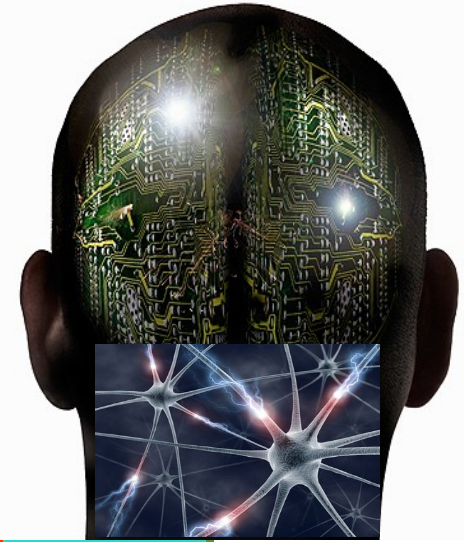
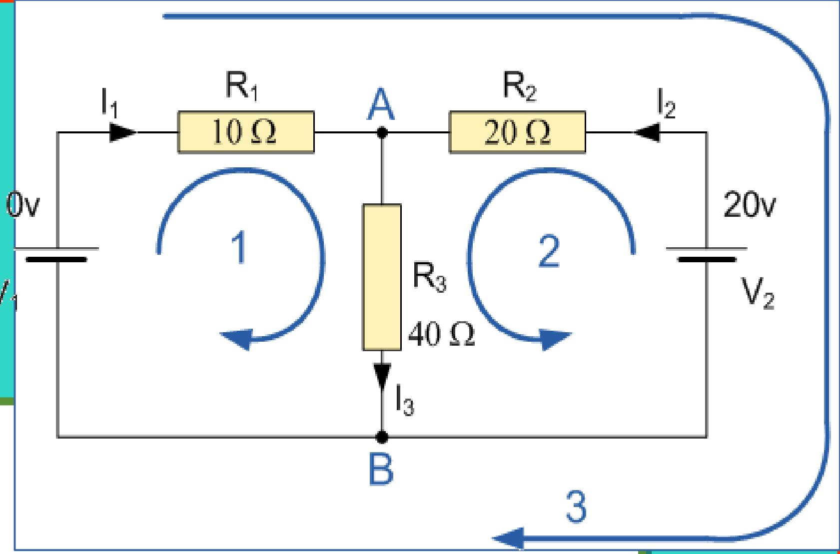
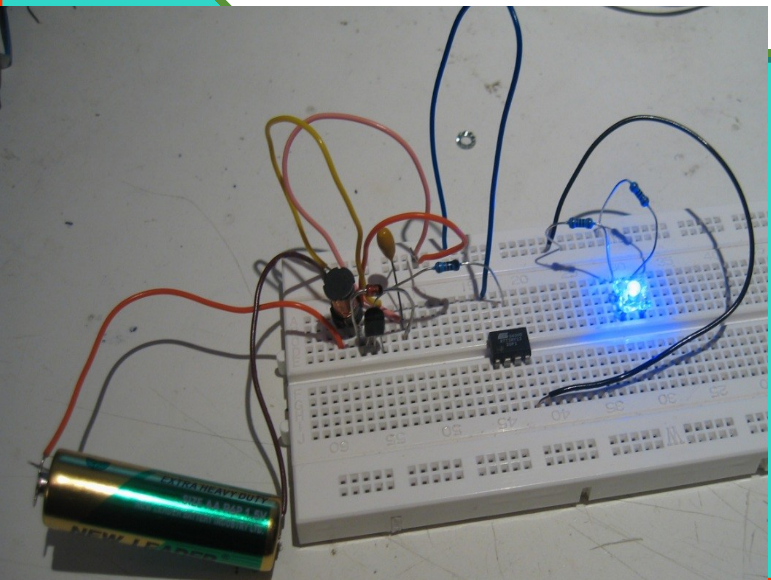


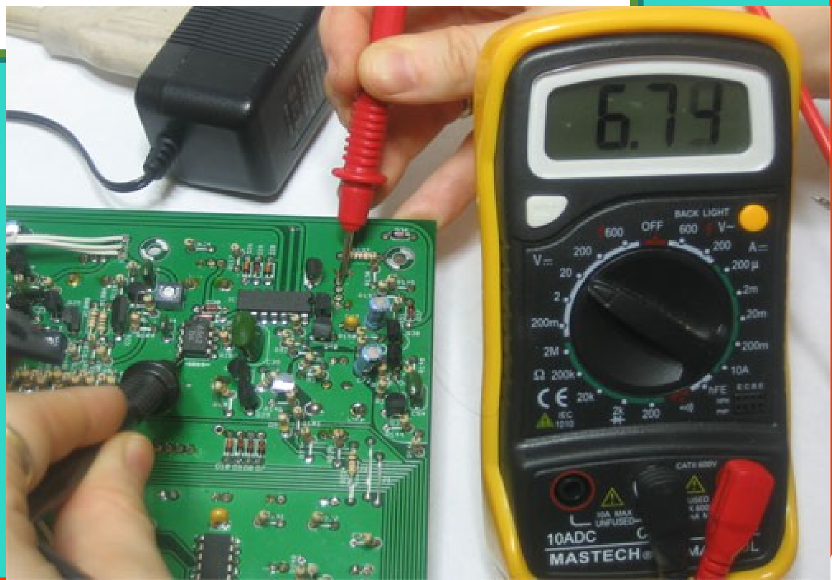


Chapter 27

Circuits



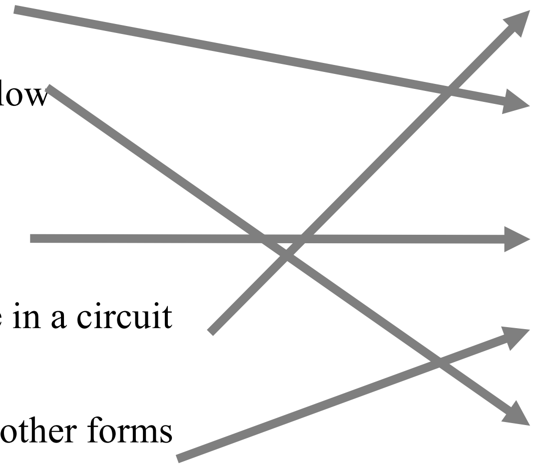
İZMİR KÂTİP ÇELEBİ ÜNİVERSİTESİ



27 CIRCUITS 705

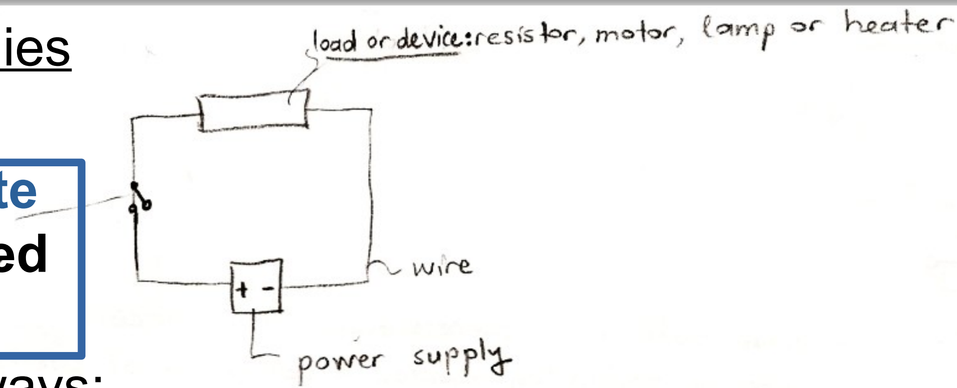
Match the following descriptions with the most appropriate terms on the right:

1. rate of flow of charge
2. A measure of the opposition to current flow
3. The path of electric current flow
4. The energy stored per coulomb of charge in a circuit
5. A device that converts electric energy to other forms



- Potential difference
- current
- circuit
- load
- resistance

- Electric circuits connect power supplies to loads.



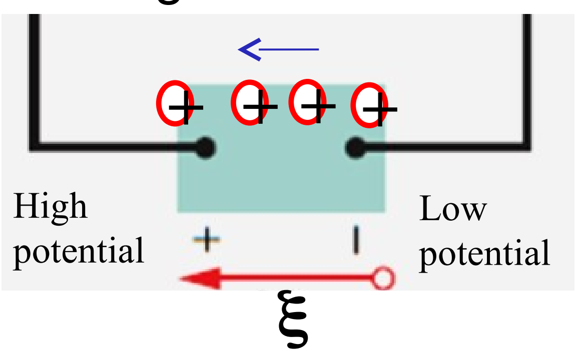
How a constant current (steady state flow of charge carriers) is maintained through the load or device?

- It can be evaluated in two different ways;

1. Electric field is needed to produce electrostatic force, F_E, on charges
2. Electrical energy should be supplied (energy is needed to do work on charge carriers)

The source of energy is called Electromotive force (emf), ξ, and the device which supply emf is called emf device.

- Emf devices can be considered as a "**charge pump**" that moves charges from lower potential to higher one in order to produce a steady flow of charge through a circuit.

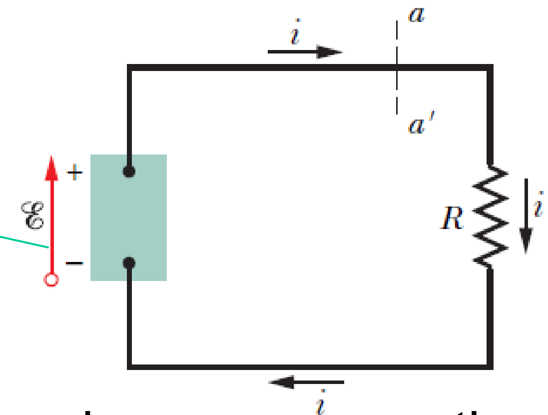


- Some emf devices;
- Battery
- Generator
- Solar Cells
- Fuel Cells
- ...

All perform the same function; *They do work on charge carriers and thus maintain a potential difference between their terminals.*

- Consider a circuit consisting of a battery as emf source and resistor of resistance R;

Emf ,ξ, of the device is shown by arrow (starting from – to + terminal)

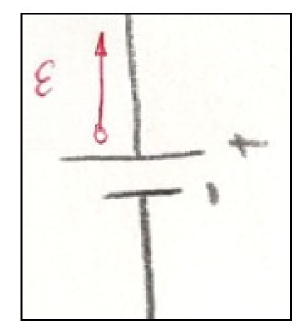


  - In any time interval dt, a charge dq passes through any cross section of the circuit shown, such as aa'.
  - This *same amount of charge* must enter the emf device at its low-potential end and leave at its high-potential end.
- Emf device must do an amount of work *dW* on the charge *dq* to force it to move in this way. $U=qV \rightarrow \Delta W/\Delta q=V=\xi$
- We define the **emf** of the emf device in terms of this work:

$$\mathscr{E} = \frac{dW}{dq} \qquad \text{(definition of } \mathscr{E}\text{)}.$$

- An **ideal emf device** is one that has no internal resistance to the internal movement of charge from terminal to terminal.
- The potential difference between the terminals of an ideal emf device is exactly equal to the emf of the device.

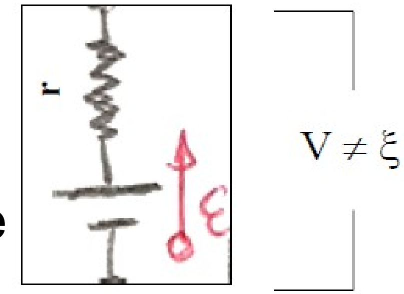
$V = \xi$



ideal emf device

27-4 Calculating the Current in a Single-Loop Circuit

- A **real emf device,** such as any real battery, has internal resistance to the internal movement of charge.
- When a real emf device is not connected to a circuit, and thus does not have current through it, the potential difference between its terminals is equal to its emf.
- However, when that device has current through it, the potential difference between its terminals differs from its emf.



Real emf device

The battery drives current through the resistor, from high potential to low potential.

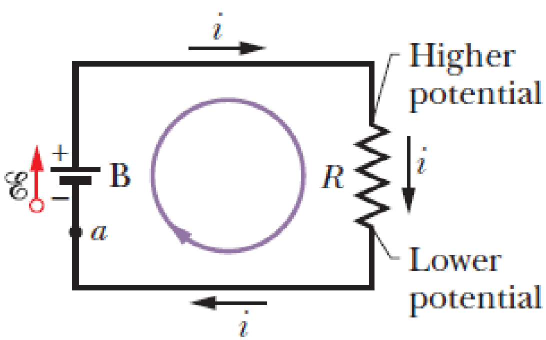


Fig. 27-3 A single-loop circuit in which a resistance R is connected across an ideal battery B with emf $\mathscr{E}$. The resulting current i is the same throughout the circuit.

Calculating the current in a Single-Loop Circuit:

Two methods used to <u>calculate current</u>;

1. Energy Method
2. Potential Method ✔

Our objective is to calculate the current at each circuit element.

1-Energy Method

The battery drives current through the resistor, from high potential to low potential.

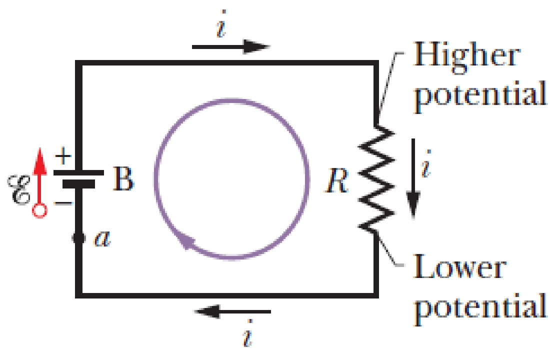


Fig. 27-3 A single-loop circuit in which a resistance R is connected across an ideal battery B with emf $\mathscr{E}$. The resulting current i is the same throughout the circuit.

- For a time interval dt, charge dq, passed from low potential to high potential point of the battery, where dq=i dt
- Work done on this charge to move;

$$dW = \mathscr{E}\,dq = \mathscr{E}i\,dt.$$

- At the same time energy is dissipated in the resistor : $i^2R\,dt.$

- For ideal battery, from the conservation of energy principle: $\mathscr{E}i\,dt = i^2R\,dt.$

$$i = \frac{\mathscr{E}}{R}.$$

2-Potential Method

The battery drives current through the resistor, from high potential to low potential.

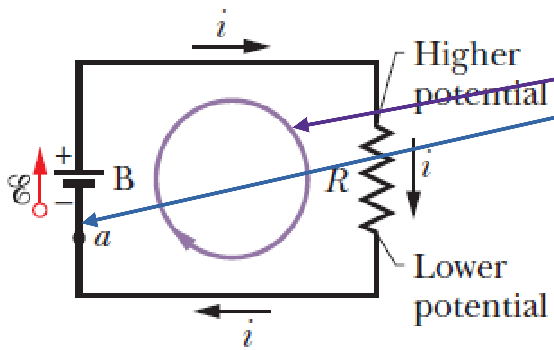


Fig. 27-3 A single-loop circuit in which a resistance *R* is connected across an ideal battery B with emf ℰ. The resulting current *i* is the same throughout the circuit.

- The algebraic sum of the *changes in potential* encountered in any loop of circuit must be zero.
- This is often referred to as Kirchhoff's Loop Rule

1. Choose a point in the circuit, i.e. Point a, whose potential is V_a
2. Draw a loop either in clockwise or counter cw
3. As passing the battery (low to high potential) the potential change is +ξ; $\mathbf{V_a + \xi}$
4. In the wires there is no potential change since we assume no resistance
5. As we pass through the resistor potential changes and decreases by –iR; $\mathbf{V_a + \xi - iR}$
6. When we complete the loop and reached at the same point, i.e. Point a,

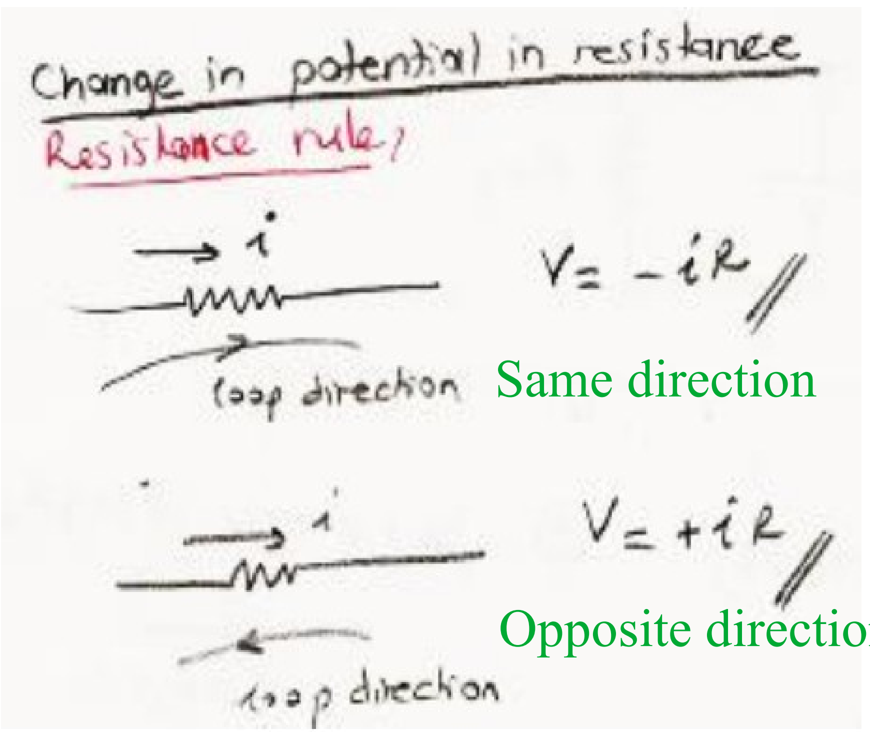
$$\mathbf{V_a + \xi - iR = V_a \Rightarrow \xi - iR = 0 \Rightarrow i = \xi / R}$$

27-4 Assignment + or – sign to potential

For circuits that are more complex than that of the previous figure, two basic rules are usually followed for finding potential differences as we move around a loop:

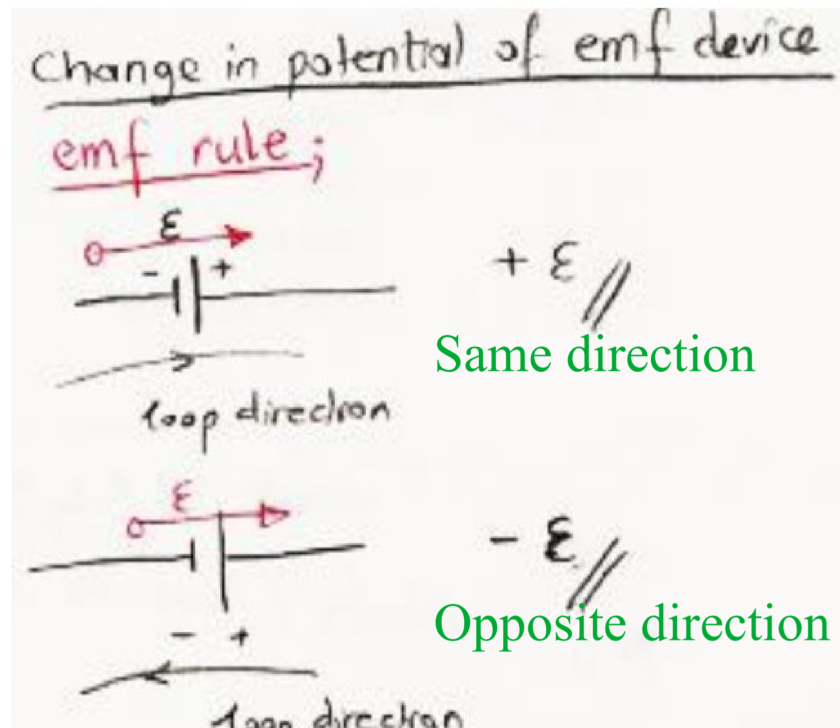
RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$.



Same direction

Opposite direction



Same direction

Opposite direction

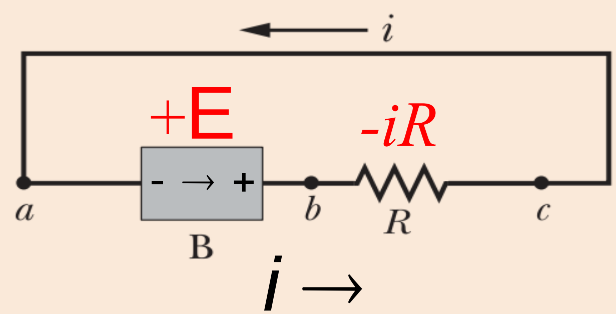
We should know these 4 rules!

RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$.

CHECKPOINT 1

The figure shows the current *i* in a single-loop circuit with a battery B and a resistance *R* (and wires of negligible resistance). (a) Should the emf arrow at B be drawn pointing leftward or rightward? At points *a*, *b*, and *c*, rank (b) the magnitude of the current, (c) the electric potential, and (d) the electric potential energy of the charge carriers, greatest first.



(a) Rightward (EMF is in direction of current)

(b) All tie (no junctions so current is conserved)

(c) b, then a and c tie (Voltage is highest near battery +)

(d) b, then a and c tie (U=qV and assume q is +)

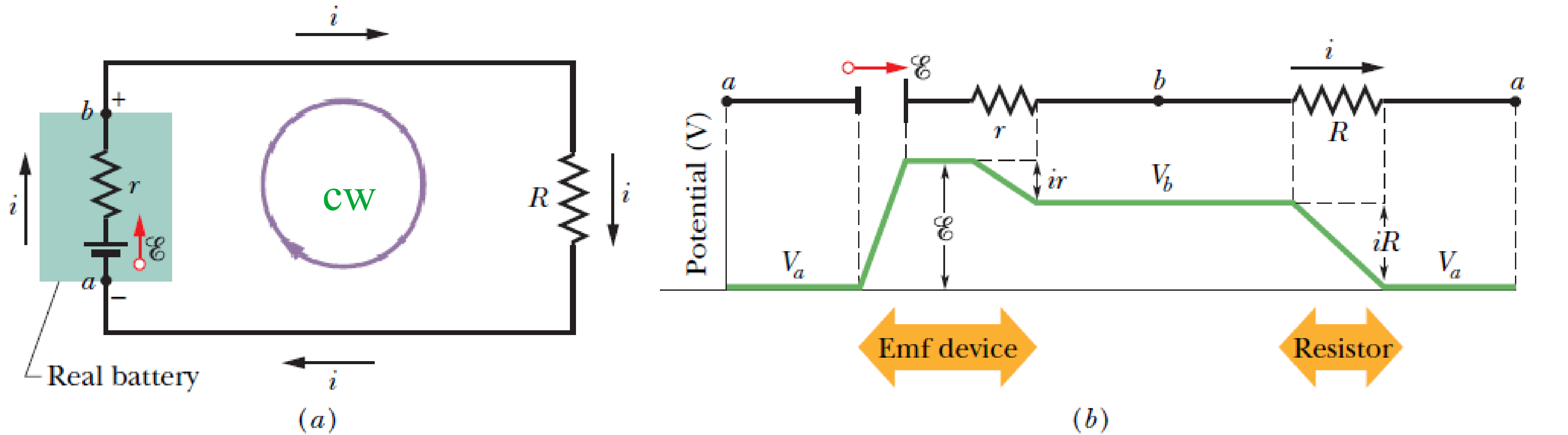


Fig. 27-4 (*a*) A single-loop circuit containing a real battery having internal resistance *r* and emf ℰ. (*b*) The same circuit, now spread out in a line. The potentials encountered in traversing the circuit clockwise from *a* are also shown. The potential V_a is arbitrarily assigned a value of zero, and other potentials in the circuit are graphed relative to V_a.

The figure above shows a real battery, with internal resistance *r,* wired to an external resistor of resistance *R.* According to the potential rule,

$$V_a + \xi - ir - iR = V_a$$

$$\mathscr{E} - ir - iR = 0.$$

$$i = \frac{\mathscr{E}}{R + r}.$$

CHECKPOINT 3

A battery has an emf of 12 V and an internal resistance of 2 Ω. Is the terminal-to-terminal potential difference greater than, less than, or equal to 12 V if the current in the battery is (a) from the negative to the positive terminal, (b) from the positive to the negative terminal, and (c) zero?

The internal resistance reduces the potential difference between the terminals.

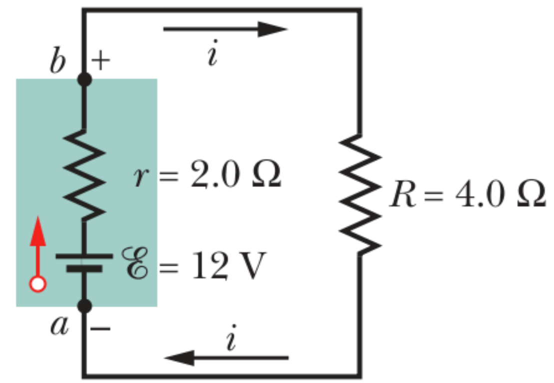
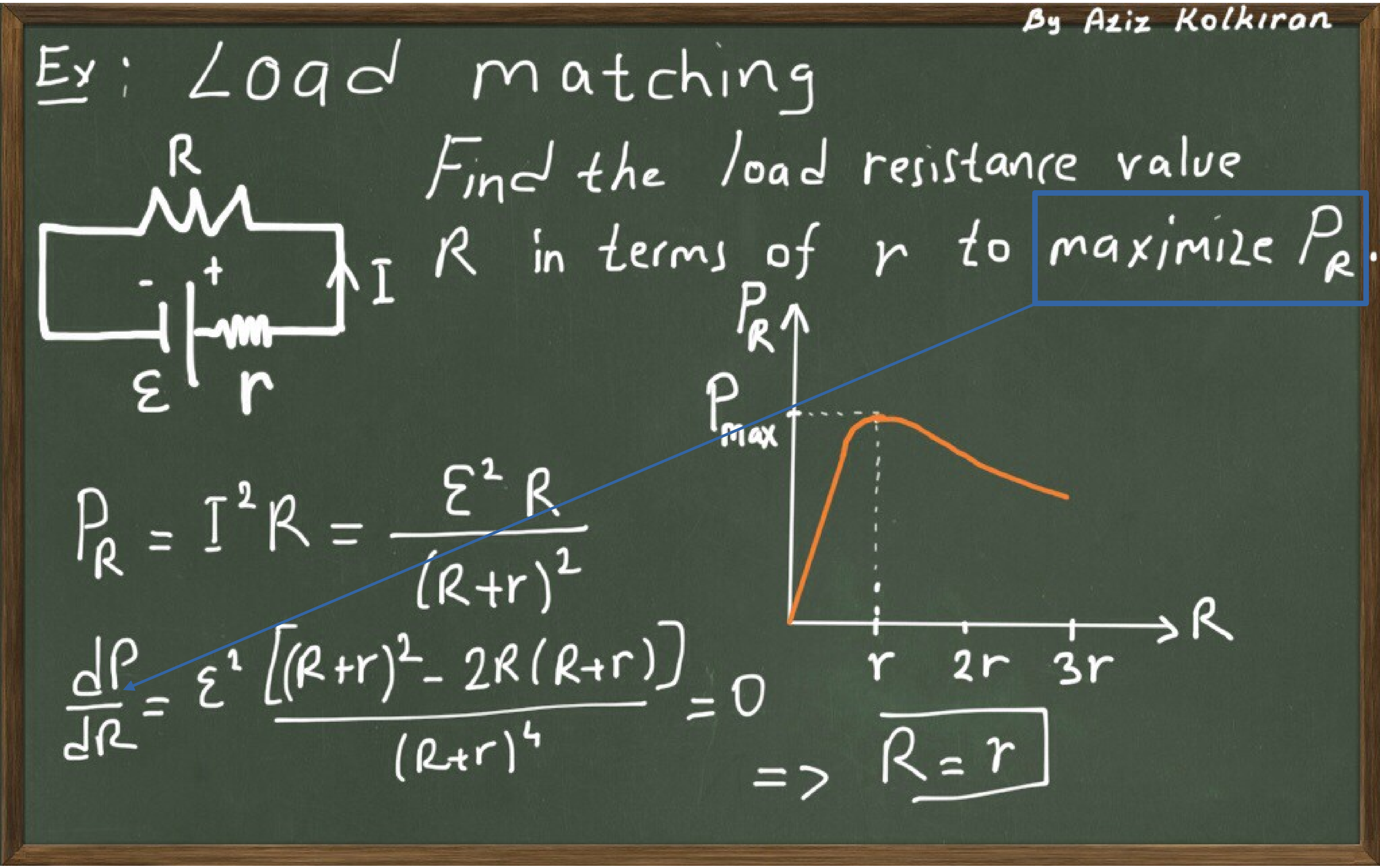


Fig. 27-6 Points *a* and *b*, which are at the terminals of a real battery, differ in potential.

(a) $V_{batt} < 12V$ (walking with current voltage drop –ir

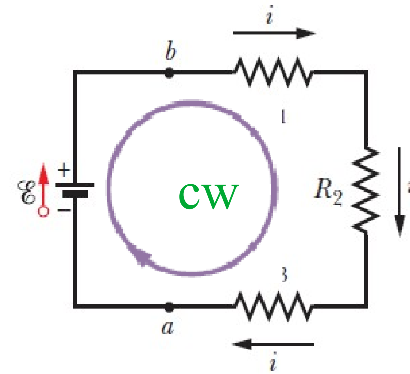
(b) $V_{batt} > 12V$ (walking against current voltage increase +ir

(c) $V_{batt} = 12V$ (no current and so ir=0)


By Aziz Kolkıran
Ex: Load matching
Find the load resistance value R in terms of r to maximize P_R.
R
I
ε
r
P_R
P_max
R
r
2r
3r
P_R = I^2 R = ε^2 R / (R+r)^2
dP/dR = ε^2 [(R+r)^2 - 2R(R+r)] / (R+r)^4 = 0
=> R = r

In Figure 27-5

$$\mathscr{E} - iR_1 - iR_2 - iR_3 = 0, \Rightarrow i = \frac{\mathscr{E}}{R_1 + R_2 + R_3}.$$



(a)

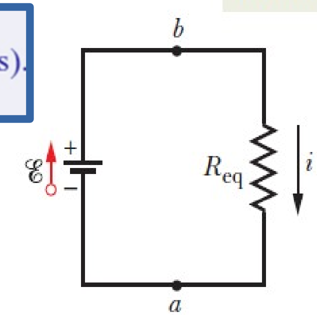
When a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V.

Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same *total* potential difference V as the actual resistances.

Series resistors and their equivalent have the same current ("ser-i").

$$\mathscr{E} - iR_{eq} = 0, \Rightarrow i = \frac{\mathscr{E}}{R_{eq}}. \Rightarrow R_{eq} = R_1 + R_2 + R_3.$$

$$R_{eq} = \sum_{j=1}^{n} R_j \qquad (n \text{ resistances in series}).$$



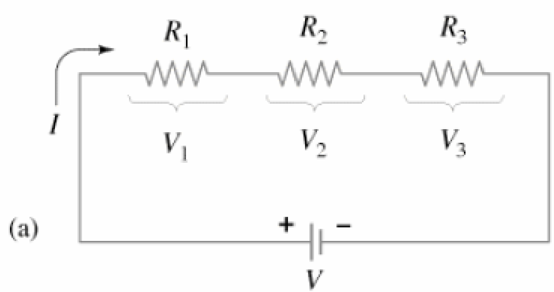
(b)

Fig. 27-5 (*a*) Three resistors are connected in series between points *a* and *b*. (*b*) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq}.

Resistors in series

→EMF of battery is 12 V, 3 identical resistors. What is the potential difference across each resistor?

- 12 V
- 0 V
- 3 V
- 4 V



What if $R_1 > R_2 > R_3$?

$\Longrightarrow V_1 > V_2 > V_3$

The internal resistance reduces the potential difference between the terminals.

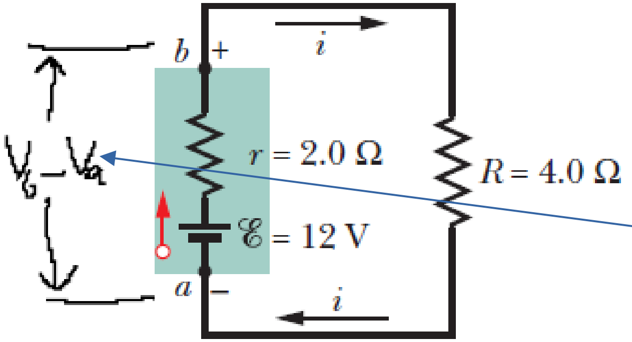


Fig. 27-6 Points a and b, which are at the terminals of a real battery, differ in potential.

Going clockwise from a:

$$V_a + \mathscr{E} - ir = V_b,$$
$$V_b - V_a = \mathscr{E} - ir.$$

$$i = \frac{\mathscr{E}}{R + r}.$$

$$V_b - V_a = \mathscr{E} - \frac{\mathscr{E}}{R + r} r$$
$$= \frac{\mathscr{E}}{R + r} R.$$

$$V_b - V_a = \frac{12\ \text{V}}{4.0\ \Omega + 2.0\ \Omega} 4.0\ \Omega = 8.0\ \text{V}.$$

Going counterclockwise from a:

$$V_a + iR = V_b$$
$$V_b - V_a = iR.$$

$+ \quad i = \frac{\mathscr{E}}{R + r}. \quad \Rightarrow \quad V_b - V_a = 8.0\ \text{V}.$

To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.

- If the internal resistance *r* of the battery in the previous case were zero, *V* would be equal to the emf *ξ* of the battery-namely, 12 V. (Fig 27.6)
- However, since $r = 2.0\ \Omega$, *V* is less than ξ.

The internal resistance reduces the potential difference between the terminals.

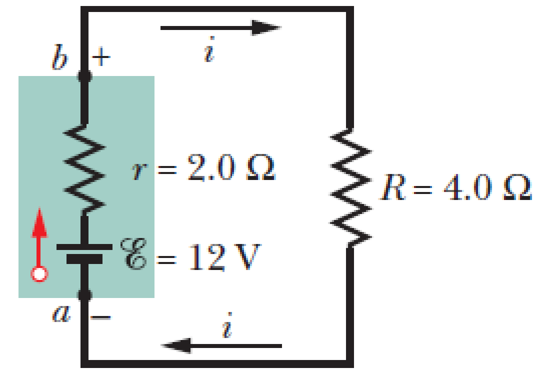
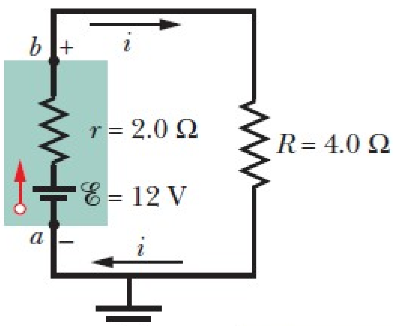


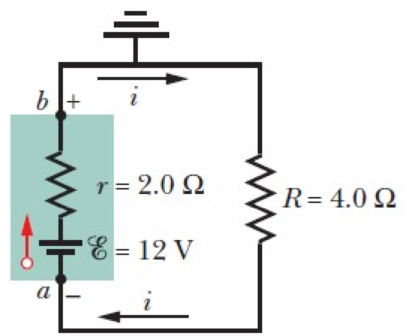
Fig. 27-6 Points *a* and *b*, which are at the terminals of a real battery, differ in potential.

- *Grounding a circuit* usually means connecting the circuit to a conducting path to Earth's surface, and such a connection means that the potential is defined to be **zero** at the grounding point in the circuit.



(*a*)

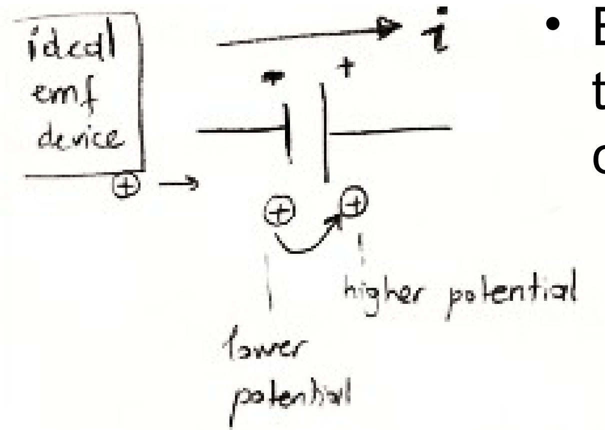
Ground is taken to be zero potential.



(*b*)

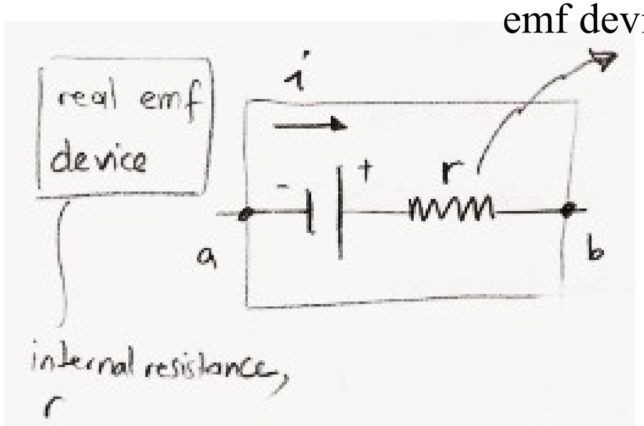
Fig. 27-7 (*a*) Point *a* is directly connected to ground. (*b*) Point *b* is directly connected to ground.

In Fig. 27-7*a*, the potential at *a* is defined to be $V_a = 0$ *and* the potential at *b* is $V_b = 8.0$ V.



- Emf device transfers its energy (stored as chemical) to the charge carrier, so it does work on the charge carriers to establish a current

internal resistance of emf device



- **The net rate *P* of energy transfer** from the emf device to the charge carriers is given by:

$P = iV$ V: potential across the emf device; $V_b - V_a$

$$P = i(\mathscr{E} - ir) = i\mathscr{E} - i^2 r.$$

Other device

P_{emf}: rate at which emf device transfers energy both to the charge carriers and to the thermal energy **(power of emf device)**

Resistor

P_r: rate of energy transfer to the thermal energy within emf device or **internal dissipation rate**

Example, Single loop circuit with two real batteries:

The emfs and resistances in the circuit of Fig. 27-8*a* have the following values:

$$\mathscr{E}_1 = 4.4\text{ V},\quad \mathscr{E}_2 = 2.1\text{ V},\qquad \xi_1 > \xi_2$$

$$r_1 = 2.3\ \Omega,\quad r_2 = 1.8\ \Omega,\quad R = 5.5\ \Omega.$$

(a) What is the current i in the circuit?

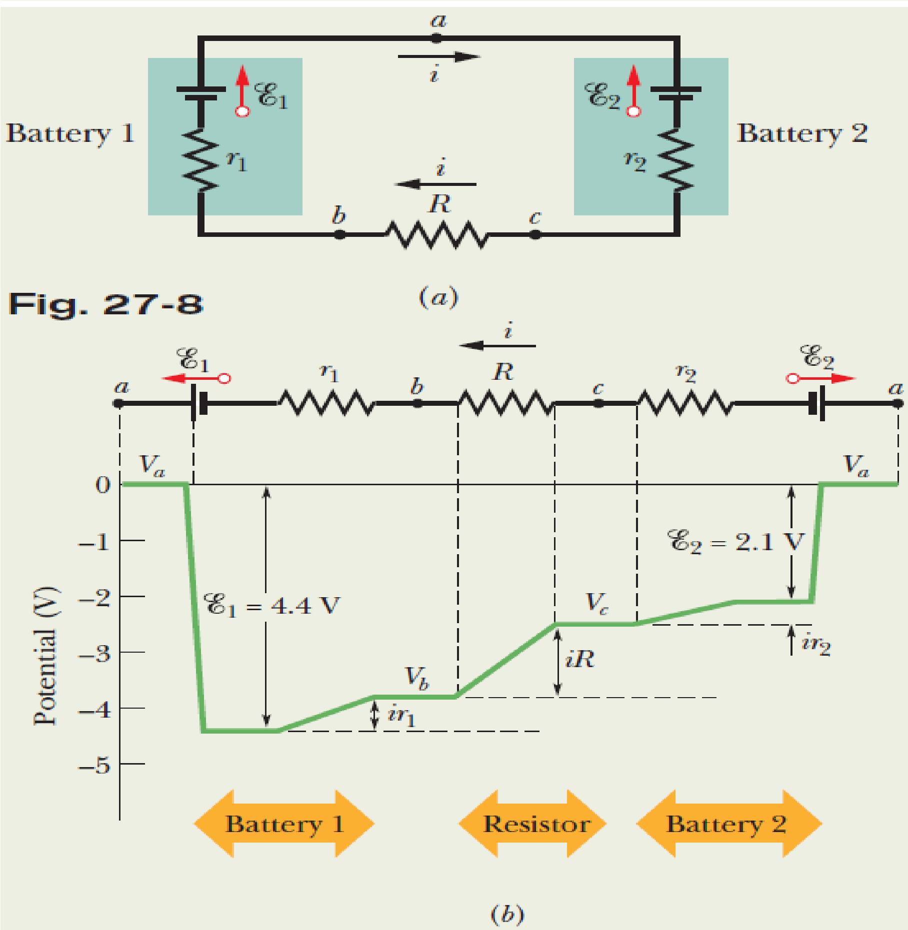


Fig. 27-8

Calculations: Although knowing the direction of i is not necessary, we can easily determine it from the emfs of the two batteries. Because $\mathscr{E}_1$ is greater than $\mathscr{E}_2$, battery 1 controls the direction of i, so the direction is clockwise. (These decisions about where to start and which way you go are arbitrary but, once made, you must be consistent with decisions about the plus and minus signs.) Let us then apply the loop rule by going counterclockwise—against the current—and starting at point a. We find

ccw $$-\mathscr{E}_1 + ir_1 + iR + ir_2 + \mathscr{E}_2 = 0.$$

Check that this equation also results if we apply the loop rule clockwise or start at some point other than a. Also, take the time to compare this equation term by term with Fig. 27-8*b*, which shows the potential changes graphically (with the potential at point a arbitrarily taken to be zero).

Solving the above loop equation for the current i, we obtain

$$i = \frac{\mathscr{E}_1 - \mathscr{E}_2}{R + r_1 + r_2} = \frac{4.4\text{ V} - 2.1\text{ V}}{5.5\ \Omega + 2.3\ \Omega + 1.8\ \Omega}$$

$$= 0.2396\text{ A} \approx 240\text{ mA}. \qquad \text{(Answer)}$$

cw $$-\xi_2 - ir_2 - iR - ir_1 + \xi_1 = 0$$

$$i = \frac{\xi_1 - \xi_2}{r_1 + r_2 + R} \quad ✓$$

Example, Single loop circuit with two real batteries, cont.:

(b) What is the potential difference between the terminals of battery 1 in Fig. 27-8*a*?

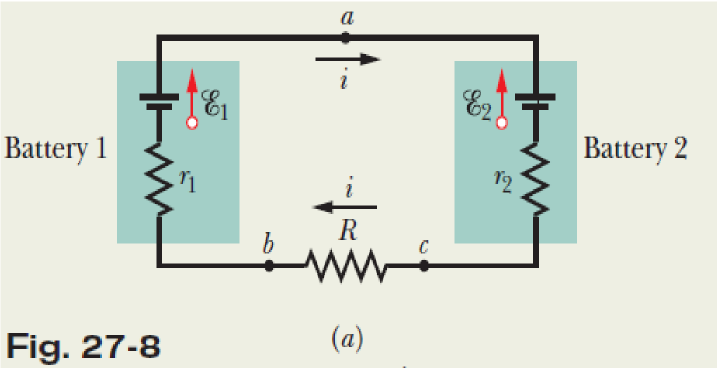
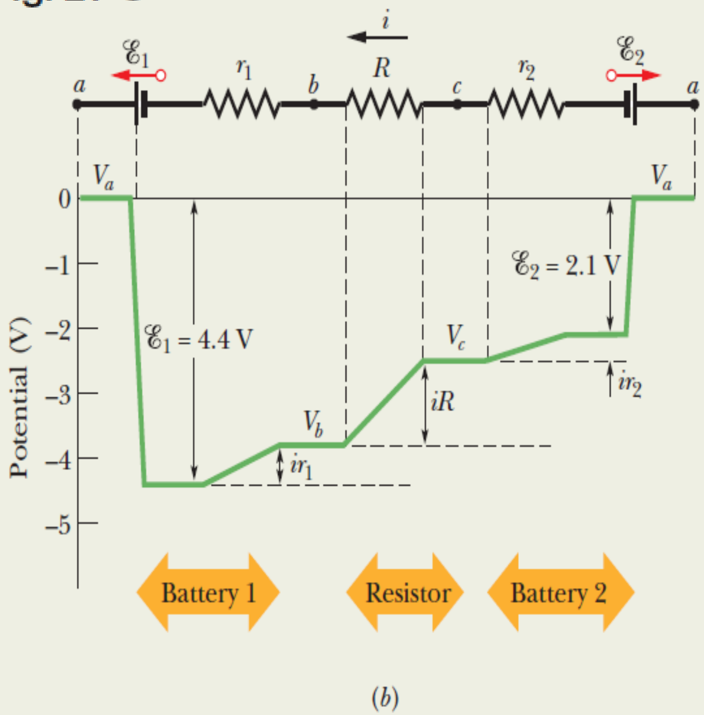


Fig. 27-8



KEY IDEA

We need to sum the potential differences between points *a* and *b*.

Calculations: Let us start at point *b* (effectively the negative terminal of battery 1) and travel clockwise through battery 1 to point *a* (effectively the positive terminal), keeping track of potential changes. We find that

$$V_b - ir_1 + \mathscr{E}_1 = V_a,$$

which gives us

$$\begin{aligned} V_a - V_b &= -ir_1 + \mathscr{E}_1 \\ &= -(0.2396\ \text{A})(2.3\ \Omega) + 4.4\ \text{V} \\ &= +3.84\ \text{V} \approx 3.8\ \text{V}, \qquad \text{(Answer)} \end{aligned}$$

which is less than the emf of the battery. You can verify this result by starting at point *b* in Fig. 27-8*a* and traversing the circuit counterclockwise to point *a*. We learn two points here. (1) The potential difference between two points in a circuit is independent of the path we choose to go from one to the other. (2) When the current in the battery is in the "proper" direction, the terminal-to-terminal potential difference is low.

When a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V.

Parallel resistors and their equivalent have the same potential difference ("par-V").

Resistances connected in parallel can be replaced with an equivalent resistance R_{eq} that has the same potential difference V and the same *total* current i as the actual resistances.

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

where V is the potential difference between a and b. From the junction rule,

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

$$\Longrightarrow \quad i = \frac{V}{R_{eq}}. \quad \Longrightarrow \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad (n \text{ resistances in parallel}).$$

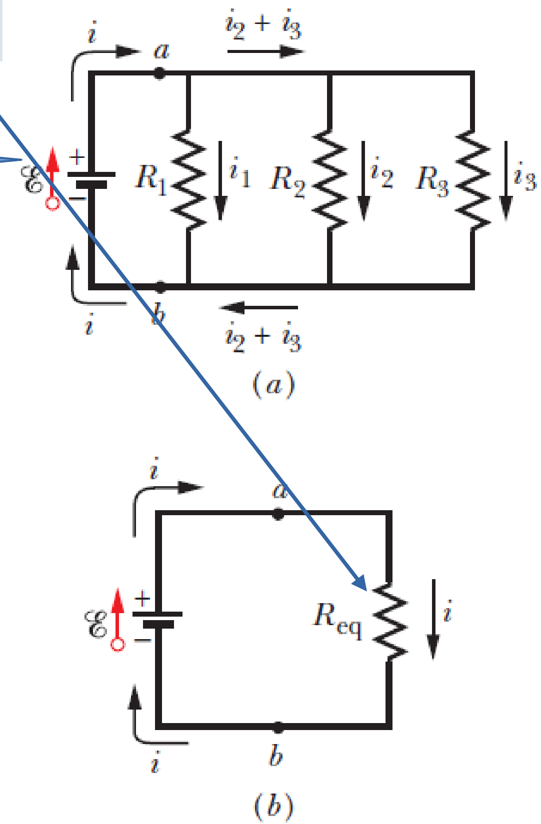
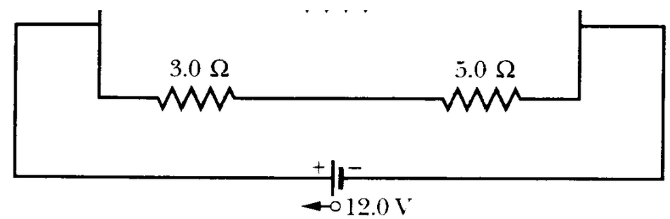


Fig. 27-10 (*a*) Three resistors connected in parallel across points *a* and *b*. (*b*) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq}.

Example

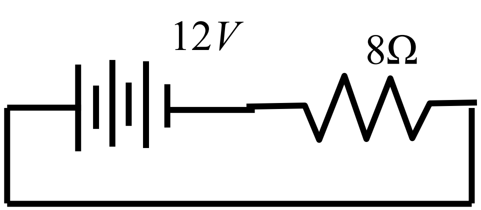
[redacted]

Bottom loop: (all else is irrelevant)
V same in parallel -- PAR-V!



$$i = \frac{V_{batt}}{R} = \frac{12V}{8\Omega} = 1.5A$$

$$E_5 = i_5 R_5 = (1.5A)(5.0\Omega) = 7.5V$$



Which resistor (3 or 5) gets hotter? $P=i^2R$

http://www.phys.lsu.edu/~jdowling/PHYS21132-SP15/lectures/index.html

Example, Resistors in Parallel and in Series:

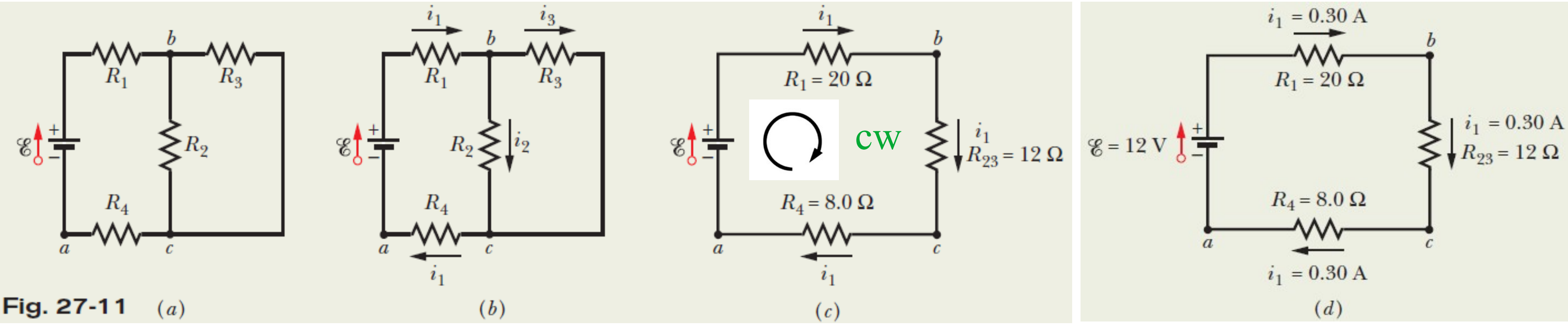


Fig. 27-11 (a) (b) (c) (d)

Figure 27-11*a* shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$$R_1 = 20\ \Omega, \quad R_2 = 20\ \Omega, \quad \mathscr{E} = 12\ \text{V},$$

$$R_3 = 30\ \Omega, \quad R_4 = 8.0\ \Omega.$$

(a) What is the current through the battery?

Note carefully that R_1 and R_2 are *not* in series and thus cannot be replaced with an equivalent resistance. However, R_2 and R_3 are in parallel, so we can use either Eq. 27-24 or Eq. 27-25 to find their equivalent resistance R_{23}. From the latter,

$$R_{23} = \frac{R_2R_3}{R_2 + R_3} = \frac{(20\ \Omega)(30\ \Omega)}{50\ \Omega} = 12\ \Omega.$$

We can now redraw the circuit as in Fig. 27-11*c*; note that the current through R_{23} must be i_1 because charge that moves through R_1 and R_4 must also move through R_{23}. For this simple one-loop circuit, the loop rule (applied clockwise from point *a* as in Fig. 27-11*d*) yields

$$+\mathscr{E} - i_1R_1 - i_1R_{23} - i_1R_4 = 0.$$

Substituting the given data, we find

$$12\ \text{V} - i_1(20\ \Omega) - i_1(12\ \Omega) - i_1(8.0\ \Omega) = 0,$$

which gives us

$$i_1 = \frac{12\ \text{V}}{40\ \Omega} = 0.30\ \text{A}. \qquad \text{(Answer)}$$

Example, Resistors in Parallel and in Series, cont.:

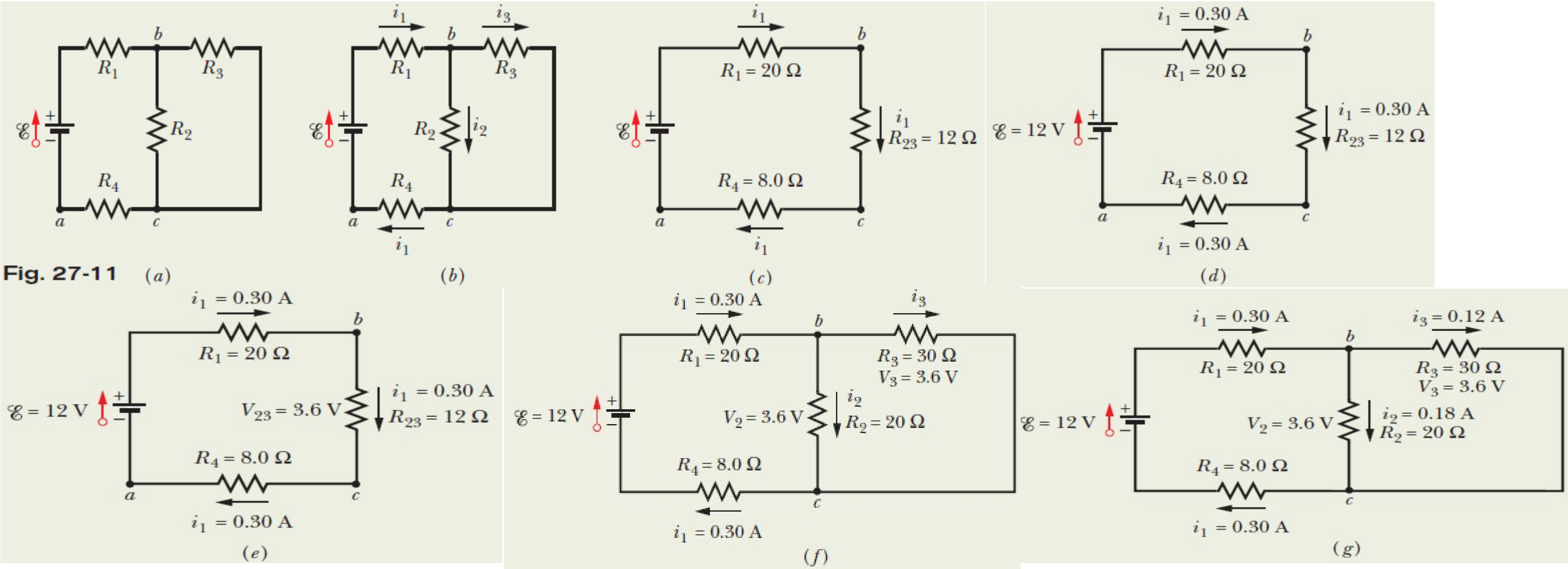


Fig. 27-11

Figure 27-11*a* shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$$R_1 = 20\ \Omega, \quad R_2 = 20\ \Omega, \quad \mathscr{E} = 12\ \text{V},$$

$$R_3 = 30\ \Omega, \quad R_4 = 8.0\ \Omega.$$

(b) What is the current i_2 through R_2?

Working backward: We know that the current through R_{23} is $i_1 = 0.30$ A. Thus, we can use Eq. 26-8 ($R = V/i$) and Fig. 27-11*e* to find the potential difference V_{23} across R_{23}. Setting $R_{23} = 12\ \Omega$ from (a), we write Eq. 26-8 as

$$V_{23} = i_1 R_{23} = (0.30\ \text{A})(12\ \Omega) = 3.6\ \text{V}.$$

The potential difference across R_2 is thus also 3.6 V (Fig. 27-11*f*), so the current i_2 in R_2 must be, by Eq. 26-8 and Fig. 27-11*g*,

$$i_2 = \frac{V_2}{R_2} = \frac{3.6\ \text{V}}{20\ \Omega} = 0.18\ \text{A}. \qquad \text{(Answer)}$$

(c) What is the current i_3 through R_3?

Calculation: Rearranging this junction-rule result yields the result displayed in Fig. 27-11*g*:

$$i_3 = i_1 - i_2 = 0.30\ \text{A} - 0.18\ \text{A} = 0.12\ \text{A}. \qquad \text{(Answer)}$$

Resistance and capacitors

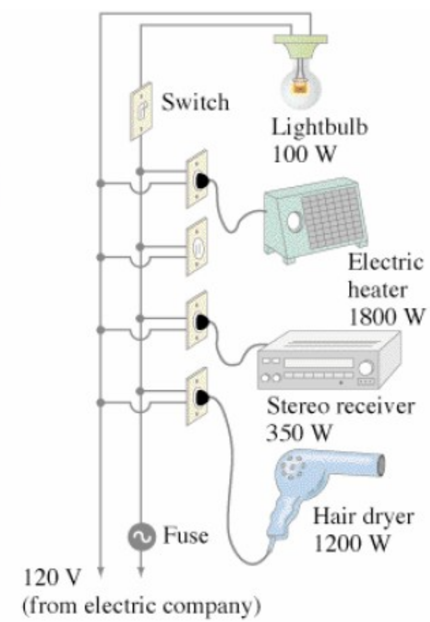
Table 27-1

Series and Parallel Resistors and Capacitors

Series	Parallel	Series	Parallel
Resistors		Capacitors	
$R_{eq} = \sum_{j=1}^{n} R_j$ Eq. 27-7	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ Eq. 27-24	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ Eq. 25-20	$C_{eq} = \sum_{j=1}^{n} C_j$ Eq. 25-19
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors

Household Circuits

- All devices are added in parallel.
- Overload: too many devices that require a lot of current can draw more current than wires can handle.
  - Overheating of wires
  - Fire hazard!



PHY2049: Chapter 27

27-7 Current in Multi-loop Circuits

- There are two **junctions**; b and d
- There are three **branches**; bad, bd, bcd
- According to **junction rule**, which is often called *Kirchoff's junction rule or Krichoff's current law* currents entering any junction must be equal to sum of the currents leaving that junction;

- At junction d; $i_3+i_1= i_2$ or at junction b; $i_2= i_1+i_3$ (1)

The current into the junction must equal the current out (charge is conserved).

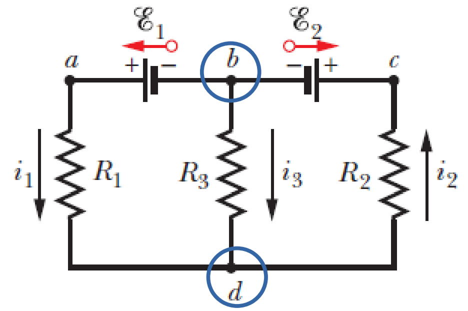
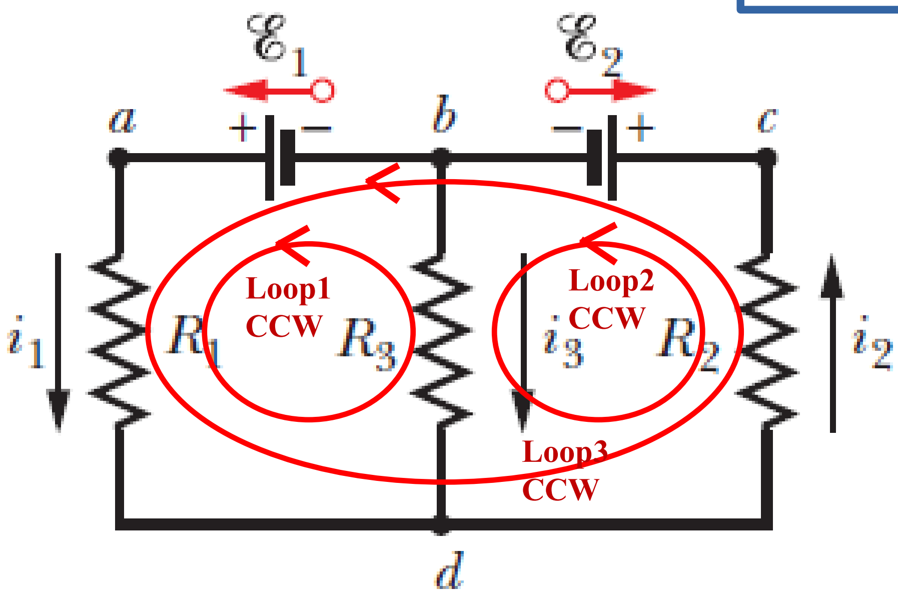


Fig. 27-9 A multiloop circuit consisting of three branches: left-hand branch *bad*, right-hand branch *bcd*, and central branch *bd*. The circuit also consists of three loops: left-hand loop *badb*, right-hand loop *bcdb*, and big loop *badcb*.

To find currents i_1,i_2 and i_3 apply **LOOP RULE**;



For the loop1, $\mathscr{E}_1 - i_1R_1 + i_3R_3 = 0.$ (2)

For the loop2, $-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0.$ (3)

and for the entire loop(3),

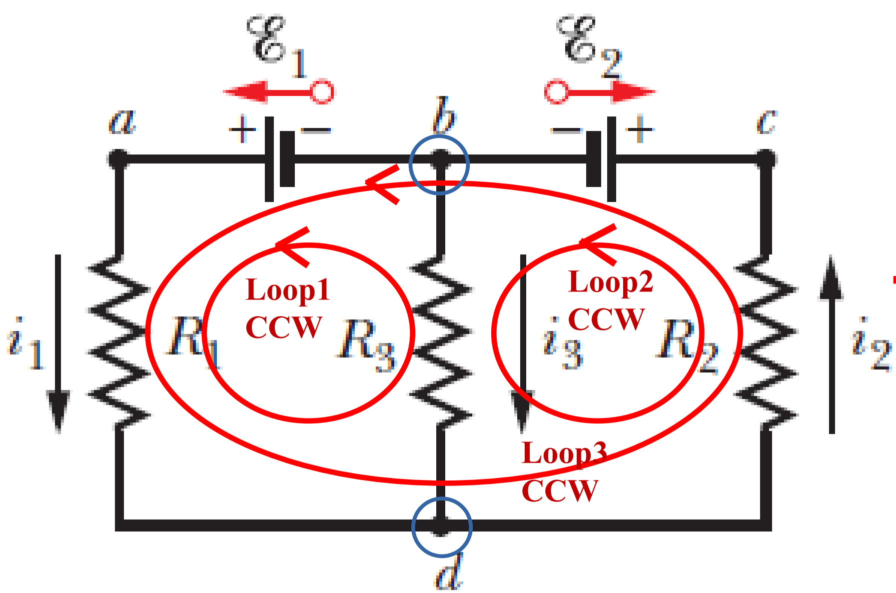
$$\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0.$$

3 unknowns (i_1, i_2, i_3) & 3 equations

Equations to Unknowns

Continue applying loop and junction rules until you have as many equations as unknowns!

Given: $\xi_1, \xi_2, \xi_3, i_2, R_1, R_2, R_3$



For the loop1, $\mathscr{E}_1 - i_1R_1 + i_3R_3 = 0.$

For the loop2, $-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0.$

~~**For the loop(3),** $\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0.$~~

loop1, loop2 and junction, $i_2 = i_1 + i_3$

Solve for i_1, i_3

Example, Multi-loop circuit and simultaneous loop equations:

Figure 27-13 shows a circuit whose elements have the following values:

$$\mathscr{E}_1 = 3.0\text{ V},\quad \mathscr{E}_2 = 6.0\text{ V},$$
$$R_1 = 2.0\ \Omega,\quad R_2 = 4.0\ \Omega.$$

The three batteries are ideal batteries. Find the magnitude and direction of the current in each of the three branches.

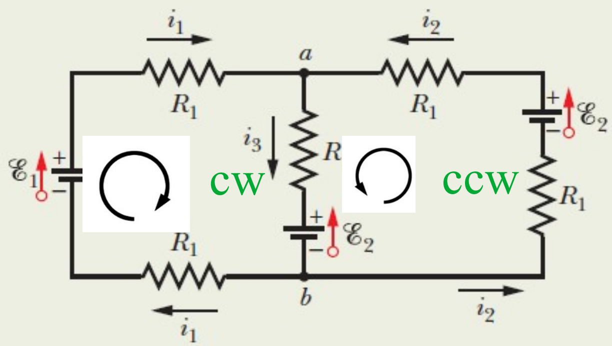


Fig. 27-13 A multiloop circuit with three ideal batteries and five resistances.

Junction rule: Using arbitrarily chosen directions for the currents as shown in Fig. 27-13, we apply the junction rule at point a by writing

$$i_3 = i_1 + i_2. \qquad (27\text{-}26)$$

Left-hand loop: We first arbitrarily choose the left-hand loop, arbitrarily start at point b, and arbitrarily traverse the loop in the clockwise direction, obtaining

$$-i_1R_1 + \mathscr{E}_1 - i_1R_1 - (i_1 + i_2)R_2 - \mathscr{E}_2 = 0,$$

where we have used $(i_1 + i_2)$ instead of i_3 in the middle branch. Substituting the given data and simplifying yield

$$i_1(8.0\ \Omega) + i_2(4.0\ \Omega) = -3.0\text{ V}. \qquad (27\text{-}27)$$

Right-hand loop: For our second application of the loop rule, we arbitrarily choose to traverse the right-hand loop counterclockwise from point b, finding

$$-i_2R_1 + \mathscr{E}_2 - i_2R_1 - (i_1 + i_2)R_2 - \mathscr{E}_2 = 0.$$

Substituting the given data and simplifying yield

$$i_1(4.0\ \Omega) + i_2(8.0\ \Omega) = 0. \qquad (27\text{-}28)$$

Combining equations: We now have a system of two equations (Eqs. 27-27 and 27-28) in two unknowns (i_1 and i_2) to solve either "by hand" (which is easy enough here) or with a "math package." (One solution technique is Cramer's rule, given in Appendix E.) We find

$$i_1 = -0.50\text{ A}. \qquad (27\text{-}29)$$

(The minus sign signals that our arbitrary choice of direction for i_1 in Fig. 27-13 is wrong, but we must wait to correct it.) Substituting $i_1 = -0.50$ A into Eq. 27-28 and solving for i_2 then give us

$$i_2 = 0.25\text{ A}. \qquad \text{(Answer)}$$

With Eq. 27-26 we then find that

$$i_3 = i_1 + i_2 = -0.50\text{ A} + 0.25\text{ A}$$
$$= -0.25\text{ A}.$$

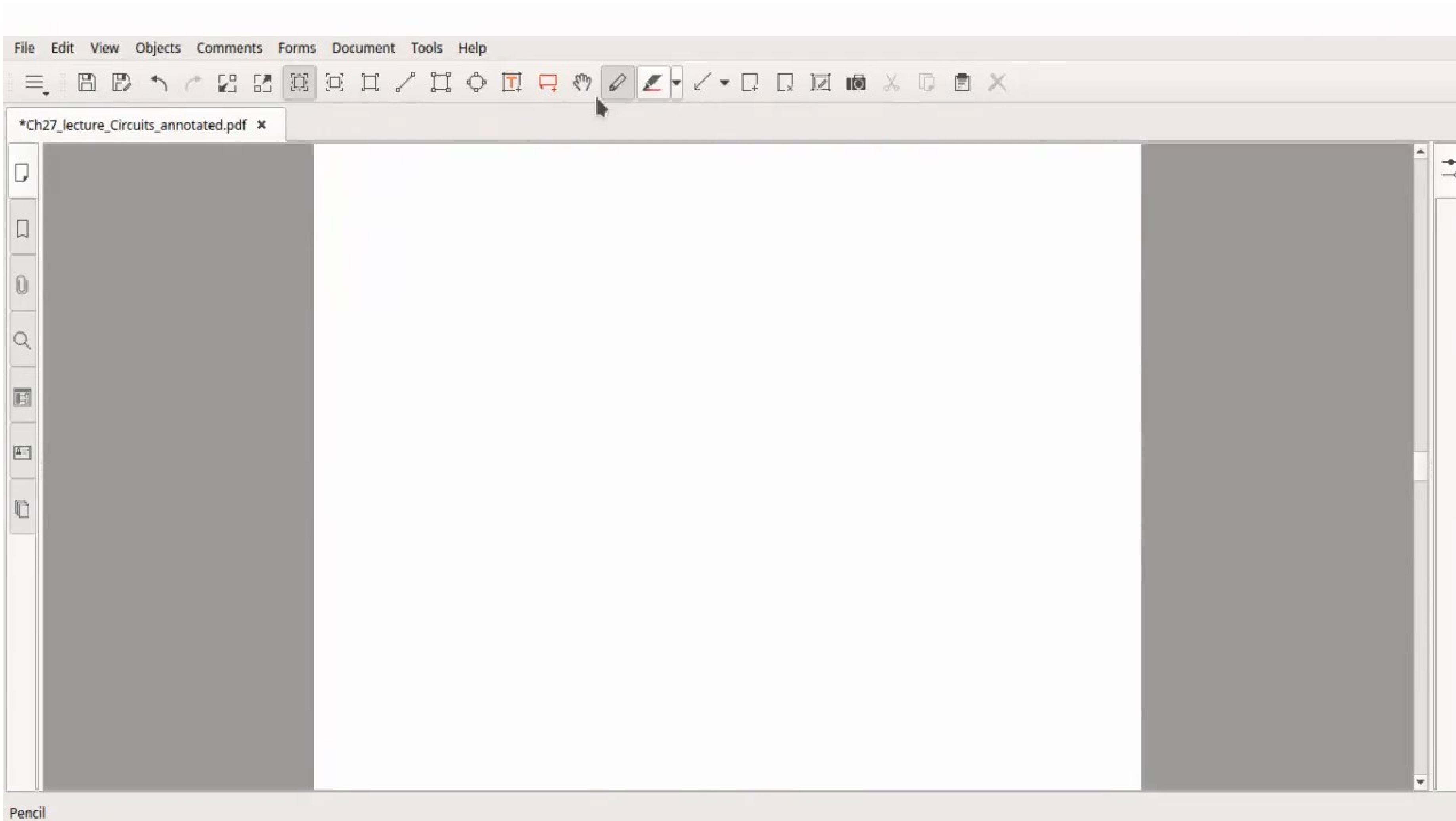
(-) indicates opposite direction

The positive answer we obtained for i_2 signals that our choice of direction for that current is correct. However, the negative answers for i_1 and i_3 indicate that our choices for those currents are wrong. Thus, as a *last step* here, we correct the answers by reversing the arrows for i_1 and i_3 in Fig. 27-13 and then writing

$$i_1 = 0.50\text{ A} \quad \text{and} \quad i_3 = 0.25\text{ A}. \qquad \text{(Answer)}$$

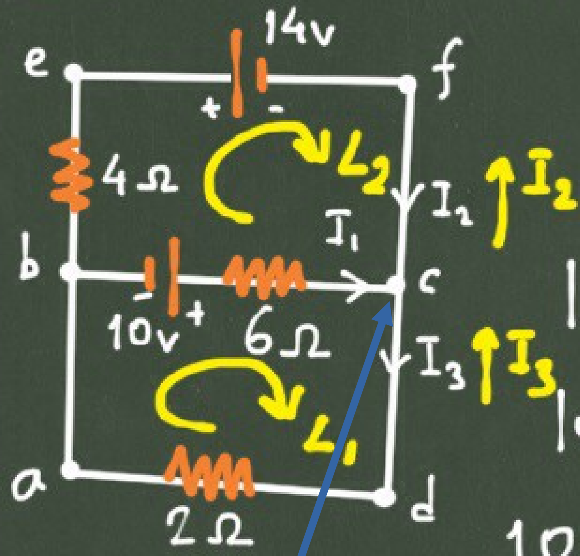
27-7 Multi-loop Circuits

Video: Example, Multi-loop circuit and simultaneous loop equations



By Aziz Kolkıran

Ex: Kirchhoff circuit - 1



Find I_1, I_2, I_3.

junction c : $I_1 + I_2 = I_3$

loop 1, abcda : $10v - 6I_1 - 2I_3 = 0$

loop 2, befcb : $-14v + 6I_1 - 10v - 4I_2 = 0$

$10 - 6I_1 - 2(I_1 + I_2) = 0 \rightarrow 10 = 8I_1 + 2I_2$

$-1 / + \quad -12 = -3I_1 + 2I_2$

$22 = 11 I_1 \rightarrow \boxed{I_1 = 2\,A}$

$\Rightarrow I_2 = -3\,A, \quad I_3 = -1\,A$

opposite direction! but values are correct.

$i_1 + i_3 = i_2$

- An instrument used to measure currents is called an ***ammeter.***
  - It is essential that the **resistance R_A** of the ammeter be **very much smaller** than other resistances in the circuit.
- Connected in series to circuit.

So that, all the charge could flow through.

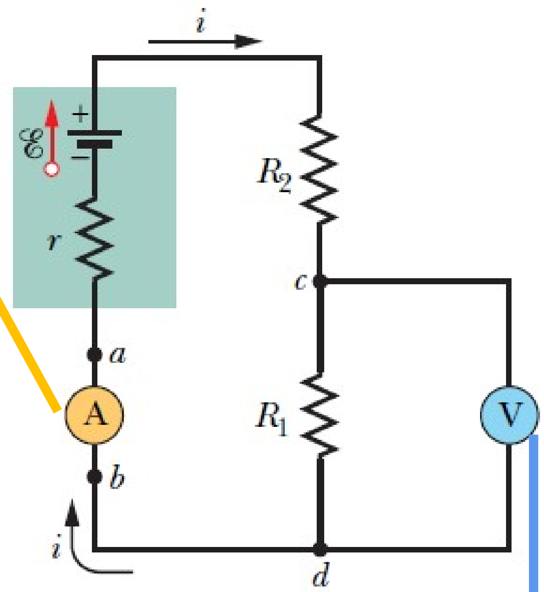
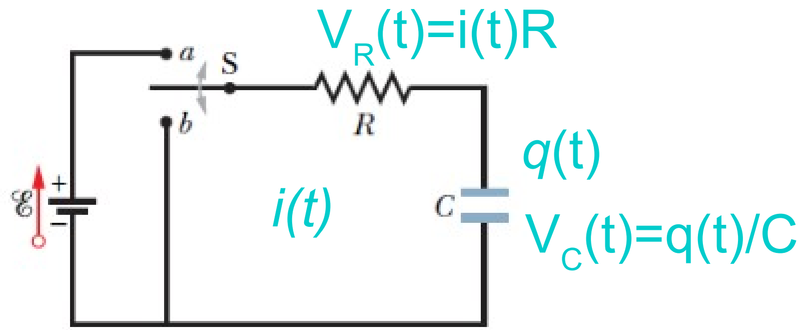


Fig. 27-14 A single-loop circuit, showing how to connect an ammeter (A) and a voltmeter (V).

- A meter used to measure potential differences is called a ***voltmeter.***
  - It is essential that the **resistance R_V** of a voltmeter be **very much larger** than the resistance of any circuit element across which the voltmeter is connected.
- Connected in parallel to circuit.

So that, no charge can flow through.

RC Circuit



- In these circuits, current will change for a while, and then stay constant.
- We want to solve for current as a function of time *i(t)=dq/dt.*
- The charge on the capacitor will also be a function of time: *q*(t)
- The voltage across the resistor ($V_R(t)$) and the capacitor ($V_C(t)$) also change with time.
- To charge the capacitor, close the switch on *a*.

Charging a Capacitor:

We know that:

$$i = \frac{dq}{dt}. \longrightarrow R\frac{dq}{dt} + \frac{q}{C} = \mathscr{E}$$

It turns out that:

$$\xi + V_R(t) + V_C(t) = 0$$

$$\mathscr{E} - iR - \frac{q}{C} = 0.$$

(charging equation).

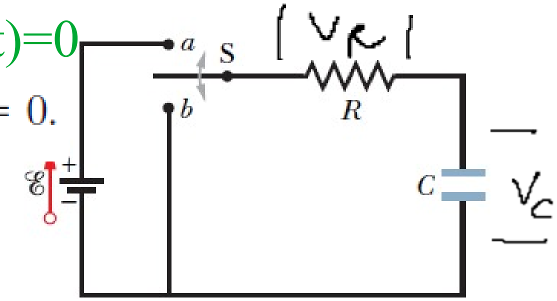


Fig. 27-15 When switch S is closed on *a*, the capacitor is *charged* through the resistor. When the switch is afterward closed on *b*, the capacitor *discharges* through the resistor.

$$q = C\mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

$$V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

- The product *RC* is called the **capacitive time constant** of the circuit and is represented with the symbol τ:

$$\tau = RC \quad \text{(time constant).}$$

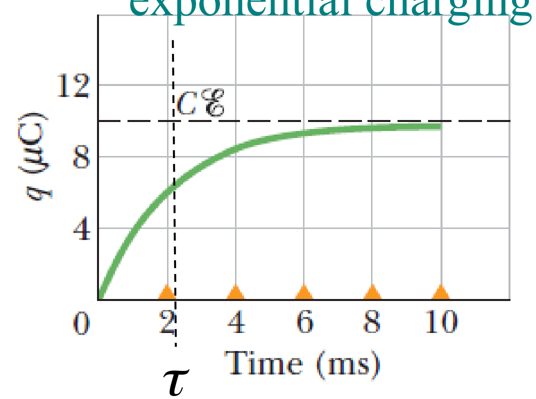
- At time $t = \tau = (RC)$, the charge on the initially uncharged capacitor increases from zero to:

$$q = C\mathscr{E}(1 - e^{-1}) = 0.63C\mathscr{E}.$$

- During the first time constant τ the charge has increased from zero to 63% of its final value $C\xi$.

The capacitor's charge grows as the resistor's current dies out.

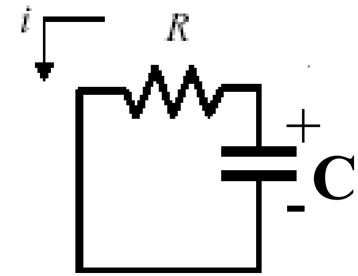
exponential charging



(*a*)

Discharging a Capacitor:

- Assume that the **capacitor** of the figure is **fully charged (switch closed for a long time)** *to a potential* V_0 *equal to the emf of the battery* ξ.
- At a new time $t = 0$, switch *S* is thrown from *a* to *b* so that the capacitor can discharge through resistance *R*.



Stored charges find their way across the circuit, as establishing a current

i(t) $\quad q(t)/C=\xi=V_C \qquad V_R(t)+V_C(t)=0$

$$R\frac{dq}{dt}+\frac{q}{C}=0 \qquad \text{(discharging equation).}$$

$$q = q_0 e^{-t/RC} \qquad \text{(discharging a capacitor),}$$

$$i(t)=dq/dt=(\xi/R)e^{-t/RC}$$

Time constant: $\tau = RC$

Time i drops to 1/e.

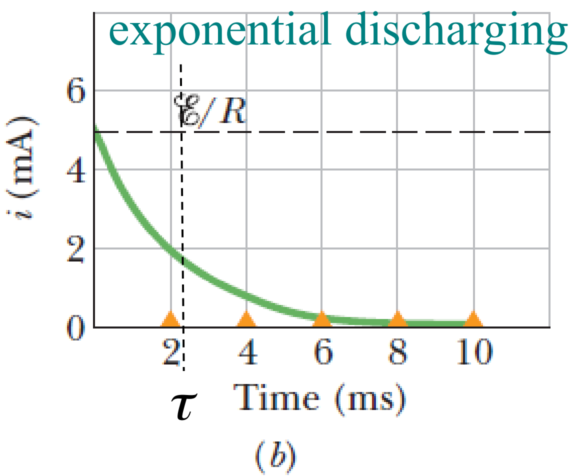
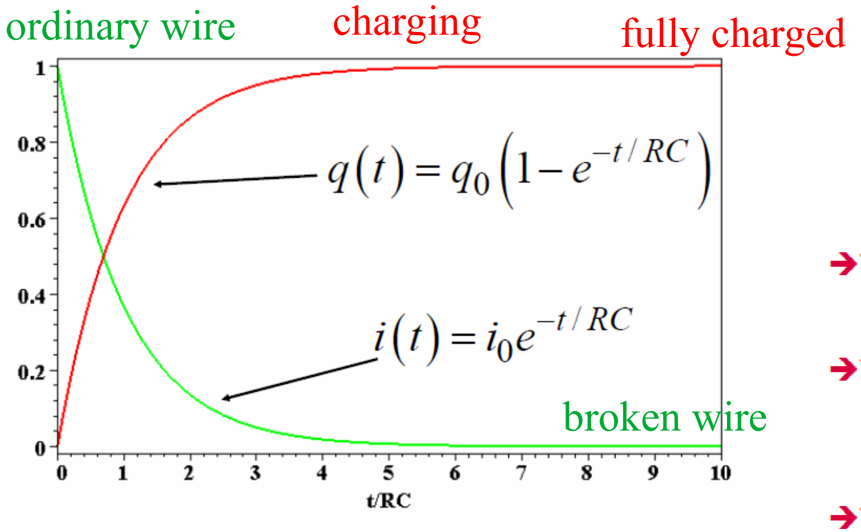


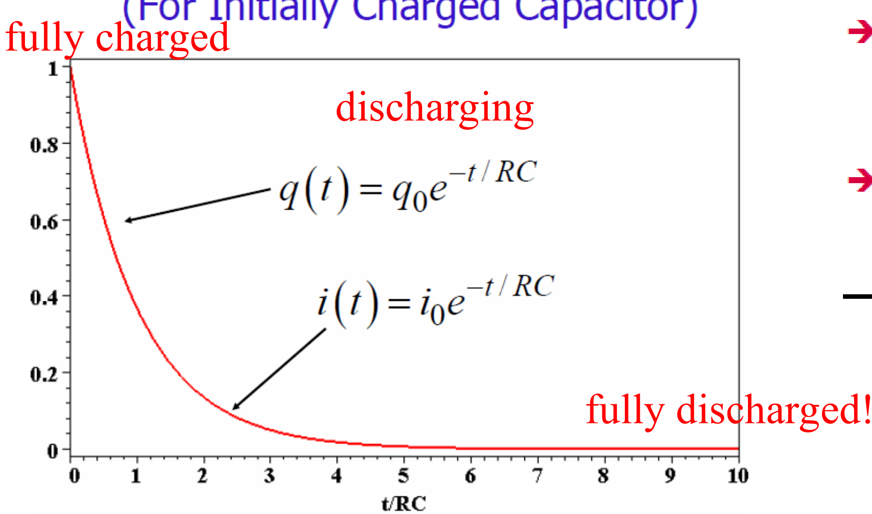
Fig. 27-16 (*b*) This shows the decline of the charging current in the circuit. The curves are plotted for *R* =2000 Ω, *C* =1 μF, *and E* =10 *V;* the small triangles represent successive intervals of one time constant τ.

Charge and Current vs Time (For Initially Uncharged Capacitor)



PHY2049: Chapter 27

Charge and Current vs Time (For Initially Charged Capacitor)



PHY2049: Chapter 27

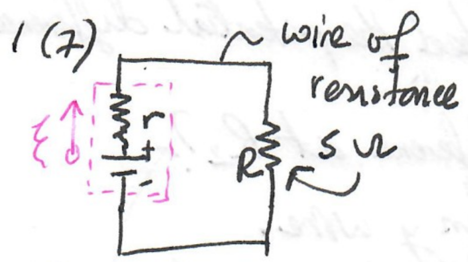
Exponential Behavior

- t = RC is the "characteristic time" of any RC circuit
  - Only t / RC is meaningful
- t = RC
  - Current falls to 37% of maximum value
  - Charge rises to 63% of maximum value
- t =2RC
  - Current falls to 13.5% of maximum value
  - Charge rises to 86.5% of maximum value
- t =3RC
  - Current falls to 5% of maximum value
  - Charge rises to 95% of maximum value
- t =5RC
  - Current falls to 0.7% of maximum value
  - Charge rises to 99.3% of maximum value

PHY2049: Chapter 27

1. (7) A wire of resistance 5.0 Ω is connected to a battery whose emf Ɛ is 2.0 V and whose internal resistance is 1.0 Ω. In 2.0 min, how much energy is (a) transferred from chemical form in the battery, (b) dissipated as thermal energy in the wire, and (c) dissipated as thermal energy in the battery

1 (7)



$\varepsilon = 2.0\,V$
$r = 1\,\Omega$
$t = 2.0\,min$

i) $U = ?$ $P = \frac{\Delta U}{\Delta t}$ $\rightarrow U = Pt = \frac{\varepsilon^2}{R_{eqv}} t = \frac{(2.0V)^2}{6\Omega} 120s = 80\,J$ ~ required (chemical) energy to set up a potential difference of 2.0V during 2.0 min.

$V = \frac{U}{q}$, $i = \frac{q}{t}$ | $P = iV = \frac{U}{t}$; $V \rightarrow \varepsilon$, $P = \varepsilon^2 / R_{eqv}$

ii) dissipated thermal energy at R (wire)

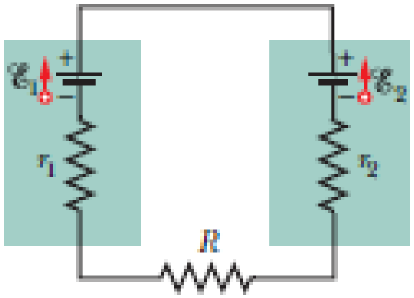
$P = i^2 R = \left(\frac{\varepsilon}{R_{eqv}}\right)^2 R = \left(\frac{2.0V}{6\Omega}\right)^2 5\Omega = \frac{5}{9} V^2/\Omega$ | $U = Pt = \frac{5}{9}\frac{V^2}{\Omega} 120s = \frac{200}{3} J = 66.7\,J$

iii) dissipated thermal energy at r (battery)

$P = i^2 r = \left(\frac{2.0V}{6\Omega}\right)^2 1\Omega = \frac{1}{9} V^2/\Omega$ | $U' = Pt = \frac{1}{9}\frac{V^2}{\Omega} 120s = 13.3\,J$

OR $80\,J - 66.7\,J = 13.3\,J$ $U - U'_R = U''_r$

2. (10) (a) In Figure, what value must R have if the current in the circuit is to be 1.0 mA? Take $\mathcal{E}_1$=2.0 V, $\mathcal{E}_2$=3.0 V, and r_1=r_2=3.0 Ω. (b) What is the rate at which thermal energy appears in R?

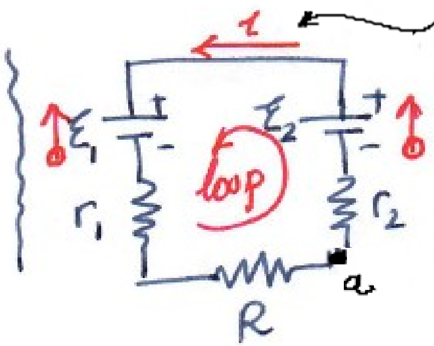


2(10)

$i = 1 \times 10^{-3}$ A

$\mathcal{E}_1 = 2$ V $\quad r_1 = 3\,\Omega$

$\mathcal{E}_2 = 3$ V $\quad r_2 = 3\,\Omega$



$\mathcal{E}_2 > \mathcal{E}_1$ & i: current at the circuit

$$-ir_2 + \mathcal{E}_2 - \mathcal{E}_1 - ir_1 - iR = 0 \quad \text{(starting point a)}$$

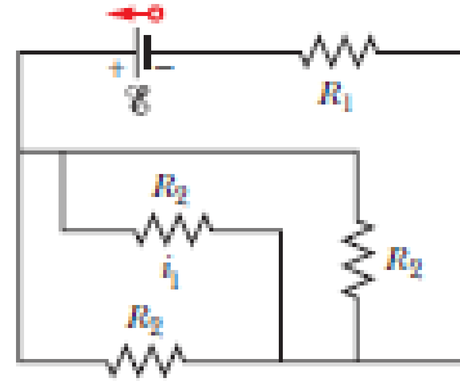
$$-(1\,mA)3\,\Omega + 3\,V - 2\,V - (1\,mA)3\,\Omega - (1\,mA)R = 0$$

$$-(2\,mA)3\,\Omega + 1\,V = (1\,mA)R$$

$$R = \frac{1\,V - (2\,mA)3\,\Omega}{1\,mA} = 994\,\Omega$$

$$P = i^2 R = (1 \times 10^{-3}\,A)^2\, 994\,\Omega = 9.94 \times 10^{-4}\,W$$

3. (29) In Figure, R_1=6.00 Ω, R_2=18.0 Ω, and the ideal battery has emf ℰ=12.0 V. What are the (a) size and (b) direction (left or right) of current i_1? (c) How much energy is dissipated by all four resistors in 1.00 min?



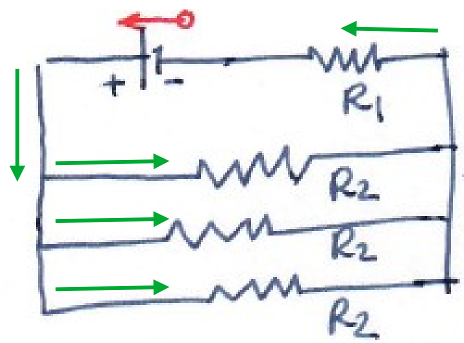
3 (29)

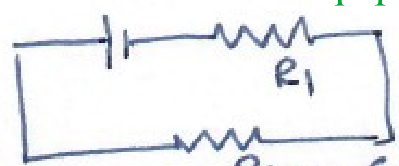
$R_1 = 6\,\Omega$

$R_2 = 18\,\Omega$

$\mathcal{E} = 12\,V$

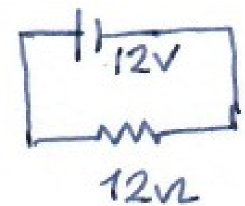
$i_1 = ?$





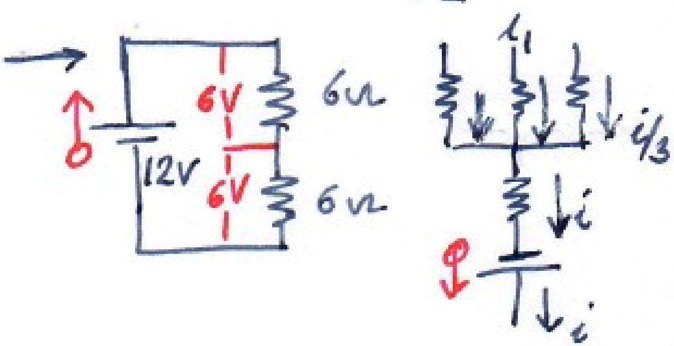
$R_{eqv} = 6\,\Omega$

$$\frac{1}{R_{eqv}} = \frac{1}{18} + \frac{1}{18} + \frac{1}{18}$$



i=1 A

$$\Rightarrow i = \frac{V}{R} = 1\,A$$

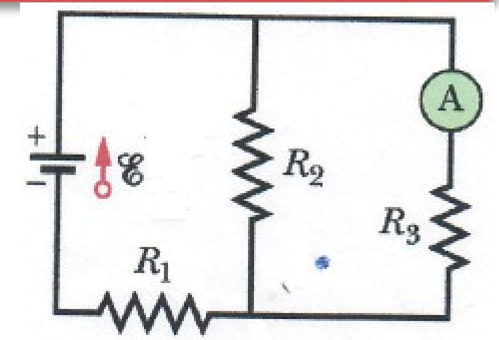


$i_1 = i/3 = 1A/3 = 0.33A$ (Rightward)

$P = i^2 R_{eqv} = (1A)^2 (12\Omega) = 12\,W$

$P = \frac{\Delta U}{\Delta t} \rightsquigarrow \Delta U = (12\,W)(60\,sec) = 720\,J$

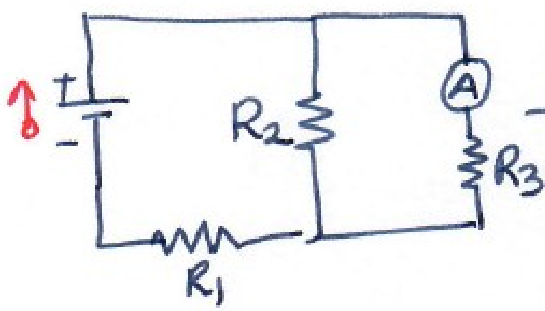
4. (37) In Figure, determine what ammeter will read, assuming $\mathcal{E}$=5.0 V (for the ideal battery), R_1=2.0 Ω, R_2=4.0 Ω, and R_3=6.00 Ω.

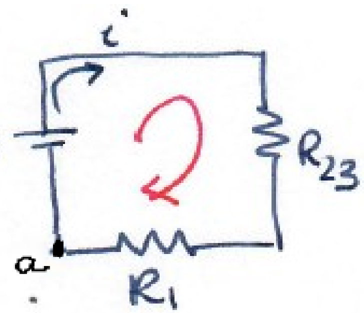


the resistance of (A) should be very small

(37) i)

$R_1 = 2\Omega$
$R_2 = 4\Omega$
$R_3 = 6\Omega$
$\mathcal{E} = 5V$
$i_3 = ?$

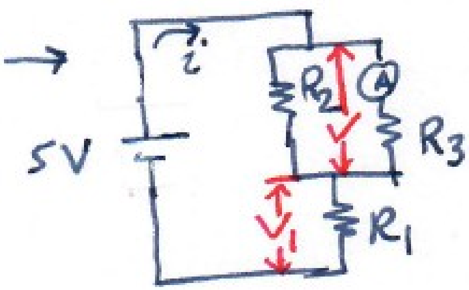


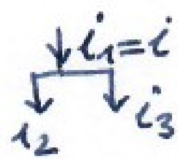


$\mathcal{E} - iR_{23} - iR_1 = 0$ (a)

$i = \frac{\mathcal{E}}{R_{23}+R_1} = \frac{5V}{\frac{R_2R_3}{R_2+R_3}+2\Omega}$

$= 1.14\,A = i_1$





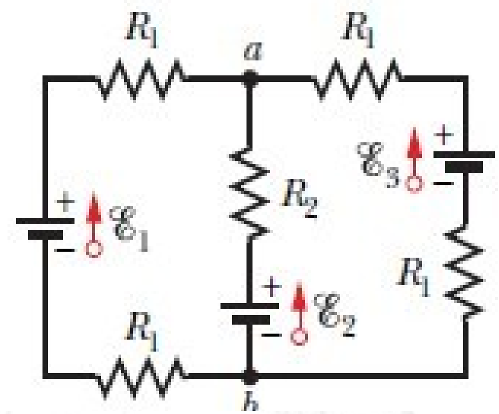
$i_3 = \frac{V}{R_3}$

$= \frac{5V - V_1}{R_3} = \frac{5V - iR_1}{6\Omega} = \frac{5V - (1.14A)2\Omega}{6\Omega} = 0.45A$

$V_{R_2} = V_{R_3} = V$

$V + V_{R_1} = 5V = \mathcal{E}$

5. (45) In Figure, the resistances are R_1 =1.0 Ω and R_2 =2.0 Ω, and the ideal batteries have emfs $\mathcal{E}_1$=2.0 V and $\mathcal{E}_2 = \mathcal{E}_3$ = 4.0 V. What are the (a) size and (b) direction (up or down) of the current in battery 1, the (c) size and (d) direction of the current in battery 2, and the (e) size and (f) direction of the current in battery 3? (g) What is the potential difference V_a - V_b?



7(45)

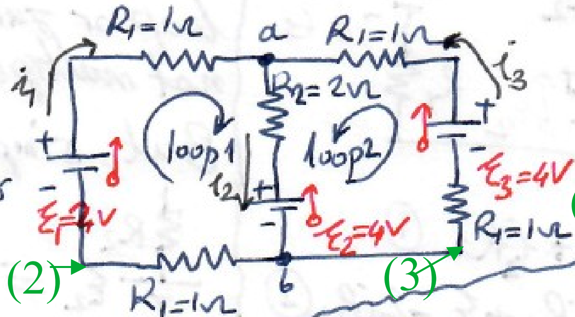
$R_1 = 1\Omega$

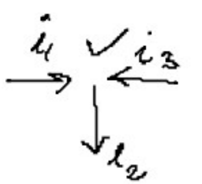
$R_2 = 2\Omega$

ideal batteries

$\mathcal{E}_1 = 2V$

$\mathcal{E}_2 = \mathcal{E}_3 = 4V$





$i_1 + i_3 = i_2$

(1) $i_1 + i_3 = i_2$: Junction Rule ✔

(2) $\mathcal{E}_1 - i_1 R_1 - i_2 R_2 - \mathcal{E}_2 - i_1 R_1 = 0$: loop 1 ✔

(3) $-i_3 R_1 + \mathcal{E}_3 - i_3 R_1 - i_2 R_2 - \mathcal{E}_2 = 0$: loop 2 ✔

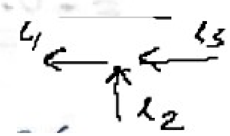
3 unknowns 3 equations

$\Rightarrow$ $\mathcal{E}_1 - \mathcal{E}_2 - 2i_1 R_1 - i_2 R_2 = 0$ | $2V - 4V - 2i_1(1\Omega) - i_2(2\Omega) = 0$ | $2i_1 + 2i_2 = -2$

$\mathcal{E}_3 - \mathcal{E}_2 - 2i_3 R_1 - i_2 R_2 = 0$ | $4V - 4V - 2i_3(1\Omega) - i_2(2\Omega) = 0$ | $2i_3 + 2i_2 = 0$

$\Rightarrow$ $2i_1 + 2i_2 = -2$, $2(i_2 - i_1) + 2i_2 = 0$ | $i_1 + i_2 = -1$, $-2i_1 + 4i_2 = 0$ | $i_1 = 2i_2$ | $2i_2 + i_2 = -1$, $i_2 = -1/3\,A$ | $i_1 = -2/3\,A$, $i_3 = 1/3\,A$

i_3 : direction is correct



$i_1 = -0.66\,A$ (downward)

$i_2 = -0.33\,A$ (upward)

$i_3 = 0.33\,A$ (upward)

$V_a - V_b = ?$

$\downarrow i_1$ $\uparrow i_2$ $\uparrow i_3$ in circuit

3 possible paths:

$V_a - i_1 R_1 - \mathcal{E}_1 - i_1 R_1 = V_b$ ①

$V_a + i_3 R_1 - \mathcal{E}_3 + i_3 R_1 = V_b$ ②

$V_a + i_2 R_2 - \mathcal{E}_2 = V_b$ ③

All gives $V_a - V_b = 3.33V$

6. (49) A 3 MΩ resistor and 1 μF capacitor are connected to in series with an ideal battery of emf Ɛ=4.0 V. At 1.0 after the connection is made, what are the rates at which (a) The charge of capacitor is increasing, (b) Energy is being stored in the capacitor, (c) Thermal energy is appearing in the resistor, (d) Energy is being delivered by the battery.

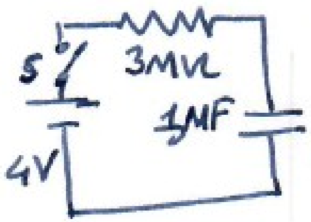
a → current dq/dt

b, c, d → $P \approx \Delta U/\Delta t$

charging the capacitor

(49)

After 1sec: What are the rates? $q(t) = C\mathcal{E}(1-e^{-t/\tau})$ & $\tau = RC = 3$ sec



i) $\frac{dq}{dt} = \frac{C\mathcal{E}}{\tau} e^{-t/\tau} = \frac{(1\times10^{-6}F)(4V)}{3s} e^{-1/3} = 9.55\times10^{-7}$ C/s : rate ($\frac{C}{s} \equiv$ Ampere)

ii) $\frac{dU_c}{dt} = ?$ $\{ U_c = \frac{q^2}{2C}$ $|$ $\frac{dU_c}{dt} = \frac{q}{C}\frac{dq}{dt} = \frac{q(t=1s)}{1\times10^{-6}F}(9.55\times10^{-7} C/s)$ $\} q(t=1) = 1.13\times10^{-6}$ C

$= 1.08\times10^{-6}$ J/s → $P = 1.08\times10^{-6}$ W

iii) $\frac{dU_R}{dt} = ?$ $\} P = i^2R = (9.55\times10^{-7} C/s)^2(3\ M\Omega) = 2.74\times10^{-6}$ W

iv) $\frac{dU_B}{dt} = ?$ $\} P = iV = i\mathcal{E} = (9.55\times10^{-7} C/s)(4V) = 3.82\times10^{-6}$ W

Check $i\mathcal{E} = \frac{q}{C}\frac{dq}{dt} + i^2R \rightarrow 3.82\times10^{-6}$ W $\stackrel{\checkmark}{=} 1.08\times10^{-6}$ W $+ 2.74\times10^{-6}$ W

(iv) (ii) (iii)

7. (65) In Figure, R_1 = 10.0 kΩ, R_2 = 15.0 kΩ, C = 0.400 μF, and the ideal battery has emf ℰ=20.0 V. First, the switch is closed a long time so that the steady state is reached. Then the switch is opened at time t= 0. What is the current in resistor 2 at t =4.00 ms?

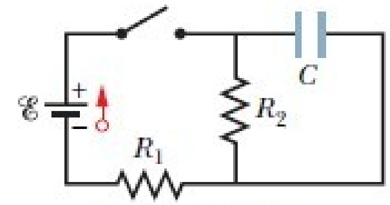
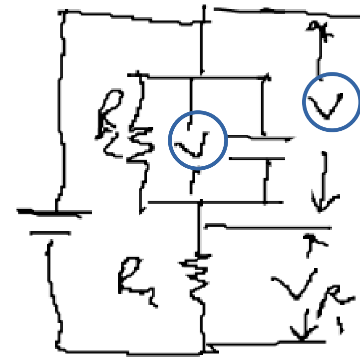


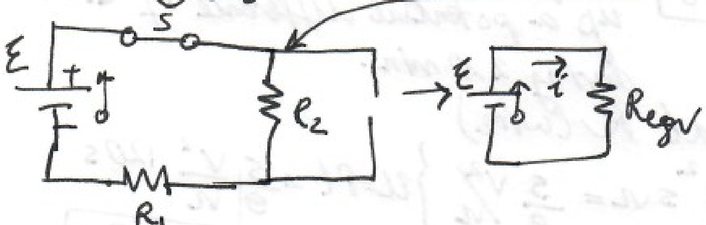
Fig. 27-66

10 (65)

R_1=10.0 kΩ
R_2=15.0 kΩ
C=0.400 μF
ℰ=20.0 V
ideal battery

switch is closed → steady state is reached
→ capacitor reached the potential difference at R_2
⇒ So, what is the potential difference at R_2?
– During charging acts as ordinary wire.
After a long time (steady state is reached) acts as broken wire





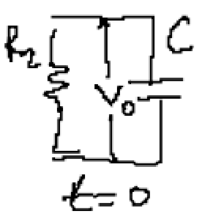
$i = \frac{ℰ}{R_1+R_2}$

Potential difference at R_2

$\Rightarrow V = iR_2 = R_2 \frac{ℰ}{R_1+R_2}$ ⟶ V_0

$= (15 kΩ)\frac{20 V}{25 kΩ}$

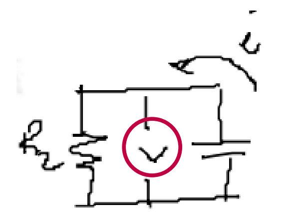
$= 12 V$



⇒ Now switch is opened (time t=0)

$i_{at R_2}$ = ? when t=4 ms

$V = V_0 \exp(-t/RC) = (12 V)\exp\left(-\frac{4\times10^{-3} s}{15\times10^3 Ω \cdot 0.400\times10^{-6} F}\right) = 6.16 V$



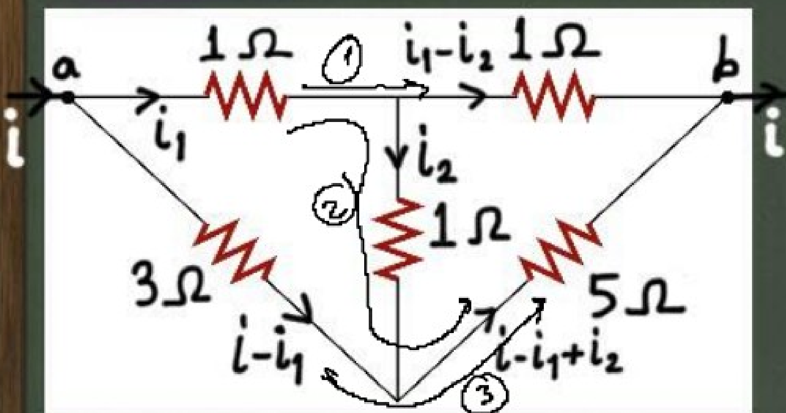
Potential drop from t=0 (12V) to t=4 ms (6.16 V)

V_0 V at 4 ms

$i = \frac{6.16 V}{15 kΩ} = 4.11\times10^{-4} A$

8.

Ex: Equivalent Resistance



Find R_{eq} between ab.

$$\Delta V_{ab} = (1\Omega) i_1 + (1\Omega)(i_1 - i_2) \quad (1)$$
$$= (1\Omega) i_1 + (1\Omega) i_2 + (5\Omega)(i - i_1 + i_2) \quad (2)$$
$$= (3\Omega)(i - i_1) + (5\Omega)(i - i_1 + i_2) \quad (3)$$

Let $i = 1A$, $i_1 = x$, $i_2 = y$

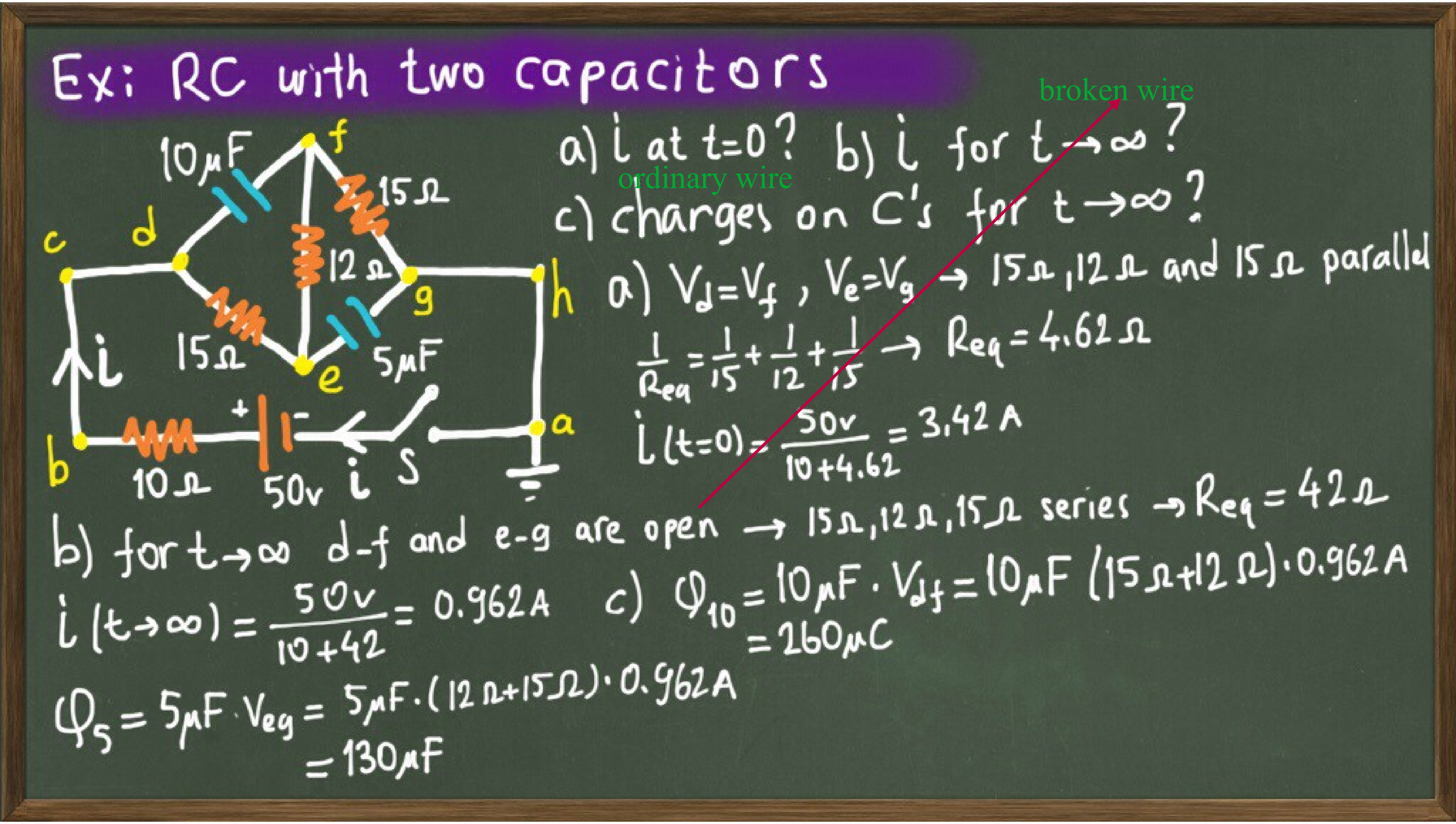
$$\Delta V_{ab} = 2x - y \rightarrow y = 2x - \Delta V_{ab}$$
$$\Delta V_{ab} = -4x + 6y + 5$$
$$\Delta V_{ab} = 8 - 8x + 5y$$

$$\Rightarrow \begin{cases} 7\Delta V_{ab} = 8x + 5 \\ 6\Delta V_{ab} = 2x + 8 \end{cases} \quad \Delta V_{ab} = \frac{27}{17} V$$

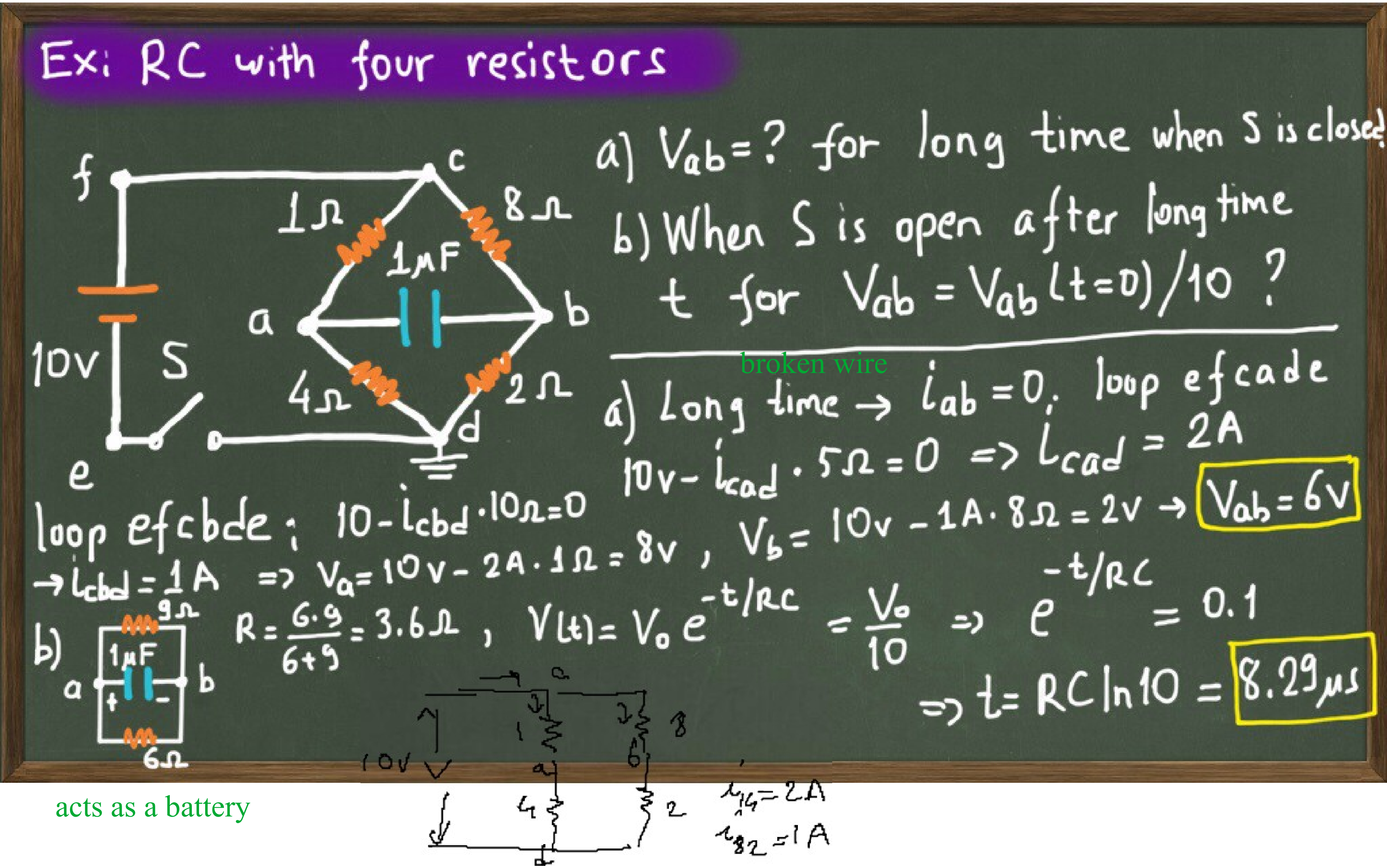
by Ohm's law:

$$R_{ab} = \frac{\Delta V_{ab}}{i} = \frac{27/17 \, V}{1 \, A} = \frac{27}{17} \Omega$$

9.


Ex: RC with two capacitors
broken wire
ordinary wire
a) i at t=0? b) i for t→∞?
c) charges on C's for t→∞?
10μF
15Ω
12Ω
15Ω
5μF
10Ω
50v
c d f g h e b a
i
S
+ -
a) Vd=Vf , Ve=Vg → 15Ω, 12Ω and 15Ω parallel
1/Req = 1/15 + 1/12 + 1/15 → Req = 4.62 Ω
i(t=0) = 50v / (10+4.62) = 3.42 A
b) for t→∞ d-f and e-g are open → 15Ω, 12Ω, 15Ω series → Req = 42Ω
i(t→∞) = 50v / (10+42) = 0.962A
c) Q10 = 10μF · Vdf = 10μF (15Ω+12Ω)·0.962A
= 260μC
Q5 = 5μF · Veg = 5μF·(12Ω+15Ω)·0.962A
= 130μF

10.


Ex: RC with four resistors
a) Vab=? for long time when S is closed
b) When S is open after long time
t for Vab = Vab(t=0)/10 ?
f
c
1Ω
8Ω
1μF
a
b
10v
S
4Ω
2Ω
d
e
broken wire
a) Long time → iab = 0, loop efcade
10v − icad · 5Ω = 0 ⇒ icad = 2A
loop efcbde ; 10 − icbd · 10Ω = 0
Va = 10v − 2A · 1Ω = 8v , Vb = 10v − 1A · 8Ω = 2v → Vab = 6V
→ icbd = 1A
b) R = 6·9/(6+9) = 3.6Ω , V(t) = V0 e^(−t/RC) = V0/10 ⇒ e^(−t/RC) = 0.1
⇒ t = RC ln10 = 8.29μs
9Ω
1μF
6Ω
10V
i14 = 2A
i82 = 1A
acts as a battery

Emf

•The **emf** (work per unit charge) of the device is

$$\mathscr{E} = \frac{dW}{dq} \quad \text{(definition of } \mathscr{E}\text{).}$$ **Eq. 27-1**

Single-Loop Circuits

•Current in a single-loop circuit:

$$i = \frac{\mathscr{E}}{R + r},$$ **Eq. 27-4**

Power

- The rate P of energy transfer to the charge carriers is

$$P = iV,$$ **Eq. 27-14**

- The rate P_r at which energy is dissipated as thermal energy in the battery is $P_r = i^2 r.$ **Eq. 27-16**

- The rate P_{emf} at which the chemical energy in the battery changes is

$$P_{\text{emf}} = i\mathscr{E}.$$ **Eq. 27-17**

Series Resistance

•When resistances are in series

$$R_{\text{eq}} = \sum_{j=1}^{n} R_j$$ **Eq. 27-7**

Parallel Resistance

•When resistances are in parallel

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j}$$ **Eq. 27-24**

RC Circuits

•The charge on the capacitor increases according to

$$q = C\mathscr{E}(1 - e^{-t/RC})$$ **Eq. 27-33**

•During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC}$$ **Eq. 27-34**

•During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$$ **Eq. 27-40**