not a conductor, and therefore the charge is assumed to be fixed in position.
A concentric spherical Gaussian surface with $r > R$ is shown in (a).
A similar Gaussian surface with $r < R$ is shown in (b).

- In Fig. 23-19a, r > R. The charge produces an electric field on the Gaussian surface as if the charge were a point charge located at the center (ST1).
- Figure 23-19b, *r* < *R*. The charge lying outside the Gaussian surface does not set up a net electric field on the Gaussian surface. *The charge enclosed by the surface sets up an electric field as if that enclosed charge were concentrated at the center.*
- Letting *q'* represent that enclosed charge:

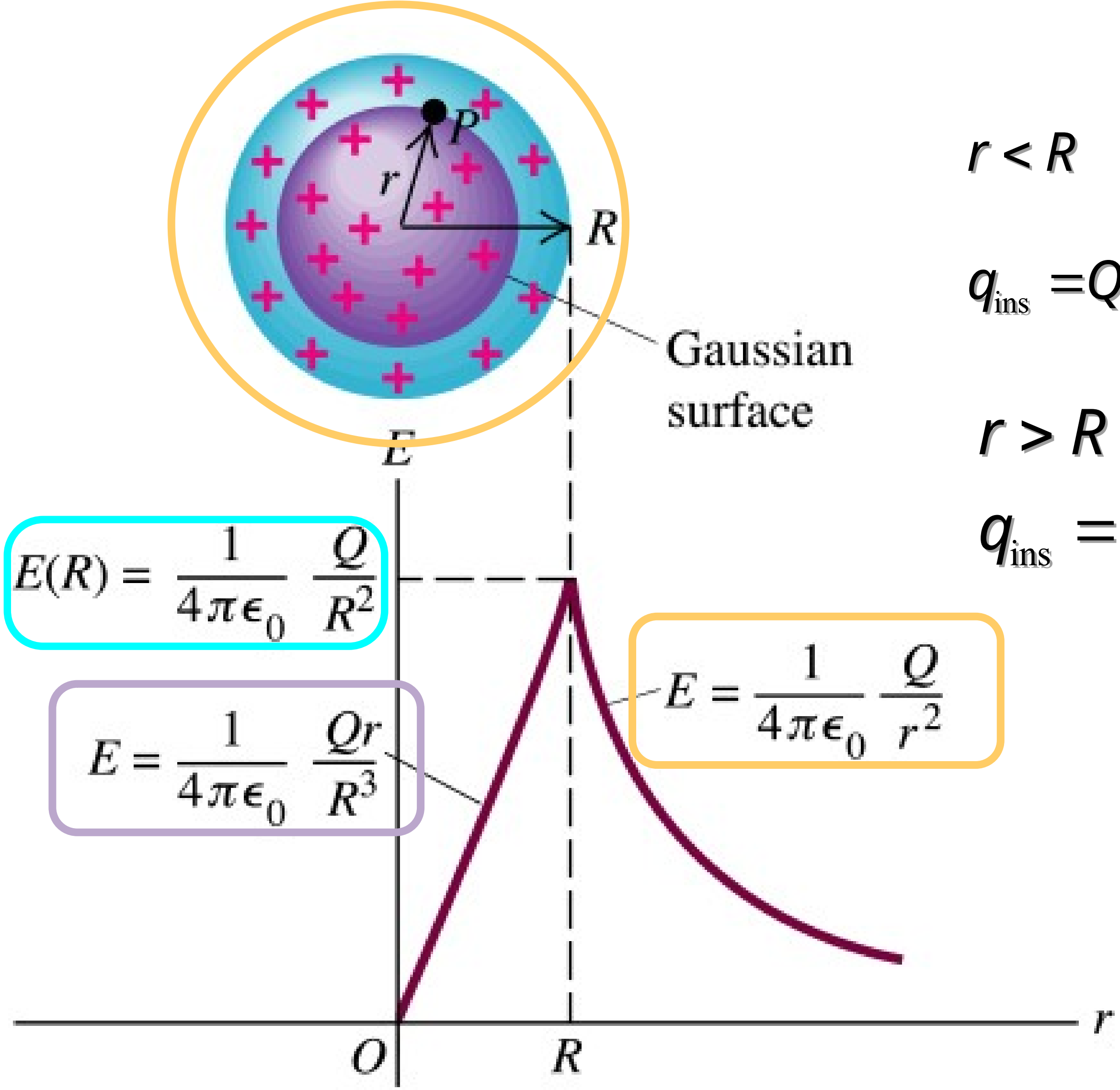
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} \qquad \text{(spherical distribution, field at } r \leq R).$$

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r \qquad \text{(uniform charge, field at } r \leq R).$$

(If the full charge *q* enclosed within radius *R* is uniform)

23-9 Applying Gauss'Law: Spherical Symmetry

Electric Fields With Insulating Sphere



$$\Phi = EA = q_{\text{ins}} / \varepsilon_0$$

$r < R$

$$q_{\text{ins}} = Q\left(\frac{V_{\text{ins}}}{V_{\text{total}}}\right) = Q\left(\frac{4\pi r^3/3}{4\pi R^3/3}\right) = Q\frac{r^3}{R^3}$$

$r > R$

$$q_{\text{ins}} = Q$$

$$r < R \rightarrow E\,4\pi r^2 = Q\frac{r^3}{R^3}/\varepsilon_0$$

$$r > R \rightarrow E\,4\pi r^2 = Q/\varepsilon_0$$

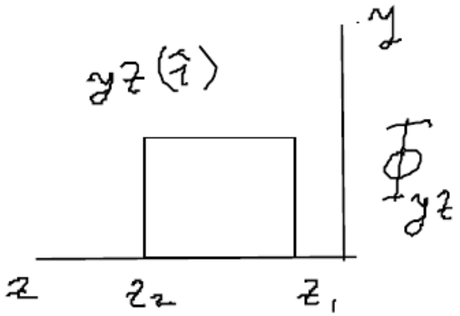
$$r=R \longrightarrow E4\pi R^2=Q/\varepsilon_0$$

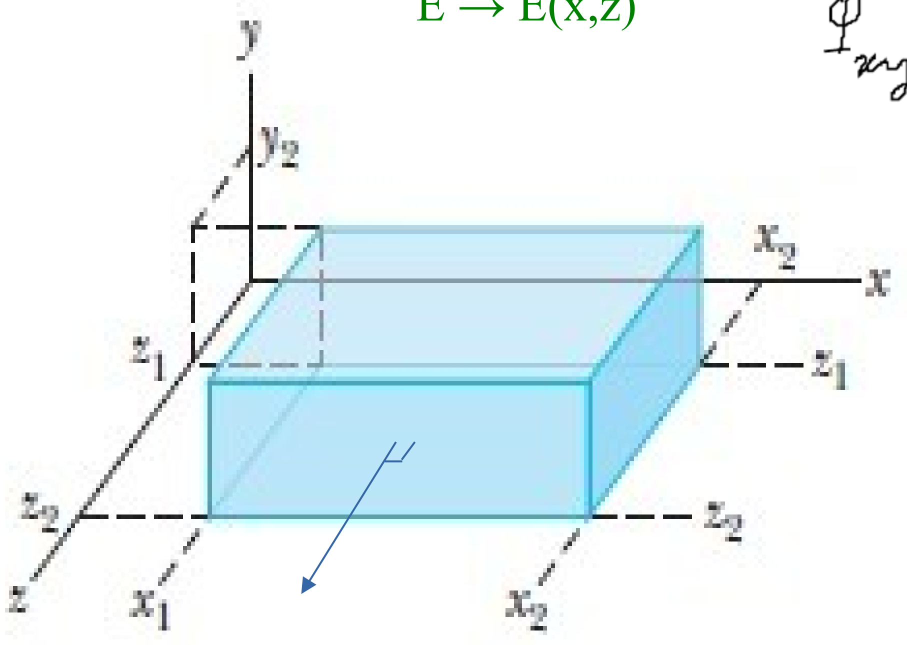
http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

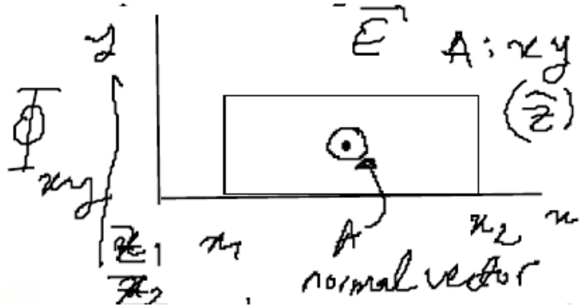
23 Solved Problems

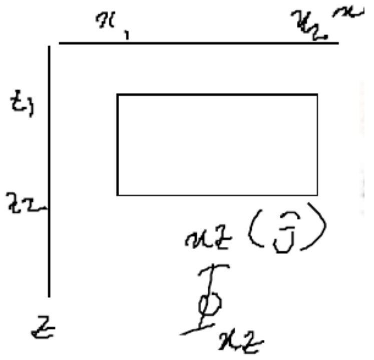
1. The box-like Gaussian surface shown in Figure encloses a net charge of $+24.0\varepsilon_0$ C and lies in an electric field given by $\boldsymbol{E} = [(10.0 + 2.00x)\mathbf{i} - 3.00\mathbf{j} + bz\mathbf{k}]$ N/C, with x and z in meters and b a constant. The bottom face is in the xz-plane; the top face is in the horizontal plane passing through y_2=1.00 m. For x_1=1.00 m, x_2=4.00 m, z_1=1.00 m, and z_2=3.00 m, what is b?

$E \rightarrow E(x,z)$









- planes xy, yz, xz

xy : normal vector in z

yz : in x

xz : in y

$(\pm)$

23 Solved Problems

3 (16) Total Electric flux $\Phi = \oint \vec{E} \cdot d\vec{A}$

$E = [(10.0 + 2.00x)\hat{i} - 3.00\hat{j} + bz\hat{k}]$

$x_1 = 1.00\,m \quad y_1 = 0.00\,m \quad z_1 = 1.00\,m$

$x_2 = 4.00\,m \quad y_2 = 1.00\,m \quad z_2 = 3.00\,m \quad b = ?$

$q_{enc} = 24.0\,\varepsilon_0\ C$

3 planes : xy, yz, xz

① Net flux through the two faces parallel to the xy-plane

$$\Phi_{xy}\Big|_{z=z_2} - \Phi_{xy}\Big|_{z=z_1} = \iint E_z(z=z_2)\,dx\,dy - \iint E_z(z=z_1)\,dx\,dy$$

bz

$\hat{k}$ $\quad -\hat{k}$

$$= \int_1^4 dx \int_0^1 dy\,(36) - \int_1^4 dx \int_0^1 dy\,(6) = 96 - 36 = 66$$

$$d\vec{A} = dxdy\hat{k}$$

② " " " " " " " yz-plane

$$\Phi_{yz}\Big|_{x=x_2} - \Phi_{yz}\Big|_{x=x_1} = \int_0^1 dy \int_1^3 dz\,(1+2x_2) - \int_0^1 dy \int_1^3 dz\,(1+2x_1) = 18 - 6 = 12$$

$$d\vec{A} = dydz\hat{i}$$

③ " " " " " " " xz-plane

$$\Phi_{xz}\Big|_{y=y_2} - \Phi_{xz}\Big|_{y=y_1} = \int_1^4 dx \int_0^1 dz\,(-3) - \int_1^4 dx \int_0^1 dz\,(-3) = 0$$

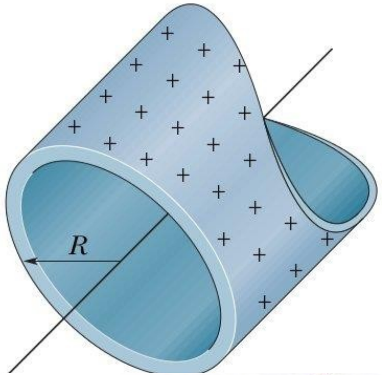
as expected

$$d\vec{A} = dxdz\hat{j}$$

Now, apply Gauss' law $\Phi = \dfrac{q_{enc}}{\varepsilon_0} \Rightarrow (\Phi_{xy} + \Phi_{yz} + \Phi_{xz}) = \dfrac{24.0\,\varepsilon_0}{\varepsilon_0}$

$$(66 + 12 + 0) = 24 \Rightarrow \boxed{b = 2.0\ \frac{N\text{-}C}{m}}$$

2. Figure shows a section of a long, thin-walled metal tube of radius *R*=3.00 cm, with a charge per unit length of λ=2.00×10⁻⁸ C/m. What is the magnitude *E* of the electric field at radial distance (a) *r* = *R*/2.00 (inside) and (b) *r*=2.00*R*? (outside) (c) Graph *E* versus *r* for the range *r*=0 to 2.00*R*.

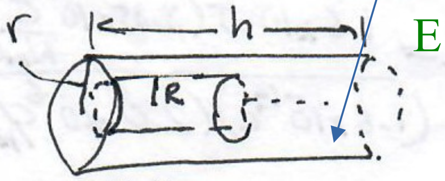


4 (24) need a cylindrical Gaussian Surface. $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ $\lambda = 2.00\times10^{-8}$ C/m

$R = 3.00$ cm

i) $r<R$ $q_{enc}=0 \Rightarrow E=0$

ii) $r>R$ $q_{enc} = \lambda h \Rightarrow E(r)\,2\pi r h = \frac{\lambda h}{\varepsilon_0} \rightarrow E(r) = \frac{\lambda}{2\pi r \varepsilon_0} = \frac{(2.00\times10^{-8}\ C/m)}{2\pi\,(0.006m)\left(8.85\times10^{-12}\frac{C^2}{Nm^2}\right)}$



E||A

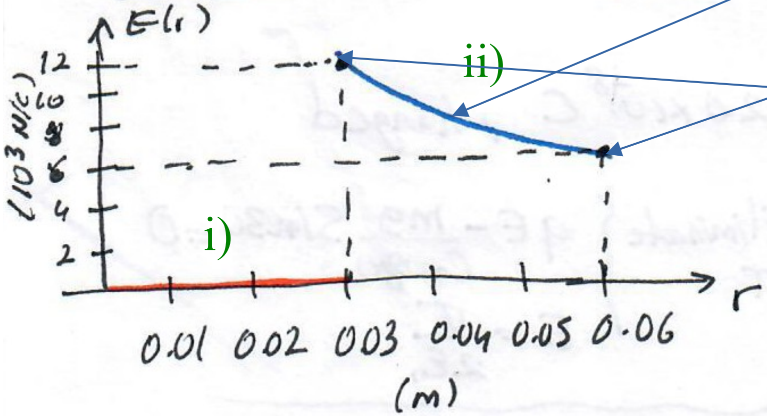
$r = 2.00R$

$= 2.00\times(3.00\ cm)\left(\frac{1m}{100cm}\right)$

$= 0.0600\ m$

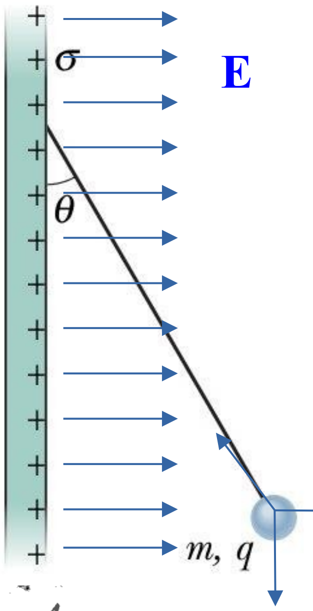
$$\boxed{E(r) = 5.99\times10^{3}\ N/C}$$

$$\boxed{E_{max} = E(r=R) = 11.99\times10^{3}\ N/C}$$

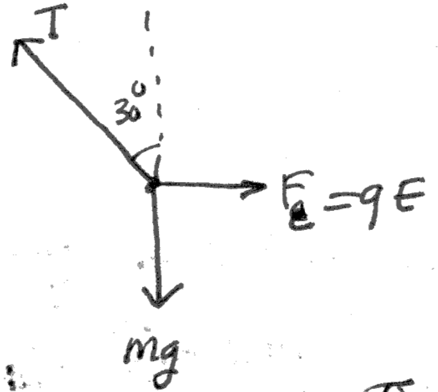


23 Solved Problems

3. In Figure, a small, nonconducting ball of mass m=1.0 mg and charge q=2.0×10^{-8} C (distributed uniformly through its volume) hangs from an insulating thread that makes an angle θ = 30°with a vertical, uniformly charged nonconducting sheet (shown in cross section). Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density σ of the sheet.



(39) small, nonconducting $m=1.0$ mg, $q=2.0\times10^{-8}$ C, hanged non-conducting sheet. $\sigma=?$



FBD

$$T\cos30-mg=ma_y=0$$

$$qE-T\sin30=ma_x=0$$

Eliminate T $\left\{ qE-\frac{mg}{\cos30}\sin30=0 \quad E=\frac{\sigma}{2\varepsilon_0} \right.$

$$\Rightarrow \frac{q\sigma}{2\varepsilon_0}=mg\tan30^\circ \rightarrow \sigma=\frac{mg2\varepsilon_0}{q}\tan30^\circ$$

$$\sigma=\frac{(1.0\times10^{-6}kg)(9.8m/s^2)2(8.85\times10^{-12}C^2/Nm^2)}{(2.0\times10^{-8}C)}\tan30=5\times10^{-9}\frac{C}{m^2}$$

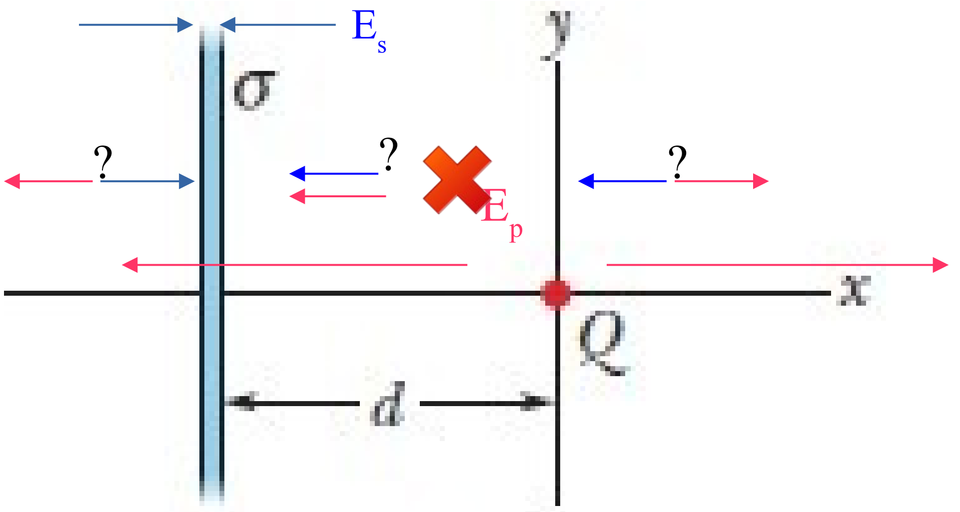
$kg\,m/s^2 \rightarrow N$

23 Solved Problems

4. Figure shows a very large nonconducting sheet that has a uniform surface charge density of σ = -2.00 μC/m^2; it also shows a particle of charge Q = 6.00 μC, at distance d from the sheet. Both are fixed in place. If d = 0.200 m, at what (a) positive and (b) negative coordinate <u>on the *x* axis</u> (other than infinity) is the net electric field $\boldsymbol{E}_{net}$ of the sheet and particle zero? (c) If d = 0.800 m, at what coordinate on the x axis is $\boldsymbol{E}_{net}$ = 0?

$$\vec{E}_{net} = \vec{E}_s + \vec{E}_p = 0$$

$$|\vec{E}_s| = |\vec{E}_p|$$



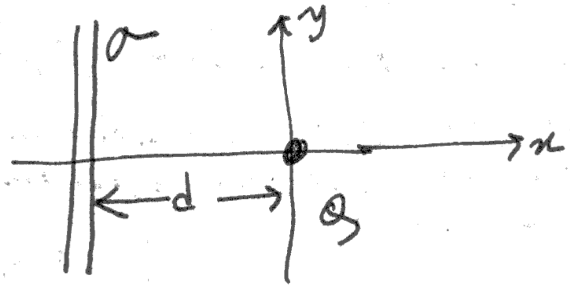
$E_p = kQ/r^2$

$E_s = \sigma/2\varepsilon_0$

Fig. 23-46 Problem 40.

23 Solved Problems

8 (40)



$\sigma = -2.00\,\frac{\mu C}{m^2}$ very lage nonconducting sheet uniform surface charge density

$Q = 6.00\,\mu C$ at distance d from the sheet

$d = 0.200\,m$

net electric field is zero. where, they cancel each other?

Btw the sheet and the particle $(-d < x < 0)$: → no cancellation

opposite charges ⇒ same direction of E

i) $x > 0$

ii) $x < 0$

$$\frac{|\sigma|}{2\varepsilon_0} = \frac{|Q|}{4\pi\varepsilon_0 R^2} \Rightarrow R = x = \sqrt{\frac{6\,\mu C}{2\,\frac{\mu C}{m^2}\,2\pi}}$$

$$\boxed{x \cong \pm 0.691\,m}$$

iii) now, $d = 0.800\,m$ means that when $x < 0$ case does not work any more. So that we have only positive 0.691 m.

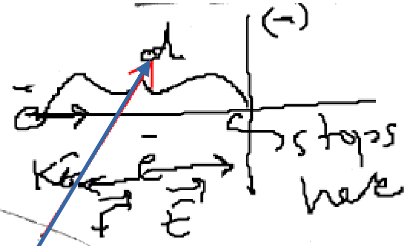
23 Solved Problems

5. An electron is shot directly toward the center of a large metal plate that has surface charge density σ = -2.0x10^{-6} C/m2. If the initial kinetic energy of the electron is 1.60x10^{-17} J and if the electron is to stop (due to electrostatic repulsion from the plate) just as it reaches the plate, how far from the plate must the launch point be?

41) large metal plate : $\sigma = -2.0 \times 10^{-6}\ C/m^2$

an electron : $q = 1.60 \times 10^{-19} C$, $KE = 1.60 \times 10^{-17}\ J = \frac{1}{2} m_e v^2$

electron is to stop! how far from the plate?



$F = qE = -eE = -e\frac{|\sigma|}{\varepsilon_0}$: Force exerted by the plate

E → constant → constant F → constant acceleration

$F = m_e a \Rightarrow m_e a = -e\frac{|\sigma|}{\varepsilon_0} \Rightarrow a = \frac{-e|\sigma|}{m_e \varepsilon_0}$; $v^2 = v_0^2 + 2ax$ → $v = 0$

deceleration

J → NM

$$0 = v_0^2 + 2\left(\frac{-e|\sigma|}{m_e \varepsilon_0}\right)x \rightarrow x = \frac{\left(\frac{1}{2} m v_0^2\right)\varepsilon_0}{e|\sigma|} = \frac{(KE)(\varepsilon_0)}{e|\sigma|} = \frac{1.6 \times 10^{-17} J\left(8.85 \times 10^{-12} \frac{C^2}{Nm^2}\right)}{(1.6 \times 10^{-19} C)(2.0 \times 10^{-6} C/m^2)}$$

$$\boxed{x = 4.4 \times 10^{-4}\ m}$$

23 Solved Problems

6. Figure shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius *R*. Point *P* lies on a line connecting the centers of the spheres, at radial distance *R*/2.00 from the center of sphere 1. If the net electric field at point *P* is zero, what is the ratio q_2/q_1 of the total charges?

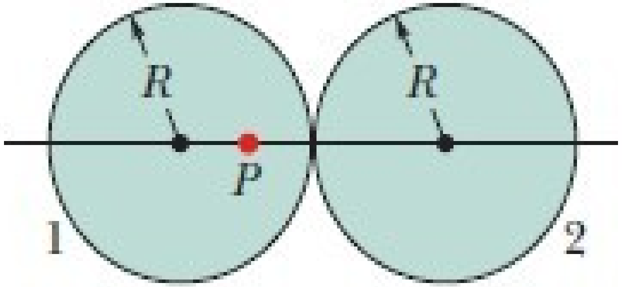


Fig. 23-54 Problem 54.

(54) net electric field is zero. Radius : R uniformly distributed charge. $\frac{q_2}{q_1} = ?$

need a spherical Gaussian Surface

sphere 2:

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ $\Big\{$ $E_2\, 4\pi (R + R/2)^2 = \frac{q_2}{\varepsilon_0}$

sphere 1

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$\frac{\frac{4\pi}{3} R^3}{\frac{4}{3}\pi (R/2)^3} \cdot \frac{q_2}{q_{enc}}$

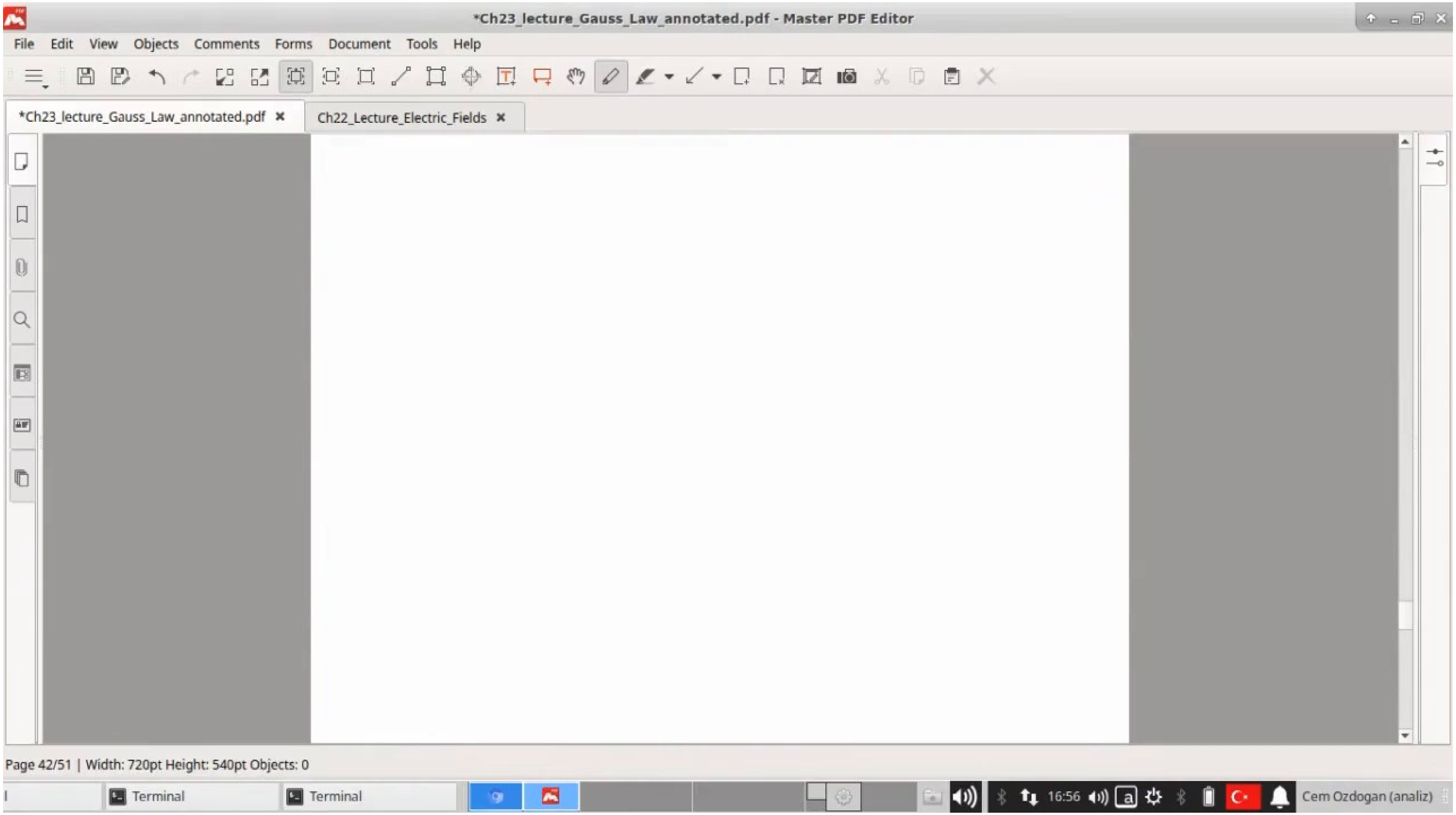
$\Big\{$ $E_1\, 4\pi \left(\cancel{\frac{R}{2}}\right)^{\cancel{2}} = \frac{\cancel{\frac{4\pi}{3}} \pi (R/2)^{\cancel{3}}}{\cancel{\frac{4\pi}{3}} R^3} \frac{q_1}{\varepsilon_0}$

$E_1 = \frac{1}{2} \frac{q_1}{4\pi\varepsilon_0 R^2}$ $\Big\{$ $E_2 = E_1$

$\Rightarrow \boxed{\frac{q_2}{q_1} = \frac{(3/2)^2 R^2}{2R^2} = 9/8 = 1.125}$

23 Solved Problems

Video: Solved Problem 6



23 Summary

Gauss' Law

- Gauss' law is

$$\varepsilon_0\Phi = q_{enc}$$ **Eq. 23-6**

- the net flux of the electric field through the surface:

$$\Phi = \oint \vec{E}\cdot d\vec{A}$$ **Eq. 23-6**

Applications of Gauss' Law

- surface of a charged conductor

$$E = \frac{\sigma}{\varepsilon_0}$$ **Eq. 23-11**

- Within the surface *E=0.*
- line of charge

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$ **Eq. 23-12**

- Infinite non-conducting sheet

$$E = \frac{\sigma}{2\varepsilon_0}$$ **Eq. 23-13**

- Outside a spherical shell of charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$ **Eq. 23-15**

- Inside a uniform spherical shell

$$E = 0$$ **Eq. 23-16**

- Inside a uniform sphere of charge

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r.$$ **Eq. 23-20**

Additional Materials