

Chapter 21 - Electric Charge

Physics of electromagnetism $\rightarrow$ combination of electric and magnetic phenomena

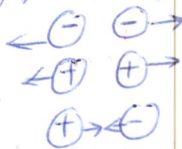
Begin with electrical phenomena; first step is to discuss the nature of electric charge and electric force.

Every object contains a vast amount of electric charge, an intrinsic characteristic.

Two kinds of charge $\left\{ \begin{array}{l} \text{positive} \\ \text{negative} \end{array} \right. \Rightarrow$ electrically neutral $\left\{ \begin{array}{l} \text{charge balance} \\ \text{equality of amounts of charge} \end{array} \right.$

Charged object $\rightarrow$ charge imbalance $\Rightarrow$ net charge $\Rightarrow$ charged object interacts $\left\{ \begin{array}{l} \text{by exerting forces on} \\ \text{one another} \end{array} \right.$

Electric Force: Charges with the same electrical sign repel each other, and charges with opposite electrical sign attract each other.
Coulomb's law of electrostatic force



Conductors & Insulators

Ability of charge to move through! Conductance

• Conductors: charge can move rather freely; metals, body, water, ...

• Nonconductors (Insulators): charge can not move freely; rubber, plastic, glass

• Semiconductors: intermediate btw conductors and insulators; (silicon, germanium)

• Superconductors: perfect conductors (creating a pathway btw object and Earth's surface)

(discharge)
We can neutralize the object (charge balance) by grounding the object.
Earth is a huge conductor

See Lecture notes

Coulomb's Law

Two charged particles are brought near each other $\rightarrow$ each exert force on the other

The force of repulsion or attraction: $\vec{F}$, electrostatic force

The equation giving the force for charged particles is called Coulomb's law.

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r} \quad \text{Coulomb's law}$$

charge q_1

charge q_2

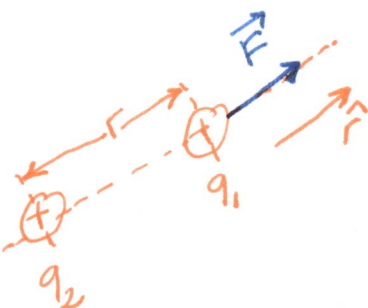
r : distance btw two charged particles

$\hat{r}$: unit vector along an axis

k : constant

See lecture notes

$\vec{F}$, force on particle 1



$$\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r} \quad \text{Newton's law}$$

Both eqns describe inverse square laws

always attractive

either attractive or repulsive

Unit of charge: Coulomb (C). Derived from electric current, $\frac{dq}{dt}$ at which charge moves past a point or through a region

$$\rightarrow 1 \text{ C} = (1 \text{ A})(1 \text{ s})$$

Electrostatic constant: k

$$k = \frac{1}{4\pi\epsilon_0} \quad \epsilon_0: \text{permittivity constant}, 8.85 \times 10^{-12} \text{ N m}^2/\text{C}^2 \text{ in vacuum}$$

The magnitude of the force in Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$$

Superposition principle for $\vec{F}$: $\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \dots + \vec{F}_{1n}$

- n particles

- interacting independently in pairs

Here, on particle 1
Then, net force on any of them

Shell theorem of electrostatics:

• A shell of uniform charge attracts or repels a charged particle that is outside the shell as if the shell's charge were concentrated at its center.

• If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

Spherical Conductors: if excess charge is placed on a spherical shell (metal), the excess charge spreads uniformly over the surface.

- place excess electrons
- they repel each other
- spreading over the surface
- uniformly distributed
- Now, first shell theorem works!

Charge is quantized

"Electrical fluid" is made up of multiples of a certain elementary charge.

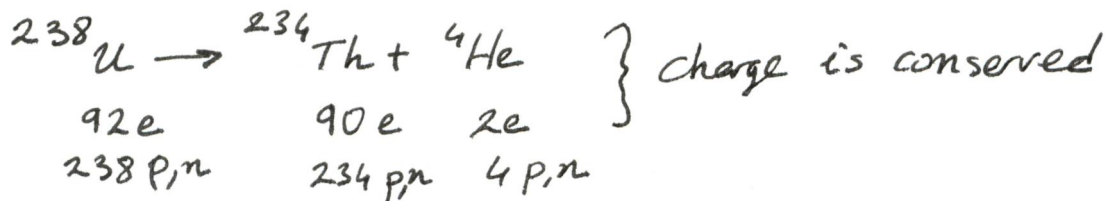
Charge is one of property of particles like mass.

multiples: $q = ne$ e : elementary charge (electron) $1.602 \times 10^{-19} \text{ C}$
 $n: \pm 1, \pm 2, \pm 3, \dots$: quantized, it has discrete values. not continuous

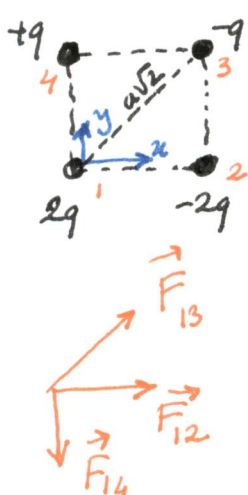
Charge is conserved! In any kind of interaction, charge is transferred. Conservation!
 i.e.

Radioactive decay of nuclei

Transformation of ^{238}U to ^{234}Th by emitting alpha particle (^4He)



Example: In figure shown, what are i) horizontal components of the net electrostatic force on the charged particle in the lower left corner of the square
 ii) vertical components of that



$q = 1.0 \times 10^{-7} \text{ C}$
 $a = 5.0 \text{ cm}$

asked $\vec{F}_{1, \text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$

$F_{1, \text{net}, x} = ?$
 $F_{1, \text{net}, y} = ?$

$$F_{12}: |\vec{F}_{12}| = k \frac{|2q||-2q|}{a^2} = k \frac{4q^2}{a^2}, \vec{F}_{12} = k \frac{4q^2}{a^2} (\hat{i})$$

$$F_{14}: |\vec{F}_{14}| = k \frac{|2q||q|}{a^2} = k \frac{2q^2}{a^2}, \vec{F}_{14} = k \frac{2q^2}{a^2} (-\hat{j})$$

$$F_{13}: |\vec{F}_{13, x}| = \frac{k |2q||-q|}{(a\sqrt{2})^2} \cos 45^\circ, F_{13, x} = k \frac{2q^2}{a^2} \frac{\sqrt{2}}{2}$$

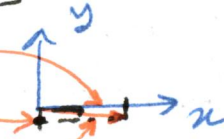
$$|\vec{F}_{13, y}| = \frac{k |2q||-q|}{(a\sqrt{2})^2} \sin 45^\circ, F_{13, y} = k \frac{2q^2}{a^2} \frac{\sqrt{2}}{2}$$

$$i) F_{1,net,x} = F_{12,x} + F_{13,x} + F_{14,x} = k \frac{q^2}{a^2} \left(4 + \frac{\sqrt{2}}{2} \right) = 8.99 \times 10^9 \frac{(1.0 \times 10^{-7})^2}{0.050^2} \left(4 + \frac{\sqrt{2}}{2} \right)$$

$$ii) F_{1,net,y} = F_{12,y} + F_{13,y} + F_{14,y} = k \frac{q^2}{a^2} \left(-2 + \frac{\sqrt{2}}{2} \right) = \boxed{0.17 \text{ N}} \quad \boxed{-0.046 \text{ N}}$$

$$\vec{F}_{1,net} = k \frac{q^2}{a^2} \left[\left(4 + \frac{\sqrt{2}}{2} \right) \hat{i} + \left(-2 + \frac{\sqrt{2}}{2} \right) \hat{j} \right]$$

$$\vec{F}_{1,net} = 0.17 \text{ N } \hat{i} + 0.046 (-\hat{j})$$



$$\tan \theta = \frac{-0.046}{0.17}$$

Chapter 22 - Electric Fields

How do charged particles "know" the presence of other particles?
How do they interact at a distance although they do not touch?

The answer is **Electric Field**. Particle 2 pushes on particle 1 not by touching it but by means of the electric field produced by particle 2.

- Define $\vec{E}$
- How to calculate it for various arrangements of charged particles

The Electric Field

Temperature, Pressure: Scalar fields

Electric field is a vector field $\leftarrow$ Distribution of vectors around a charged object
place a test charge near by

SLN Fig 22-1

Define the electric field $\vec{E}$ at point P due to the charged object as } $\boxed{\vec{E} = \frac{\vec{F}}{q_0}} \quad |\vec{E}| = \frac{|\vec{F}|}{q_0}$

$\vec{E}$ exists independently of the test charge.

SLN

Unit: N/C
magnitude same as F/q_0 direction

Electric Field Lines

To visualize patterns in $\vec{E}$.

- direction of line $\Rightarrow$ direction of $\vec{E}$
- number of lines per unit area $\Rightarrow$ magnitude of $\vec{E}$
 - lines: close $\rightarrow$ large $|\vec{E}|$
 - lines: away $\rightarrow$ small $|\vec{E}|$
- extends away from positive charge
- toward negative charge

SLN Fig 22-4

Fig 22-5: Electric Dipole

$\vec{E}$ is tangent to the field line

SLN Fig 22-2

Fig 22-3

sphere of uniform negative charge
nonconducting sheet with uniformly distributed positive charge

$\vec{E}$ Due to a Point Charge

To find $\vec{E}$ due to a point charge at any point a distance r { put a test charge, q_0 }



Then, electrostatic force acting on q_0

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \hat{r}, \quad \boxed{\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}}$$

magnitude: $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

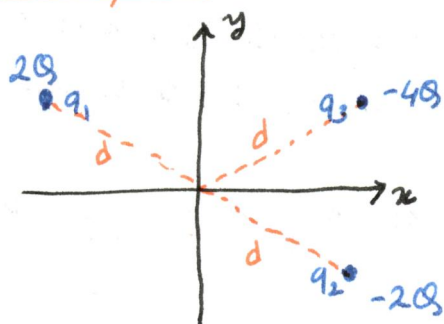
direction: same as the direction of $\vec{F}$

More than one point charge! Net force, acting on test charge, q_0

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \dots + \vec{F}_{0n} \quad \left\{ \begin{array}{l} \text{net} \\ \vec{E} \end{array} \right\} \quad \vec{E} = \frac{\vec{F}_0}{q_0} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n$$

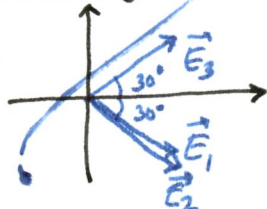
principle of superposition
(acting one by one in pairs)

Example

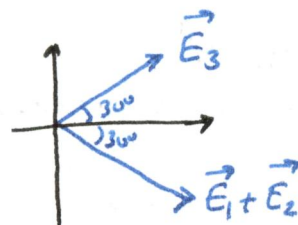


Find the net $\vec{E}$ at the origin

①



②



in magnitude $E_1 + E_2 = \frac{1}{4\pi\epsilon_0} \frac{|2Q|}{d^2} + \frac{1}{4\pi\epsilon_0} \frac{|-2Q|}{d^2}$; $E_3 = \frac{1}{4\pi\epsilon_0} \frac{|-4Q|}{d^2}$

in vector $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$$\vec{E}_{\text{net},x} = E_1 \cos 30^\circ \hat{i} + E_2 \cos 30^\circ \hat{i} + E_3 \cos 30^\circ \hat{i}$$

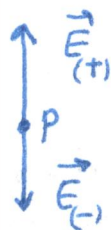
$$\vec{E}_{\text{net},y} = E_1 \sin 30^\circ (-\hat{j}) + E_2 \sin 30^\circ (-\hat{j}) + E_3 \sin 30^\circ (\hat{j})$$

$$\vec{E}_{\text{net},x} = \frac{1}{4\pi\epsilon_0} \frac{8Q}{d^2} \frac{\sqrt{3}}{2} \hat{i} = \boxed{\frac{6.93Q}{4\pi\epsilon_0 d^2} \hat{i}}$$

$$\vec{E}_{\text{net},y} = \frac{1}{4\pi\epsilon_0} (-2Q - 2Q + 4Q) \sin 30^\circ \hat{j} = \boxed{0 \hat{j}}$$

$\vec{E}$ Due to an Electric Dipole

Two charged particles of magnitude q but opposite sign separated by a distance d . SLN Fig 22-8 $\vec{E}$ due to dipole at a point P , a distance z from the midpoint of the dipole and on the axis through the particles (dipole axis)



$$E = E_{(+)} - E_{(-)}$$

at point P (magnitude)

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(-)}^2}$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{(z - d/2)^2} - \frac{1}{(z + d/2)^2} \right)$$

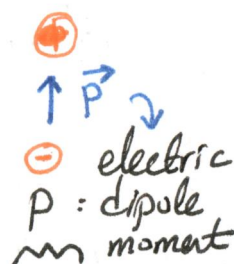
$$E = \frac{q}{4\pi\epsilon_0 z^3} \left(\frac{d}{(1 - (\frac{d}{2z})^2)^2} \right)$$

at large distances $\Rightarrow$

$$\frac{d}{2z} \ll 1 \quad (z \gg d)$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3} = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3}$$

Direction of $\vec{p}$: $- \rightarrow +$
specify orientation of



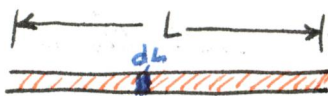
$\vec{E}$ Due to a Line of Charge

A single point charge! not realistic. Usually we have charge distributions
(discrete)
Many closely spaced point charges spread $\left\{ \begin{array}{l} \text{along a line} \\ \text{over a surface} \\ \text{within a volume} \end{array} \right\}$ continuous

Now, we need charge density rather than total charge

λ, σ, ρ (linear, surface, volume)
charge density

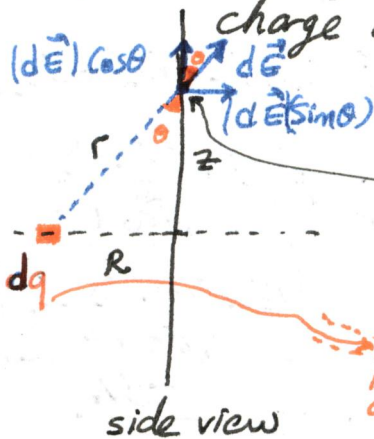
SLN Table 22-



$$\lambda = \frac{Q}{L}$$

$$\lambda = \frac{dq}{dL}$$

Example: A ring of radius R with a uniform positive linear charge density. (Plastic or insulator so that charges are fixed, not moving)



SLN Fig 22-10

what is $\vec{E}$ at point P

Previously

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$\frac{dq}{ds} = \lambda$ charge per unit length
now, we have dq and we should find $d\vec{E}$.

dq : charge of the differential element
 ds : length of any differential element

$\Rightarrow d\vec{E}$ at point P. Now, dq can be treated as point charge.

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(R^2 + z^2)} \quad \text{First step is completed}$$

Second step is to calculate total electric field. $2\pi R$ ~ length of ring

$$|\vec{E}| = \int dE \cos\theta \quad \left\{ \cos\theta = \frac{z}{r} = \frac{z}{(R^2 + z^2)^{1/2}} \right\} \quad E = \int_0^{2\pi R} \frac{1}{4\pi\epsilon_0} \frac{z \lambda ds}{(R^2 + z^2)^{3/2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{\lambda z}{(R^2 + z^2)^{3/2}} \int_0^{2\pi R} ds = \frac{z \lambda 2\pi R}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} \quad \text{--- } q$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q z}{(R^2 + z^2)^{3/2}}}$$

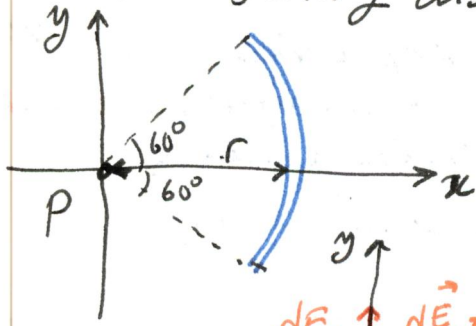
charged ring

when $z \gg R$ $E = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2}$

At very large distances, ring looks like a point charge.



Example A 120° circular arc rad of radius r with having a uniformly distributed charge $-Q$. SLN Fig. 22-11



What is $\vec{E}$ at point P? in terms of Q and r

First step: Find $d\vec{E}$ with knowns!

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{r}, \quad |d\vec{E}| = dE \text{ magnitude}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}, \quad \lambda = \frac{dq}{ds}, \quad dq = \lambda ds$$

$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2} \quad \text{First step is complete}$$

Second step: Calculate total $\vec{E}$

$$\vec{E} = E_x \hat{i} + E_y \hat{j}, \quad |\vec{E}| = E_x = E$$

y -components of $d\vec{E}$ are cancelled out.

$$E = \int dE_x = \int dE \cos \theta = \int_{60^\circ}^{300^\circ} \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2} \cos \theta$$

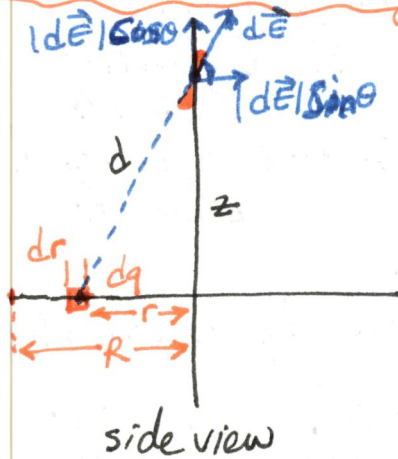
$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r} \int_{60^\circ}^{300^\circ} \cos \theta d\theta \Rightarrow \frac{\lambda}{4\pi\epsilon_0} [\sin \theta]_{60^\circ}^{300^\circ} = \frac{\lambda}{4\pi\epsilon_0} [\sin 60^\circ - \sin(-60^\circ)]$$

$$\Rightarrow \boxed{E = \frac{1.73\lambda}{4\pi\epsilon_0 r}} \quad \text{Total } \vec{E} \text{ Not finished. } \lambda \text{ should be removed}$$

$$\lambda = \frac{Q}{\frac{2\pi}{3}r} \Rightarrow \boxed{E = \frac{0.83Q}{4\pi\epsilon_0 r^2}}$$

$$\boxed{\vec{E} = \frac{0.83Q}{4\pi\epsilon_0 r^2} \hat{i}}$$

$\vec{E}$ Due to a Charged Disk



What is $\vec{E}$ at point P?

$$\frac{dq}{dA} = \sigma = \frac{dq}{2\pi r dr}$$

$$d = \sqrt{r^2 + z^2}$$

$$\cos \theta = \frac{z}{d}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{d^2} = \frac{1}{4\pi\epsilon_0} \frac{\sigma dA}{d^2}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\sigma 2\pi r dr}{(r^2 + z^2)} \quad \text{1st step}$$

$$\Rightarrow \text{2nd step: } dE \cos \theta = dE_z$$

$$dE_z = \frac{1}{4\pi\epsilon_0} \sigma 2\pi \int \frac{r dr}{(r^2 + z^2)} \frac{z}{(r^2 + z^2)^{1/2}}$$

$$dE_z = \frac{\sigma z}{4\epsilon_0} \int \frac{2r dr}{(r^2 + z^2)^{3/2}}$$

3rd step: Calculate total $\vec{E}$

$$E = \int dE_z = \frac{\sigma z}{4\epsilon_0} \int_0^R (z^2 + r^2)^{-3/2} (2r) dr$$

$$E = \frac{\sigma z}{4\epsilon_0} \left[\frac{(z^2 + r^2)^{-1/2}}{-1/2} \right]_0^R$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)} \quad \text{charged disk}$$

if $R \rightarrow \infty \Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$ infinite sheet

A Point Charge in an $\vec{E}$

A charged particle located to $\vec{E}$. what happens? An electrostatic force acts on the particle: $\boxed{\vec{F} = q\vec{E}}$ $\vec{E}$: External electric field
 q : charge of the particle

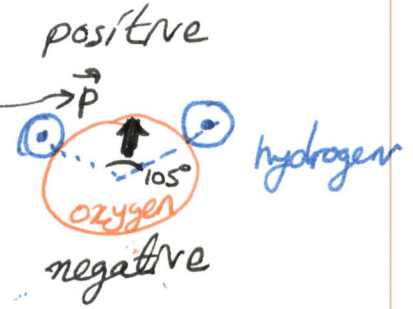
* The direction of $\vec{F}$ and $\vec{E}$ are the same if charge, q , is positive
 " " " " opposite " " " negative

SLN. Example

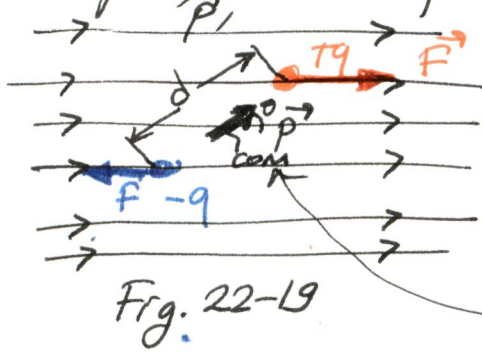
A dipole in an $\vec{E}$

The behavior of a dipole in an $\vec{E}$ can be described completely in terms of $\vec{E}$ and $\vec{p}$.

No need any information about dipole's structure



A dipole in a uniform external $\vec{E}$.



- Forces act in opposite directions.
- Net force on the dipole is zero.
- But, forces produce a net torque, $\vec{\tau}$ on the dipole ~~ends~~ about its com.

$$\tau = Fd \sin \theta$$

$$\tau = pE \sin \theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$p = qd$ & $F = qE$
 potential energy of a dipole.

SLN

Chapter 23 - Gauss' Law

Simple way of solving complex problem. $\Leftarrow$ due to symmetry
For certain charge distributions $\rightarrow$ use Gauss' law instead of considering the fields $d\vec{E}$.

Hypothetical closed surface enclosing charge distribution.

Gaussian Surface

$\left\{ \begin{array}{l} \text{can have any shape} \\ \text{minimizes our calculations of } \vec{E} \\ \text{mimics the symmetry of the charge distribution} \end{array} \right.$

Gauss' law relates the electric fields at points on a (closed) Gaussian surface to the net charge enclosed by that surface. SLN Fig. 23-1 (OR reverse)

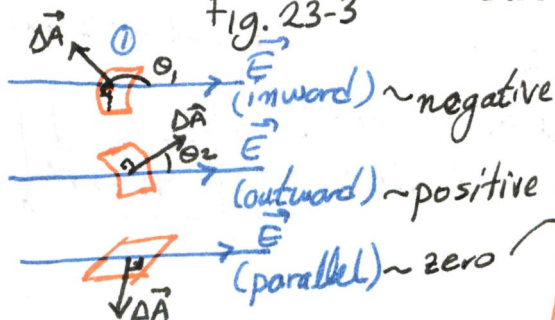
Flux

How much charge is enclosed? need a way $\rightarrow$ Flux (intercepted field).
Flux: product of an area and the field across that area. SLN

Flux of an $\vec{E}$: Consider an arbitrary Gaussian surface immersed in a nonuniform electric field.



Fig. 23-3



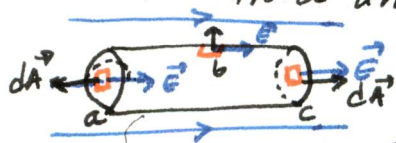
Divide the surface into small flat squares each having area of ΔA . Since squares are very small electric field can be taken as constant.

$$\Phi = \sum \vec{E} \cdot \Delta \vec{A} \quad \left\{ \begin{array}{l} \Delta A \text{ is smaller and smaller} \end{array} \right. \quad \left\{ \begin{array}{l} \Phi = \oint \vec{E} \cdot d\vec{A} \end{array} \right. \quad \left\{ \begin{array}{l} \text{Electric flux through a Gaussian surface} \end{array} \right.$$

The electric flux Φ through a Gaussian surface is proportional to the net number of electric field lines passing through that surface.

Example

Gaussian surface in the form of a cylinder of radius R immersed in a uniform $\vec{E}$. What is Φ ?



Gaussian surface

$$\begin{aligned} \Phi &= \oint \vec{E} \cdot d\vec{A} = \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A} \\ &= \int E(\cos 180^\circ) dA + \int E(\cos 90^\circ) dA + \int E(\cos 0^\circ) dA \\ &= -EA + 0 + EA \quad (A = \pi R^2) \\ \Phi &= 0 : \text{net flux zero} \end{aligned}$$

$\left\{ \begin{array}{l} \text{input are same in output magnitude, but opposite in direction.} \end{array} \right.$

SLN

Gauss' law

Relates the net flux Φ of $\vec{E}$ through a closed surface to the net charge q_{enc} enclosed charge

$$\epsilon_0 \Phi = q_{enc} \xrightarrow[\text{flux}]{\text{subs.}} \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

Gauss' law

- charge is placed in vacuum or air
- net charge $\leq +$
- q_{enc} is (+) $\Rightarrow \Phi$ is outward
- q_{enc} is (-) $\Rightarrow \Phi$ is inward

SLN Fig. 23-6

S_1 : $\vec{E}$ is outward $\rightarrow \Phi$ is positive $\rightarrow$ net charge is (+) as Gauss' law

S_2 : $\vec{E}$ is inward $\rightarrow \Phi$ is negative $\rightarrow$ net charge is (-)

S_3 : no charge is enclosed, $q_{enc} = 0 \rightarrow \Phi = 0$: Field enter and also leave the surface

S_4 : net charge is zero $\rightarrow \Phi = 0$ (net flux is zero)

What would happen if a large Q is placed near the surface S_4 ?
 $\rightarrow$ The pattern of $\vec{E}$ lines would change, but flux would not change.

SLN Fig. 23-7 & Fig 23-5

Gauss' Law and Coulomb's Law

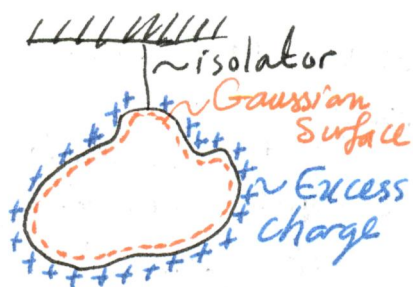
Gauss' law and Coulomb's law are different ways of describing the relation b/w $\vec{E}$ and q .
 Coulomb's law $\Rightarrow$ So that, derive each from the other is possible. SLN Fig. 23-8

Consider a positive charge q a Gaussian surface $\rightarrow \oint \vec{E} \cdot d\vec{A} = q_{enc}$

$$\left. \begin{array}{l} \text{a differential area from Gaussian surface (spherical).} \\ \vec{E} \parallel d\vec{A} \Rightarrow \theta = 0^\circ \\ q_{enc} = q \end{array} \right\} \left. \begin{array}{l} \oint \vec{E} \cdot d\vec{A} = q \\ \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q \end{array} \right\} \left. \begin{array}{l} \epsilon_0 E \oint dA = q \\ \epsilon_0 E (4\pi r^2) = q \end{array} \right\} \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}}$$

A charged Isolated Conductor

Gauss' law permits us to provide { If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of these excess charge will be found within the body of the conductor.



GS: just below the surface
 $\vec{E} \rightarrow 0$ since net charge inside is 0

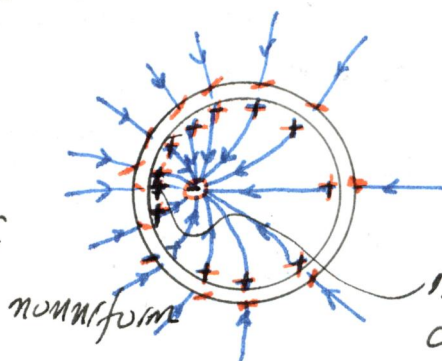
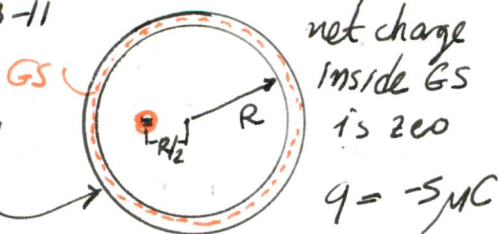


$\vec{E} \rightarrow 0$ again

Example Spherical metal shell, $\vec{E}$ and q_{enc}
 A negatively charged particle is placed inside the neutral metal shell.
 Draw $\vec{E}$ lines and charge distribution.

SLN Fig. 23-11

neutral metal shell

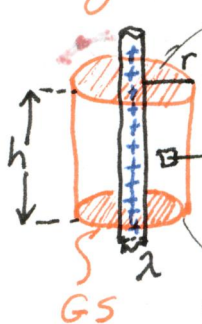


Total charge in shell is conserved

more positive charge is near negative charge.

Applying Gauss' law

1) Cylindrical Symmetry



- An infinitely long cylindrical plastic rod, with a
- a uniform positive linear charge density, λ
- What is the magnitude of $\vec{E}$ at a distance r ?
 - Directed radially outward. (+ charge)
 - Flux through end caps is zero since $\vec{E}$ is parallel to these caps.

Fig. 23-12

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Flux through cylindrical surface, line of charge $\Phi = E A \cos\theta = E(2\pi r h) \cos 0 = E(2\pi r h)$

Gauss' law ($\vec{E} \cdot d\vec{A}$)

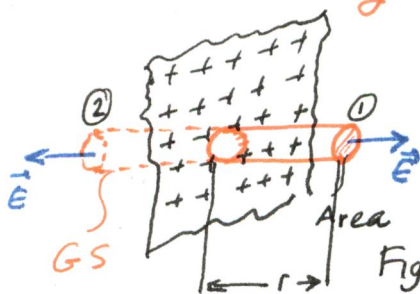
$$\epsilon_0 \Phi = q_{enc}$$

$$\epsilon_0 E 2\pi r h = \lambda h$$

charge enclosed by GS

2) Planar Symmetry

2a Nonconducting sheet



- Infinite nonconducting sheet
- a uniform positive surface charge density, σ
- What is the magnitude of $\vec{E}$ at a distance r ?
 - Again cylindrical GS

Fig. 23-15

Directed away from the sheet. (+ charge)

Flux through cylindrical surface is zero since $\vec{E} \parallel$ surface

Flux through end caps $\epsilon_0(EA + EA) = \sigma A$

$$E = \frac{\sigma}{2\epsilon_0}$$

sheet of charge

2b Two Conducting Plates

Thin infinite conducting plate

(+) charged plate (-) charged plate

They come close

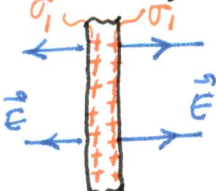
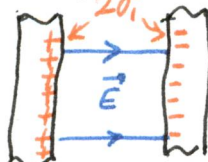
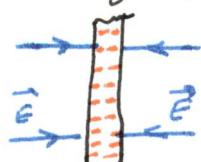


Fig. 23-16



$$\epsilon_0(E_1 A \cos 0 + E_2 A \cos 0) = 2\sigma_1 A \Rightarrow E = \frac{2\sigma_1}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

The excess charge on one plate will attract the excess charge on the other plate, and all the excess charge will move to the inner surfaces.

charge density on each surface: 2σ

3) Spherical Symmetry

shell theorems SLN. charged spherical shell, q
Radius, R .

• Two Gaussian Surfaces $GS1$ & $GS2$

• $GS2$ $r \geq R$ (Remember shell theorem 1)

• $GS1$ $r < R$

Apply Gauss' Law

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$E = 0$$

• Since $GS1$ encloses no charge

• Remember shell theorem 2

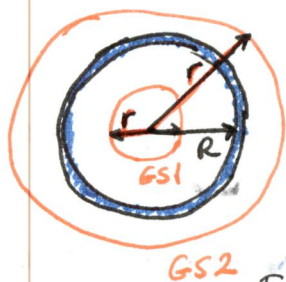


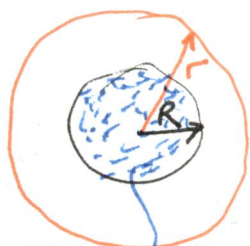
Fig. 23-18

$\vec{E}$ in Spherically Symmetric Charge Distribution

charge density, ρ . which is a function of only distance from center.

• Entire charge lies within GS

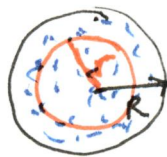
• Entire charge doesn't lie within GS



GS enclosed charge is $q = \rho \cdot \frac{4}{3}\pi r^3$

Fig. 23-19a

• $\vec{E}$: as if all the charge were a point charge at the center



q' : the charge enclosed by GS

$$E = \frac{1}{4\pi\epsilon_0} \frac{q'}{r^2}$$

Fig. 23-19b

To find q' : $\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}$

$$\Rightarrow q' = q \frac{r^3}{R^3}$$

$$\Rightarrow E = \frac{q}{4\pi\epsilon_0 R^3} r$$

Chapter 24 - Electric Potential

A force is conservative? $\left\{ \begin{array}{l} \text{thus, has an associated electrical potential energy} \\ \text{Path independence (i.e. Gravitational force)} \\ \text{Apply the principle of the conservation of mechanical energy.} \end{array} \right.$
 height $\leftrightarrow$ equipotential surfaces

Electric Potential Energy

Charged particles $\rightarrow$ Electrostatic force acting btw them $\rightarrow$ Assign an electric potential energy
 $W = \vec{F} \cdot \vec{d}$, $\vec{F} = q\vec{E}$

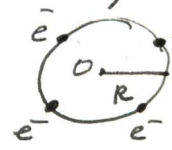
system has a configuration $\rightarrow$ initial state, $i \Rightarrow U_i$
 a change $\downarrow$
 final state, $f \Rightarrow U_f$ } $\Delta U = U_f - U_i = -W$
 change in potential energy

Since electrostatic force is conservative $\Rightarrow$ path independence
 which means that work done on the particle is the same for all paths.

Electric Potential

- The potential energy per unit charge ($\frac{U}{q}$) is independent of the charge q .
- Characteristic only of the electric field. Example

$V = \frac{U}{q}$ $\div$ Electric Potential. A scalar not a vector
 (OR Potential)


 $V = -4 \times \frac{1}{4\pi\epsilon_0} \frac{e}{R}$
 $\vec{E} = 0$: vector quantity, symmetry cancelled

- Potential energy per unit charge
- Electric Potential Difference, $\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = \frac{-W}{q}$
 btw two points SW

Take $U_i = 0$ at infinity then V at infinity is equal to zero.

$V = -\frac{W_{\infty}}{q}$ work done by $\vec{E}$ on a charged particle to move from infinity to point f .
 Unit of potential
 $\frac{\text{Joule}}{\text{Coulomb}} \equiv 1 \text{ Volt}$

- Redefine the unit of $\vec{E}$ ($\frac{N}{C}$): $1 \frac{N}{C} = 1 \frac{N}{J/V} = 1 \frac{NV}{Nm} = 1 \frac{V}{m}$

- Work to move an electron through 1V is called one electron-volt (1eV)

Magnitude of the work: $q\Delta V \Rightarrow 1 \text{ eV} = e(1V) = (1.6 \times 10^{-19} \text{ C})(1 \text{ J/C})$
 $\Rightarrow 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Work Done by an Applied Force

a charged particle, q in an $\vec{E} \rightarrow$ experiences force $\left\{ \begin{array}{l} \text{move from point } i \text{ to } f \\ \text{Change in KE} \end{array} \right\} \Delta K = K_f - K_i = W_{app} + W$

if $K_f = K_i \Rightarrow W_{app} = -W$
 Applied one From $\vec{E}$ itself
 W_{app} : work done by the applied force
 W : work done by $\vec{E}$
 stationary particle is stationary before and after move.
 $\Delta U = -W$
 $\Delta U = W_{app}$
 $\Delta U = U_f - U_i = W_{app} = q\Delta V$
 (+), (-) or zero

Equipotential Surfaces

- Adjacent points that have the same electric potential form an equipotential surface.
- Work done on a particle on a given equipotential surface is zero.
 $-W = q\Delta V = 0 \Rightarrow \Delta V = 0$

SLN Fig. 24-2, Fig. 24-3

① Calculating the Potential from the Field

What is the potential difference btw two points ① and ② in an $\vec{E}$?

SLN Fig. 24-4. Consider a positive charge q_0 moving btw points ① and ② along the path shown

$$\begin{aligned} dW &= \vec{F} \cdot d\vec{s} \\ dW &= q\vec{E} \cdot d\vec{s} \end{aligned} \quad \left\{ \begin{array}{l} \text{Differential} \\ \text{work} \end{array} \right\} \quad \left\{ \begin{array}{l} \text{Total work} \\ W = q_0 \int \vec{E} \cdot d\vec{s} \end{array} \right\} \rightarrow \frac{W}{q_0} = \int \vec{E} \cdot d\vec{s} \Rightarrow \boxed{V_f - V_i = - \int \vec{E} \cdot d\vec{s}}$$

Since $\vec{F}$ is conservative, all paths btw 1-2 yield the same work.
 $q\Delta V = -W$

② Potential due to a Point Charge

Consider a positive charged particle, q and a positive test charge, q_0 at point P at a distance R from the fixed charged particle. What is the potential at point P? SLN Fig. 24-6

$$\begin{aligned} i: q_0 \text{ at } R & \left\{ \begin{array}{l} V_i = V \\ V_f = V_{\infty} = 0 \end{array} \right\} \\ f: q_0 \text{ at } \infty & \left\{ \begin{array}{l} V_f = V_{\infty} = 0 \end{array} \right\} \end{aligned} \quad \left\{ \begin{array}{l} V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s} \\ 0 - V_i = - \int_R^{\infty} E ds \cos \theta \end{array} \right\}$$

$\rightarrow V_i = V(\text{at } R) = + \int_R^{\infty} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr \rightarrow \boxed{V = + \frac{1}{4\pi\epsilon_0} \frac{q}{R}}$
 + charge produces +V
 - charge produces -V

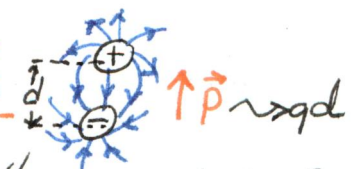
③ Potential due to a Group of Point Charges

Net Potential. Superposition principle. Potential is equal to sum of potential resulting from each charge at the given point

$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

Example 
 $V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \frac{q_4}{r_4} \right)$

④ Potential due to an Electric Dipole



SLN Fig. 24-10 Potential at an arbitrary point P, $V = \sum_{i=1}^2 V_i$

At point P: + charge sets a +V
 - charge sets a -V } $V = V_{(+)} + V_{(-)}$ Net potential

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}} \right) = \frac{q}{4\pi\epsilon_0} \left(\frac{r_{(-)} - r_{(+)}}{r_{(-)} r_{(+)}} \right)$$

Since naturally occurring dipoles are small, $r \gg d$
 $\Rightarrow r_{(-)} - r_{(+) } \sim d \cos \theta$
 $r_{(-)} r_{(+)} \sim r^2$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \frac{d \cos \theta}{r^2} \Rightarrow \boxed{V = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}}$$

p: magnitude of electric dipole moment

SLN Fig. 24-11

⑤ Potential due to a Continuous Charge Distribution

Not point charges anymore. But Continuous charge distribution.

$q \rightarrow V$

Line of Charge:

non-conducting thin rod.

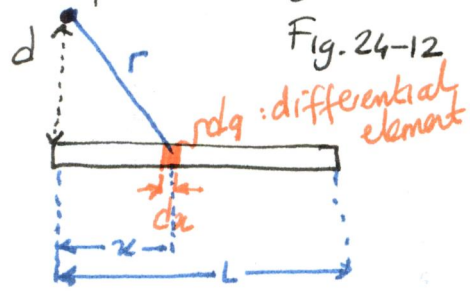


Fig. 24-12

$dq \rightarrow dV$ $dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$
 Then, integrate over the entire charge distribution to find the potential, V

$V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$

What is V at point P? positive charge linear density, λ length, L

Start by $dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} \rightarrow V = \int dV$

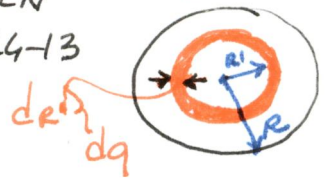
$dq = \lambda dx$
 $r = \sqrt{x^2 + d^2}$

$\Rightarrow V = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(x^2 + d^2)^{1/2}} \Rightarrow \boxed{V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{L + (L^2 + d^2)^{1/2}}{d} \right]}$

Charged Disk:

non-conducting (plastic) disc

SLN Fig. 24-13



What is V at point P? surface charge density, σ

Start by

$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} \rightarrow V = \int dV$ $dq = \sigma dA = \sigma 2\pi R' dr'$
 $r = \sqrt{R'^2 + z^2}$

$\Rightarrow V = \frac{1}{4\pi\epsilon_0} \int_0^R \frac{\sigma 2\pi R' dr'}{\sqrt{R'^2 + z^2}} \Rightarrow \boxed{V = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z)}$

Calculating the Field from the Potential

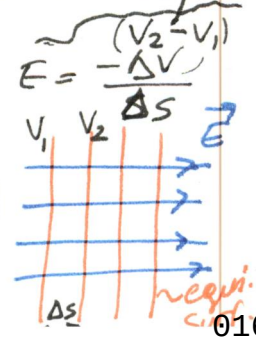
SLN Fig. 24-14

$\Delta U = \Delta V, \Delta U = -W$
 $\frac{\Delta U}{q_0} = \frac{\Delta V}{q_0}$

$W = \vec{F} \cdot \vec{ds}, W = q_0 \vec{E} \cdot \vec{ds}$ } $q_0 \vec{E} \cdot \vec{ds}$

$-q_0 dV = q_0 \vec{E} \cdot \vec{ds}$ } $\vec{E} = -\frac{dV}{ds}$ or $\frac{\partial V}{\partial s}$

in general $E_x = -\frac{\partial V}{\partial x}$
 $E_y = -\frac{\partial V}{\partial y}$
 $E_z = -\frac{\partial V}{\partial z}$



Electric Potential Energy of a System of Point Charges

SLN Fig. 24-15

When we bring q_2 from infinity to a point near q_1 , we must do a work since q_1 exerts electrostatic force on q_2 .

That work should be equal to $q_2 V$, where V is the potential that is created by q_1 .

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$$

$$U = W = q_2 V = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

U
 q_1, q_2
(+) $\leftarrow$ same charges
(-) $\leftarrow$ opposite charges

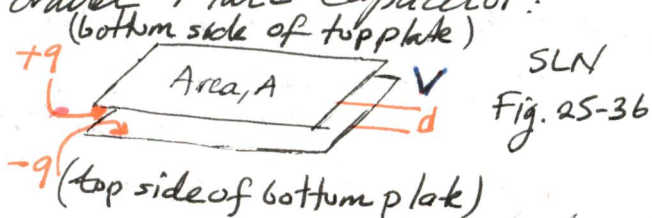
Chapter 25 - Capacitance

The Capacitor is a device $\rightarrow$ electrical energy can be stored.
How much $\rightarrow$ capacitance.

Capacitance SLN Fig. 25-2

Basic elements of any capacitor $\Rightarrow$ Two isolated conductors of any shape
 $\rightarrow$ Plates (flat or not)

Parallel Plate Capacitor:



Two parallel conducting plates of area A , separated by a distance d .
Equipotential Surface
+ | - symbol; used for capacitors of all geometries

* No material medium (glass, plastic) is present btw the plates for now
* Capacitor is charged $\Rightarrow$ plates have charges of equal magnitudes but opposite signs; $+q$ & $-q$

* Potential difference $\Rightarrow$ btw two plates $\rightarrow \Delta V$, we V
 $\rightarrow$ However, charge of capacitor is q

The charge q and the potential difference V for a capacitor are proportional to each other. Proportionality Constant: $C (= \frac{q}{V})$
 $\Rightarrow \boxed{q = CV}$ 1 farad, 1 F, 1 Coulomb per volt
(usually μF or pF)

Charging a capacitor Place the capacitor in an electric circuit with a battery
SLN, Fig. 25-4 $\rightarrow$ open; incomplete circuit
 $\rightarrow$ closed; complete circuit
electrons are driven through the wires by an $\vec{E}$ that the battery sets up in the wire.
maintains a potential difference btw its terminals
• higher potential $\rightarrow$ + terminal
• lower potential $\rightarrow$ - terminal

	Initially	During Charge	Fully charged
- plate h; becomes + (losing e^-)	no charge $V=0$ $\vec{E}=0$	$+q$ & $-q$	V equals to battery
- plate l; becomes - (gaining e^-)		V increases $\vec{E}$ increases	$\vec{E}=0$ charge, q

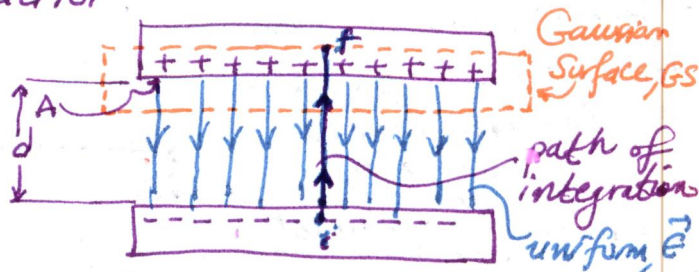
$$\boxed{\frac{q}{V} = C}$$

Calculating the Capacitance

Known geometry $\Rightarrow$ capacitance of a capacitor

- 1) assume a charge q on the plates
- 2) calculate $\vec{E}$ using Gauss' law
- 3) calculate V (knowing $\vec{E}$)
- 4) calculate C (knowing q & V)

Fig. 25-5



(Fringing is neglected)

② Calculating $\vec{E}$, using Gauss' law. $\boxed{\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q}$ net charge enclosed by a GS
 knowing $\vec{E} \parallel d\vec{A} \Rightarrow \boxed{q = \epsilon_0 EA}$ net electric flux through the surface

③ Calculating V
 $\boxed{V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}}$ - choose a path that follows an $\vec{E}$ line } $\vec{E} \cdot d\vec{s}$
 - from negative to positive plate } opposite directions
 $\Rightarrow -E ds$
 $\boxed{V = \int_-^+ E ds}$ $V = E \int_0^d ds = Ed$ } $\boxed{C = \frac{\epsilon_0 A}{d}}$ Parallel Plate Capacitor
 $q = CV \Rightarrow \epsilon_0 EA = CED$ } ϵ_0 : permittivity constant of air. $8.85 \times 10^{-12} \frac{F}{m}$

Capacitance does only depend on geometrical factors; A, d

$$\begin{matrix} C \uparrow \Leftarrow A \uparrow \\ C \downarrow \Leftarrow d \uparrow \end{matrix}$$

A Cylindrical Capacitor

SLN Fig. 25-6 - A cylindrical capacitor of length L } $L \gg b \rightarrow$ neglect fringing of $\vec{E}$
 - Two coaxial cylinders of radii a & b
 - Charge, q
 radius of GS

② $q = \epsilon_0 EA = \epsilon_0 E(2\pi r L) \Rightarrow E = \frac{q}{2\pi \epsilon_0 L r}$
 ③ $V = \int_-^+ \vec{E} \cdot d\vec{s} = \int_6^a -\frac{q}{2\pi \epsilon_0 L} \frac{dr}{r} = -\frac{q}{2\pi \epsilon_0 L} \ln\left(\frac{b}{a}\right)$ (ds = dr)
 ④ $q = CV \Rightarrow \boxed{C = \frac{2\pi \epsilon_0 L}{\ln(b/a)}}$ Cylindrical Capacitor

A Spherical Capacitor

SLN Fig. 25-6 Same figure but two concentric spherical shells of radii a & b .

② $q = \epsilon_0 E(4\pi r^2) \Rightarrow E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2}$
 ③ $V = -\frac{q}{4\pi \epsilon_0} \int_6^a \frac{dr}{r^2} = \frac{q}{4\pi \epsilon_0} \left(\frac{1}{b} - \frac{1}{a}\right)$
 ④ $\boxed{C = 4\pi \epsilon_0 \frac{ab}{b-a}}$ Spherical Capacitor

An Isolated Sphere

- Single isolated spherical conductor of radius, R
 - Assuming missing plate is a conducting sphere of infinite radius ($b \rightarrow \infty$)
 $0 \leq a \rightarrow R$

$$C = 4\pi \epsilon_0 \frac{a}{1 - \frac{a}{b}} = 4\pi \epsilon_0 R$$

Example

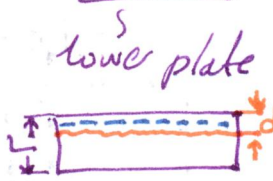


Fig. 25-7

$$C = 0.25 \mu F$$

$$V = 12 V$$

$$L = 0.5 cm$$

$$A = 2.0 \times 10^{-4} m^2$$

$$n = 8.49 \times 10^{18} \frac{e^-}{m^3}$$

(in copper)

$$\left. \begin{matrix} q = CV = 3.0 \times 10^{-6} C \\ \text{Number of } e^- \end{matrix} \right\} N = \frac{q}{e}$$

$$N = \frac{3.0 \times 10^{-6} C}{1.602 \times 10^{-19} C} = 1.87 \times 10^{13}$$

$$n = \frac{N}{\text{Volume}} = \frac{N}{Ad} \Rightarrow d = \frac{N}{An}$$

$$\Rightarrow d = 1.1 \times 10^{-12} m = 1.1 pm$$

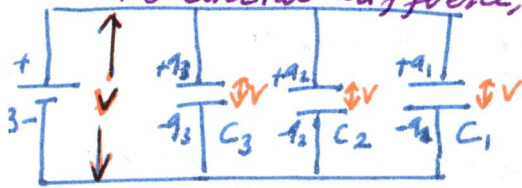
- Electrons very close to the plate face.

Capacitors in Parallel and in Series

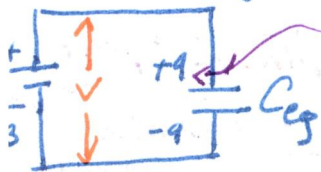
An equivalent capacitor: a single capacitor that has the same capacitance as the actual combination of capacitors

-Capacitors in Parallel

Potential difference, V : applied across several capacitors in parallel



Equivalent

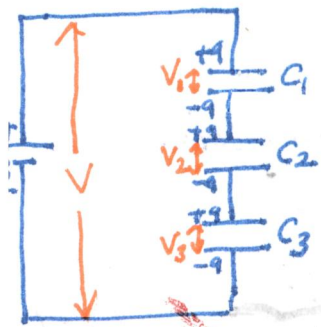


- Same V is applied across each capacitor
- total charge q , is the sum of the charges stored on all the capacitors

$$\begin{aligned} C_1 V &= q_1 \\ C_2 V &= q_2 \\ C_3 V &= q_3 \\ q &= q_1 + q_2 + q_3 \\ &= (C_1 + C_2 + C_3) V \\ &= C_{eq} V \end{aligned}$$

$$\Rightarrow C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3 = \sum_{j=1}^n C_j \quad n \text{ capacitors in parallel}$$

- Capacitors in Series



A chain reaction $\Rightarrow$ charging of each capacitor causes the charging of the next capacitor (due to...)

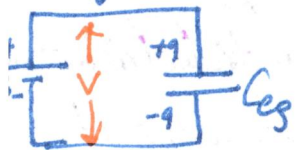
From C_3 to C_1

B produces charge $-q$ on the bottom plate of $C_3 \Rightarrow +q$ on the top plate of C_3

$C_3 - \text{ " " " " " " " } C_2 \Rightarrow$

$C_2 \rightarrow$ " " " " " " " " $C_1 \Rightarrow$ " C_1

q - supplies $+q$ to the battery $\Rightarrow V$ is sustained



- Same charge q across each capacitor
- same total potential difference, V

$$\left. \begin{aligned} V_1 &= \frac{q}{C_1} \\ V_2 &= \frac{q}{C_2} \\ V_3 &= \frac{q}{C_3} \end{aligned} \right\} \begin{aligned} V &= V_1 + V_2 + V_3 \\ &= q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \\ &= q C_{eq} \Rightarrow \frac{1}{C_{eq}} = \sum_{j=1}^n \frac{1}{C_j} \end{aligned}$$

Energy Stored in an Electric Field

n capacitors in series

To charge a capacitor $\Leftarrow$ Work must be done by an external agent

e^- s are removed from one plate $\rightarrow$ transferred to the other plate

$\vec{E}$ (btw the plates) tends to oppose further transfer

- Thus, as charge accumulates on the capacitor plates $\Rightarrow$ more work is required to transfer e^- s

Resistor
→ Chemical energy stored in battery $\Rightarrow$ Electric Potential Energy

Incremental work, dW } $dW = V' dq' = \frac{q'}{C} dq'$ } $W = \int dW = \frac{1}{C} \int_0^q q' dq' = \boxed{\frac{q^2}{2C} = U}$
 potential difference, $\frac{q'}{C}$ } $U = \frac{q^2}{2C} = \frac{1}{2} CV^2$ } stored potential energy, U
 btw plates

Capacitor with a Dielectric

An insulating material (dielectric; plastic...) $\rightarrow$ btw the plates of a capacitor } what happens? } Capacitance increased by a numerical factor κ
 κ : Dielectric constant of the insulating material

Limits the potential difference btw the plates to V_{max} $\rightarrow$ breakdown potential

SLN Table 25-1

when $V \gg V_{max} \rightarrow$ dielectric material will breakdown and form a conducting path between the plates

Dielectric strength: E_{max} that material can tolerate without breakdown
 $C = \epsilon_0 \frac{A}{d} \Rightarrow C = \kappa \epsilon_0 \frac{A}{d} = \kappa C_{air}$ } $\epsilon_0 \Leftrightarrow \kappa \epsilon_0$ } $E = \frac{1}{4\pi\kappa\epsilon_0} \frac{q}{r^2}$: point charge
 $E = \frac{\sigma}{\kappa\epsilon_0}$: isolated conductor

For a fixed distribution of charges, the effect of a dielectric is to weaken the electric field.

Dielectric and Gauss' Law

SLN Fig. 25-16 q' : induced charges, q : free charge on plate

without dielectric material

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 EA = q$$

$$\rightarrow E_0 = \frac{q}{\epsilon_0 A}$$

with dielectric material

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 EA = q - q'$$

$$\rightarrow E = \frac{q - q'}{\epsilon_0 A}$$

we know $E = \frac{E_0}{\kappa}$: weaken original field

$$\frac{q - q'}{\epsilon_0 A} = \frac{q}{\kappa \epsilon_0 A} \Rightarrow q - q' = \frac{q}{\kappa}$$

induced surface charge $\rightarrow q' = 0 \rightarrow \kappa = 1$ no dielectric present
 $\rightarrow q' < q \rightarrow \kappa > 1 \Rightarrow$ weakens $\vec{E}$

$$\Rightarrow \epsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q$$

Gauss' law with dielectric

Example

SLN Fig 25-17

$$A = 115 \text{ cm}^2 = 115 \times 10^{-4} \text{ m}^2$$

$$d = 1.24 \text{ cm} = 1.24 \times 10^{-2} \text{ m}$$

$$V_0 = 85.5 \text{ V}$$

$$b = 0.78 \text{ cm} = 0.78 \times 10^{-2} \text{ m}$$

$$\kappa = 2.61$$

without slab

i) $C_0 = ?$

$$C_0 = \frac{\epsilon_0 A}{d}$$

$$= \frac{(8.85 \times 10^{-12} \frac{\text{F}}{\text{m}})(115 \times 10^{-4} \text{ m}^2)}{1.24 \times 10^{-2} \text{ m}}$$

$$C_0 = 8.21 \text{ pF}$$

ii) $q = ?$

$$q = C_0 V_0 = (8.21 \times 10^{-12} \text{ F}) 85.5 \text{ V}$$

$$q = 702 \text{ pC}$$

iii) $E_0 = ?$ btw the plates and slab

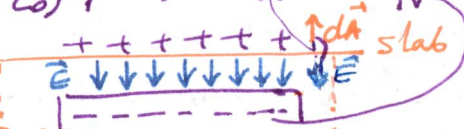
$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q \rightarrow \epsilon_0 \int \vec{E} \cdot d\vec{A} \cos 0 = q \rightarrow \epsilon_0 E \kappa A = q$$

$$E = \frac{q}{\kappa \epsilon_0 A} = \frac{702 \times 10^{-12} \text{ C}}{2.61 \times 8.85 \times 10^{-12} \frac{\text{F}}{\text{m}} \times 115 \times 10^{-4} \text{ m}^2} = 6.90 \text{ kV/m}$$

with slab

iv) $E_s = ?$ in slab

$$\epsilon_0 \oint \vec{E}_s \cdot d\vec{A} = q_{enc}$$

$$\epsilon_0 \int \vec{E}_s \cdot \kappa d\vec{A} \cos 180^\circ = -q$$


$$\epsilon_0 \kappa A (-1) = -q$$

$$E_s = \frac{q}{\kappa \epsilon_0 A} = 2.64 \text{ kV/m}$$

v) $V = \int \vec{E} \cdot d\vec{s} = E_0 (d+b) + E_s b$

$$V = 52.3 \text{ V}$$

iv) $C = \frac{q}{V} = 13.4 \text{ pF}$
 slab $12.6 \text{ pF} > 8.21 \text{ pF}$

Chapter 26 - Current and Resistance

Up to now: Electrostatics $\rightarrow$ physics of stationary charges.

From now on: physics of electric currents $\leftarrow$ charges in motion

Electric Current

streams of moving charges

Not all moving charges constitute an electric current $\rightarrow$ Net flow of charge is needed.
 $\rightarrow$ e^- s are in random motion with $10^6 \text{ m/s}!!$ $\leftarrow$ no net transport of charge (cancel each other!) SLN Fig. in Slide 2

$\rightarrow$ Connect a battery $\rightarrow$ Create a potential difference $\Rightarrow$ Now, we have a net transport of charge.
 SLN Fig. in Slide 3

Steady Currents of conduction e^- s moving through metallic conductors.
 SLN Fig. in Slide 4

Fig. 26-1 Consider a battery in the loop

- $\vec{E}$ act inside the material. $+ - \vec{E}$ loop
- $\vec{E}$ creates a force which acts on conduction e^- . SLN Fig. in Slide 4
- $\rightarrow$ This force causes e^- s to move and electric current is established.
- $\Rightarrow$ After a very short time, e^- flow reaches a constant value and the current is in its steady state (does not vary with time)

$i = \frac{dq}{dt}$ SI Unit
 1 ampee = 1 A = 1 C/s

Total charge from 0 to t $q = \int dq = \int_0^t i dt$ SLN

Fig. 26-2 $i(aa') = i(bb') = i(cc')$ $\leftrightarrow$ charge is conserved $\leftrightarrow$ Fig. 26-3
 SLN The current has same value at each plane $(i_0 = i_1 + i_2)$

Direction of Current: $i \leftarrow$ direction of current (conventionally accepted!!)
 Battery high voltage low voltage SLN
 Actual charge carriers are negative and move in opposite direction. (electrons)

Current Density: Flow of charge through a cross section $\rightarrow$ Current density
 $\vec{J} = \frac{i}{A} = \frac{dq/dt}{A} = \frac{\Delta q}{\Delta A \Delta t} \rightarrow \frac{dq}{dA dt} = \vec{J}$
 Unit A/m^2
 $i = \int \vec{J} \cdot d\vec{A} = JA$ charge carrier (cc) $\vec{J}$
 Magnitude & direction $\rightarrow$ + same with (cc) $-$ opposite to (cc)
 SLN Fig. 26-4

Charge is conserved. However, current density ($\vec{J}$) changes.
 (same i) $\rightarrow$ (different area)

Drift Speed: Random motion with 10^6 m/s + applied $\vec{E} \Rightarrow$ Drift motion with v_d
 SLN Fig. 26-5 Type of charge carriers! (or V) drift speed ($\sim 10^{-5} \text{ m/s}$)

$i \leftarrow$ con. acc. $\vec{J}$
 $\leftarrow + + + + \rightleftarrows \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow$
 $\leftarrow \vec{E} \leftarrow \vec{J}$

- A conductor of length L
- a current in which
- n : # of cc / volume (number density)
- A : cross sectional area ($1.6 \times 10^{-19} \text{ C}$)

$q = (nAL)e$: Total cc in length L
 $t = \frac{L}{v_d}$: Time interval that charges move through L
 $i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d \rightarrow v_d = \frac{i}{nAe}$
 $\Rightarrow v_d = \frac{I}{ne} \Rightarrow \boxed{\vec{J} = n e \vec{v}_d}$ $\frac{A}{m^2} = \frac{C}{m^3} \frac{m}{s}$
 charge carrier (cc) density

Example: Current density: uniform & nonuniform

A cylindrical wire with radius $R = 2.0 \text{ mm}$

i) $\vec{J}$: uniform $2.0 \times 10^5 \text{ A/m}^2$. What is the current btw $\frac{R}{2}$ & R ?

SLN Fig. 26-6a $\vec{J} = \frac{i}{A}$ $\left\{ \begin{array}{l} A = ? \\ \text{not total cross-section area but effective} \\ \text{one } A' = \pi R^2 - \pi (\frac{R}{2})^2 = \frac{3\pi R^2}{4} \end{array} \right.$
 $\Rightarrow 2.0 \times 10^5 \text{ A/m}^2 = \frac{i}{\frac{3\pi (2.0 \times 10^{-3} \text{ m})^2}{4}} \Rightarrow \boxed{i = 1.9 \text{ A}}$

ii) $\vec{J} = ar^2$: non uniform ($\vec{J} \rightarrow J(r)$). What is the current btw $\frac{R}{2}$ & R ?

$3 \times 10^{11} \text{ A/m}^2$
 SLN Fig. 26-6b-e $\vec{J} = \frac{di}{dA} \rightarrow i = \int di = \int \vec{J} \cdot d\vec{A}$
 $\Rightarrow \int_{R/2}^R \vec{J} \cdot d\vec{A} = \int_{R/2}^R ar^2 \cdot 2\pi r dr = 2\pi a \int_{R/2}^R r^3 dr = 2\pi a \left[\frac{r^4}{4} \right]_{R/2}^R = 2\pi a \left(\frac{R^4}{4} - \frac{(R/2)^4}{4} \right) = 2\pi a \left(\frac{15}{64} R^4 \right)$
 $\Rightarrow \boxed{i = 7.1 \text{ A}}$

Sample Problem SLN In a current, the conduction e s move very slowly ($v_d = 4.9 \times 10^{-4} \text{ m/s}$) for sample problem

Resistance and Resistivity

Copper & Glass (similar shapes) $\left\{ \begin{array}{l} \text{same } \vec{E} \text{ (or } V) \\ \text{applied} \end{array} \right. \Rightarrow \text{different currents are obtained}$ } why? The answer is **resistance of the material!**
 SLN Characteristics of the material
 $R = \frac{V}{i}$ SI unit $V_L = \frac{\text{Volts}}{\text{Ampere}} (1 \Omega = 1 \frac{V}{A})$ The conductor which provide resistance is called **resistor**.

Resistance

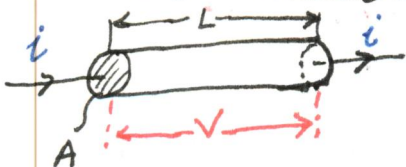
In case of having $V \Rightarrow$ deal with i & R . Resistance is a property of an **object**. (Shape dependency)

In case of having $\vec{E} \Rightarrow$ deal with $\vec{J}$ & ρ : Resistivity is a property of a **material**. (Shape does not matter)

$R = \frac{V}{i} \Rightarrow \left\{ \begin{array}{l} \rho = \frac{E}{J} \\ \text{Resistivity } (\frac{V \cdot m}{A/m^2}) \end{array} \right.$ SI unit $\Omega \cdot m$
 $\left\{ \begin{array}{l} \sigma = \frac{1}{\rho} \\ \text{Conductivity} \end{array} \right.$ SI unit $\frac{1}{\Omega \cdot m}$
 In vector forms $\vec{E} = \rho \vec{J}$
 $\vec{J} = \sigma \vec{E}$ SLN Table 26-1

Calculating Resistance from Resistivity

We have a conductor as below



Assumption:
 $\vec{J}$ is uniform
 $\vec{E}$ is constant all over the wire

$\rho = \frac{E}{J} = \frac{V/L}{i/A}$
 $\rho = \frac{R A}{L}$
 $\boxed{R = \rho \frac{L}{A}}$

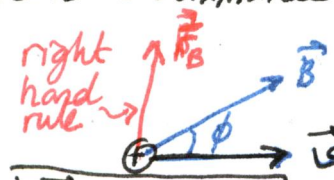
Resistivity vary with temperature
 SLN Fig. 26-10
 $\rho - \rho_0 = \rho_0 \alpha (T - T_0)$



Chapter 28 - Magnetic Fields

- How an $\vec{E}$ can produce an $\vec{F}_E$ on a charged object?
- How an $\vec{B}$ can produce a $\vec{F}_B$ on a (moving) charged particle or on a magnet?
- $\vec{E}$ is produced by an electric charge.
- $\vec{B}$ is produced by moving electrically charged particles $\rightarrow$ current $\rightarrow$ electromagnets
 - by means of e^- 's (intrinsic $\vec{B}$ around them): spinning motion $\rightarrow$ permanent magnet
- The Definition of $\vec{B}$
 - Put a charged particle in an $\vec{E}$ and measure the $\vec{F}_E$ on it. 
 - Put a charged moving particle in an $\vec{B}$ and measure $\vec{F}_B$

Experiment

- $\vec{F}_B$ is proportional to particle velocity $\vec{v}$, and q
 - Magnitude and direction of $\vec{F}_B$ depends on $\vec{v}$ and $\vec{B}$
 - When $\vec{v} \parallel \vec{B}$, $\vec{F}_B \rightarrow 0$
when $\vec{v}$ makes an angle ϕ with $\vec{B}$, then $\vec{F}_B \perp \vec{v}$, $\vec{B}$
Magnitude of $\vec{F}_B \propto \sin \phi$
- All these experimental results can be summarized as
- 

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

Definition of $\vec{B}$

$$|\vec{F}_B| = F_B = |q| v B \sin \phi$$

Magnitude
- * if $q=0$ or $v=0 \Rightarrow F_B=0$

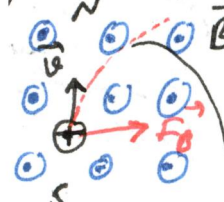
* if $\vec{v} \parallel \vec{B}$ ($\phi=0$ or $\phi=180$) $\Rightarrow F_B=0$

* if $\vec{v} \perp \vec{B} \rightarrow F_B$ is maximum

 - Direction is found by Right Hand Rule (SLN (1,2,3/1,2,3))
 - Direction of $\vec{F}_B$ is always perpendicular to plane of $\vec{v}$ and $\vec{B}$ SLN (4,5/5)
 - SI Unit: 1 Tesla = 1 $\frac{N}{C \cdot m/s}$ = 1 $\frac{N}{A \cdot m}$
- $\vec{F}_B$ never has a component parallel to $\vec{v}$
 $\Rightarrow \vec{F}_B$ can not change the particle's speed, it can only change its direction.

Example

Consider a uniform $\vec{B}$ of 1.2 mT directed towards out of the paper. What will be the magnetic force and direction of motion of proton if the proton has a KE of 5.3 MeV and moves initially from S to N?



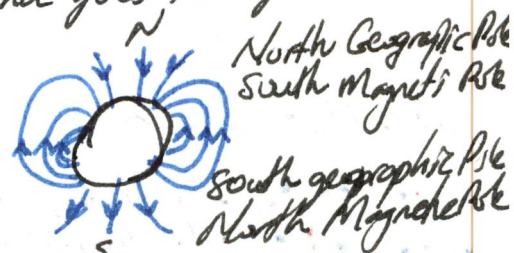
$$K = \frac{1}{2} m v^2 \rightarrow v = \sqrt{\frac{2(5.3 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(1.67 \times 10^{-27} \text{ kg})}} = 3.2 \times 10^7 \text{ m/s}$$

$$F_B = |q| v B \sin \phi = (1.6 \times 10^{-19} \text{ C})(3.2 \times 10^7 \text{ m/s})(1.2 \times 10^{-3} \text{ T}) \sin 90^\circ = 6.1 \times 10^{-16} \text{ N}$$

Proton motion direction is changed and goes to right.

Magnetic Field Lines.

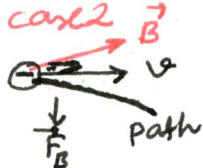
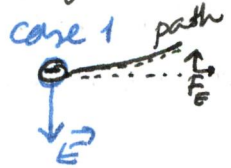
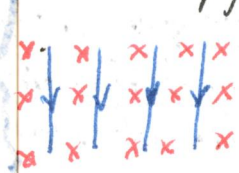
- Similar to electric field lines. } SLN
- North Pole & South pole. } Fig. 28-4
- (emerge from) (enter into) } Fig. 28-5
- Magnet has two poles $\Rightarrow$ magnetic dipole
- Like poles repel, opposite poles attract
- SLN Sample Problem for large acceleration



Crossed Fields: Discovery of the Electron

$\vec{E}$ produces $\vec{F}_E$ on a charged particle. When $\vec{E} \perp \vec{B}$: crossed fields

what happens if e^- move in cross-fields? SLN Fig. 28-7



when $\vec{B} \neq 0$ and $|\vec{F}_B| = |\vec{F}_E|$

$$qE = |q|vB \sin 90^\circ \rightarrow \boxed{v = \frac{E}{B}}$$

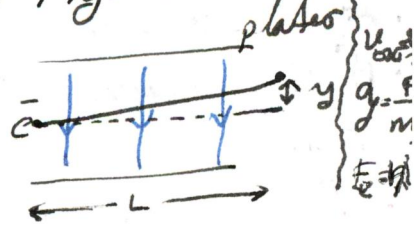
velocity of the charged particle can be measured.

When $\vec{B} = 0$

$$y = \frac{1}{2} a_y t^2$$

$$x = L = v_{0x} t \Rightarrow t = \frac{L}{v}$$

$$\boxed{y = \frac{1}{2} \frac{|q|E}{m_e} \frac{L^2}{v^2}}$$



Deflecting

When $F_E > F_B$



When $F_E < F_B$



if we substitute $v = \frac{E}{B}$ into deflection equation
 $\rightarrow$ all the quantities at the right hand side can be measured

$$\boxed{\frac{m}{|q|} = \frac{B^2 L^2}{2 y E}}$$

$\rightarrow$ Allowed us to measure $\frac{m}{|q|}$ of moving particle: Discovery of electron

Crossed Fields: The Hall Effect

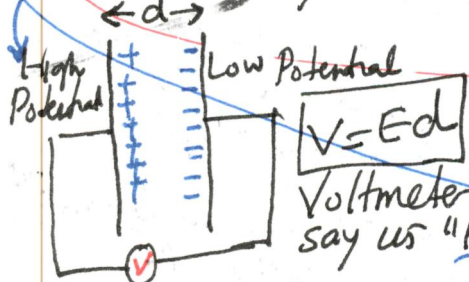
Previously, e^- was in vacuum and deflected by $\vec{B}$ (and/or $\vec{E}$)

Now, e^- are in a copper wire! $\left\{ \begin{array}{l} \text{Drifting} \\ \text{Deflected by } \vec{B} \end{array} \right\}$ Hall effect

Hall effect allows us to determine

type of charge carriers

number of carriers per unit volume



SLN Fig. 28-8

$$\text{When } F_E = F_B \rightarrow eE = ev_d B \rightarrow v_d = \frac{E}{B}$$

we know $v_d = \frac{J}{neA} = \frac{i}{neA} \Rightarrow n = \frac{i}{v_d e A} \left[\frac{E}{B} \right]$

$$n = \frac{iB}{EeA} = \frac{iB}{\frac{V}{d} e d l} = \boxed{\frac{iB}{V e l}}$$

A circulating Charged Particle

(Uniform Circular Motion)

Consider a beam of e^- s projected into a chamber $\left\{ \begin{array}{l} \text{speed } v \\ \text{uniform } B \\ \text{directed out of page} \end{array} \right.$



Magnetic force
 A circular path
 Continuous Deflection

$$F_B = |q|vB \sin 90^\circ$$

$$F = m \frac{v^2}{r}$$

$$m \frac{v^2}{r} = |q|vB$$

$$\boxed{r = \frac{m v}{|q| B}}$$

Radius of Circular Path

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q| B}$$

$$f = \frac{1}{T} = \frac{|q| B}{2\pi m}$$

$$\omega = 2\pi f = \frac{|q| B}{m}$$

$$\frac{|q|}{m} !!$$

Example SLN Fig. 28-12 Uniform Circular Motion of a charged particle in a $\vec{B}$

$$B = 80.000 \text{ mT}$$

$$V = 1000.0 \text{ V}$$

$$q = 1.6022 \times 10^{-19} \text{ C}$$

$\alpha = 1.6254$: strikes

$m = ?$ in atomic mass unit

$$r = \frac{mv}{|q|B}$$

We need $\vec{v}$ when enters the chamber.

$$K_2 + U_2 = K_1 + U_1$$

$$\frac{1}{2}mv^2 - qV = 0$$

$$\Rightarrow v = \sqrt{\frac{2qV}{m}}$$

$$r = \frac{m}{|q|B} \sqrt{\frac{2qV}{m}} = \frac{1}{B} \sqrt{\frac{2mV}{q}} = \frac{r}{2}$$

$$\Rightarrow m = \frac{B^2 q r^2}{8V}$$

$$m = 3.3863 \times 10^{-25} \text{ kg} = 203.93 \text{ amu}$$

divided by $1.6605 \times 10^{-27} \text{ kg}$ (atomic mass unit)

Magnetic Force on a Current-Carrying Wire

Current Carrying wire
Magnetic field

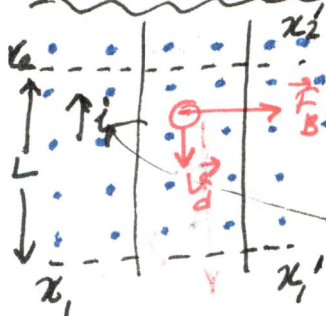
Force on electrons

This force is transmitted to the wire

So that, a current carrying wire will also experience a $\vec{F}_B$

SLN Fig. 28-14

Inside the wire



All conduction e 's move through planes $(xx')_1$ & $(xx')_2$
Time required, $t = \frac{L}{v_d}$ & Charge $q = it$

$$\Rightarrow q = i \frac{L}{v_d}$$

$$F_B = |q| v_d B \sin \phi = i \frac{L}{v_d} v_d B \sin \phi$$

$$\Rightarrow F_B = iLB$$

$$\vec{F} = i \vec{L} \times \vec{B}$$

Generalization

Magnetic force acting wire of length L in $\vec{B}$
 $\vec{L}$: length vector L // $\vec{i}$

Wire of arbitrary shape.

Divide the wire into small segments having a length of $d\vec{L}$
 $\Rightarrow d\vec{F}_B = i d\vec{L} \times \vec{B}$ Magnetic force acting on this segment, $d\vec{F}_B$

Total Magnetic Force

$$\vec{F}_B = i \int_a^b d\vec{L} \times \vec{B} = i \left(\int_a^b d\vec{L} \right) \times \vec{B}$$

If wire form a closed loop: $\vec{F}_B = 0$ ($\oint_a^a d\vec{L} = 0$)

Example

Magnetic Force on a wire carrying current Fig. 28-17 SLN

straight, horizontal copper wire
 $i = 28 \text{ A}$

$B_{\min} = ?$

to suspend the wire to balance $\vec{F}_g$

$$\frac{m}{L} = 46.6 \text{ g/m}$$

$$\vec{F}_B = i \vec{L} \times \vec{B} \Rightarrow iLB \sin \phi$$

$$\vec{F}_g = m\vec{g} \Rightarrow mg$$

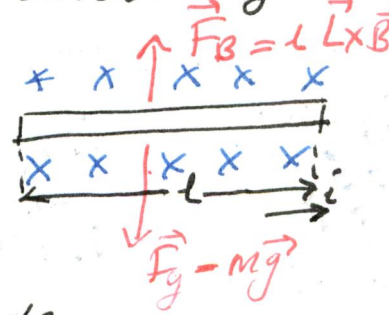
$$iLB \sin \phi = mg$$

for minimum B ; $\phi = 90^\circ$

$$B = \frac{mg}{iL} = \frac{46.6 \times 10^{-3} \text{ kg/m} \cdot 9.8 \text{ m/s}^2}{28 \text{ A}}$$

$$B = 1.6 \times 10^{-2} \text{ T}$$

~ 160 times the strength of Earth's magnetic field



Torque on a Current Loop

Rotation in electric motors is created by torque produced by $\vec{F}_B$
 SLN Fig. 28-18 A rectangular loop in $\vec{B}$.

Fig. 28-19 $\tau' = \underbrace{(i a B \sin \theta) \frac{b}{2}}_{\vec{r} \times \vec{F}_1} + \underbrace{(i a B \sin \theta) \frac{b}{2}}_{\vec{r} \times \vec{F}_3} \left\{ |\vec{r}| = \frac{b}{2} \right.$
 $\left. \right\} \tau' = i a b B \sin \theta$

$\tau = i a b B \sin \theta \rightarrow \tau = N \tau' = (N i A) B \sin \theta$

• For a coil containing N loop (turns)
 $\tau = N \tau' = (N i A) B \sin \theta \quad \{ A = ab \}$

• For a coil containing N loop (circular turns)
 $\tau = N \tau' = (N i A) B \sin \theta \quad \{ A = \pi r^2 \}$

The Magnetic Dipole Moment

A current-carrying coil placed in a $\vec{B} \rightarrow$ a torque acts
 is said to be a magnetic dipole magnetic dipole moment, $\vec{\mu}$

we know that the magnitude of the torque in a N -loop coil
 $\tau = N i a b B \sin \theta = N i (\text{Area}) B \sin \theta = \mu B \sin \theta = \vec{\mu} \times \vec{B}$

$\vec{\mu} = N i \vec{A}$
 $\vec{A} \uparrow$

$\left\{ \begin{array}{l} \text{Unit} \\ A m^2 \end{array} \right\}$

$\left\{ \begin{array}{l} \text{Direction} \\ \text{Right Hand Rule} \end{array} \right\}$

• Magnetic moment vector tries to align with $\vec{B} \rightarrow$ minimum energy
 SLN Fig. 28-20

• Torque on the coil, τ : $\tau = \mu \times B$

• Energy of magnetic dipole in an external $\vec{B}$: $U(\theta) = -\vec{\mu} \cdot \vec{B}$
 $\left\{ \begin{array}{l} U: \text{max } \theta = 180^\circ \\ U: \text{min } \theta = 0 \end{array} \right\}$ dipole's orientation

• Work done on dipole: There is torque $\rightarrow$ Rotates the dipole $\theta_i \rightarrow \theta_f$
 $\Rightarrow W_a = U_f - U_i$

Example

Rotating a magnetic dipole in a $\vec{B}$

SLN Fig 28.21

$\vec{B} \rightarrow$

 circular coil
 $N = 250$
 $A = 2.52 \times 10^{-4} m^2$
 $i = 100 \mu A$
 $B = 0.85 T$
 $\vec{B} \parallel \vec{\mu}$

i) Direction of the current?
 RHR $\rightarrow \downarrow i$
 ii) How much work: $\theta = 0 \rightarrow \theta = 90^\circ$ ~ initial orientation $\rightarrow$ final orientation
 $W_a = U(90^\circ) - U(0) = -\mu B \cos 90 - (-\mu B \cos 0)$
 $= 0 + \mu B = \mu B = (N i A) B$
 $= (250)(100 \times 10^{-6} A)(2.52 \times 10^{-4} m)(0.85 T)$
 $= 5.355 \times 10^{-6} J \approx 5.4 \mu J$

Chapter 30 Induction and Inductance

A current produces Magnetic Field } More Surprisingly { A magnetic field can produce an electric field that can drive a current.

$\vec{B}$ induces $\vec{E} \rightarrow$ Faraday's law of induction.

Experiment 1

SLN Fig. 30-1

- Initially, no current passing through the wire
- Move magnet $\rightarrow$ current on wire
 - move towards the loop $\rightarrow$ in one direction
 - move away from the loop $\rightarrow$ in another direction

Experiment 2

SLN Fig. 30-2

- Initially, no current
- S is closed $\rightarrow$ a sudden current in one direction
- S is opened $\rightarrow$ a sudden current in another direction
- Steady state current $\rightarrow$ No current in right hand loop in left hand loop

- The current produced in the loop is called induced current.
- The work done per unit charge to produce that current is called induced emf.
- The process of producing the current and emf is called induction.

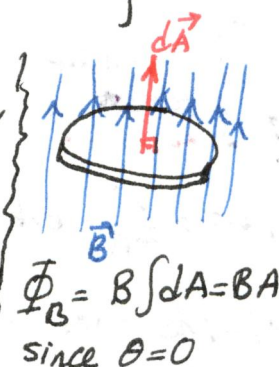
Faraday's law of induction

Electro
motive
force

Faraday realized that an emf and a current can be induced in a loop by changing the amount of magnetic field passing through the loop.

of $\vec{B}$ field lines that pass through the loop $\xrightarrow{\text{changing}}$ An emf is induced in the loop } Faraday's law of Induction.

$\Phi_B = \int \vec{B} \cdot d\vec{A}$: Magnetic flux. Magnetic field through area A.
 ($\Phi_E = \int \vec{E} \cdot d\vec{A}$)
 Weber (Wb) $1 \text{ Wb} = 1 \text{ Tm}^2$
 $\Phi_B = \int B dA \cos \theta$



$\mathcal{E} = - \frac{d\Phi_B}{dt}$
 minus means that emf, $\mathcal{E}$, tends to oppose flux change

How we can change the Φ_B through a coil?

- 1) Change the magnitude of B,
- 2) Change the area of the coil, or
 " " " " " by sliding in or out of $\vec{B}$
- 3) Change the angle btw $\vec{B}$ and $d\vec{A}$

Example

Induced $\mathcal{E}$ in a coil due to a solenoid. SLN Fig. 30-3

$N_s = 220 \text{ turn/cm}$
 $i = 1.5 \text{ A}$
 $D = 3.2 \text{ cm}$
 $N_c = 130 \text{ turn/cm}$
 $d = 2.1 \text{ cm}$
 $\Delta t = 25 \text{ ms}$
 $\mathcal{E} = N_c d\Phi - 25 \text{ ms}$

$B_s = \mu_0 i n = \mu_0 (1.5 \text{ A}) (22000 \text{ turn/m})$
 $\Phi = B_s A = B (\pi (\frac{d}{2})^2) = 1.44 \times 10^{-5} \text{ Wb}$
 $\frac{d\Phi_B}{dt} = \frac{\Delta \Phi_B}{\Delta t} = \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} = \frac{1.44 \times 10^{-5} \text{ Wb}}{25 \times 10^{-3} \text{ s}}$
 $\mathcal{E} = -5.76 \times 10^{-4} \text{ V}$

Lenz's Law

Determining the direction of induced current in the loop.

An induced current has a direction such that the magnetic field due to current opposes the change in the magnetic flux that induces the current.

SLN Lenz law can be evaluated in two different ways.

SLN Fig. 30-5

Example Changing Uniform B

A conducting loop
half-circle $r = 0.20\text{m}$
three straight sections

$$B = 4.0t^2 + 2.0t + 3.0$$

$$\mathcal{E}_{\text{bat}} = 2.0\text{V}$$

$$R = 2.0\Omega$$

Fig. 30-6

i) $\mathcal{E}_{\text{ind}} = ?$ at $t = 10\text{s}$

$$\mathcal{E}_{\text{ind}} = \frac{d\Phi_B}{dt} = A \frac{dB}{dt}$$

$$= \pi r^2 (8.0t + 2.0)$$

at $t = 10\text{s}$

$$\mathcal{E}_{\text{ind}} = 5.152\text{V}$$

ii) current = ? at $t = 10\text{s}$

$$i = \frac{\mathcal{E}_{\text{net}}}{R} = \frac{\mathcal{E}_{\text{ind}} - \mathcal{E}_{\text{bat}}}{R}$$

$$= \frac{5.152\text{V} - 2.0\text{V}}{2.0\Omega} = 1.58\text{A}$$

CW

Example Changing nonuniform B

Fig. 30-7

Rectangular loop

$$B = 4t^2 x^2$$

$$W = 3.0\text{m}$$

$$H = 2.0\text{m}$$

$$\mathcal{E}_{\text{ind}} = ?$$
 at $t = 0.15$

$$\mathcal{E} = \frac{d\Phi_B}{dt}$$

$$= \frac{d}{dt} \int B dA$$

$$= \frac{d}{dt} \int 4t^2 x^2 H dx$$

$$= \frac{d}{dt} (4t^2 H \frac{W^3}{3})$$

$$= 144t^2$$

$$= 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

$$\mathcal{E} = 14\text{V}$$

Induction and Energy Transfers

A rectangular loop $\left\{ \begin{array}{l} \text{one end in a uniform external magnetic field} \\ \text{has a resistance of } R \\ \text{pull this loop to the right at a constant velocity} \end{array} \right\}$ Flux change with time
current will be induced in CW.

Applied force $\rightarrow$ energy transferred to $\rightarrow$ thermal energy
loop + magnet system

Mechanical work done during pulling $\left\{ \begin{array}{l} \text{Magnetic force} \\ \text{Apply a constant force} \end{array} \right\}$ The rate of work done: Power $P = Fv$ speed of the loop

Apply Faraday's law: $\Phi_B = BA = BLx$ $\mathcal{E} = \frac{d\Phi}{dt} = BL \frac{dx}{dt} = BLv$

Representation of the loop as a circuit: SLN Fig. 30-9 $i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$

Deflection Forces in Fig. 30-8: $\vec{F}_2$ & $\vec{F}_3$ cancels each other. $\vec{F}_1$ is opposing force $\vec{F} = i\vec{l} \times \vec{B} \rightarrow F_1 = i\ell B \sin 90^\circ = (\frac{BLv}{R})BL = \frac{B^2 \ell^2 v}{R}$ $P = \frac{B^2 \ell^2 v^2}{R} = i^2 R$: Power Thermal energy rate

$\Rightarrow$ Work done on the loop transformed to thermal energy in the loop

SLN Fig. 30-17. Physical meaning of τ_L : when $t = \tau_L \Rightarrow i = 0.63 \mathcal{E}/R$ { since $e^{-1} = 0.63$
 $\Rightarrow \tau_L$ is the time it takes current in the circuit to reach about 63% of its final equilibrium value ($\mathcal{E}/R$)

- When S is on "b" (Fig. 30-15, Fig. 30-16)

$L \frac{di}{dt} + iR = 0 \rightarrow i = \frac{\mathcal{E}}{R} e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$ Decay of current

Example RL circuit immediately after switching and after a long time

SLN Fig. 30-18a
 $R = 9 \Omega$
 $L = 2.0 \text{ mH}$
 $\mathcal{E} = 18 \text{ V}$
 (ideal battery)

i) $i = ?$ just after the switch is closed
 .. Initially, an inductor acts like broken wire Fig. 30-18b
 .. Long later, it acts like ordinary wire Fig. 30-18c

$\mathcal{E} - iR = 0 \rightarrow i = \mathcal{E}/R = \frac{18 \text{ V}}{9.0 \Omega} = 2.0 \text{ A}$

ii) $i = ?$ long after the switch is closed
 $i = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{18 \text{ V}}{3.0 \Omega} = 6.0 \text{ A}$

$U_B = \frac{1}{2} Li^2$: Energy stored in Magnetic Fields (SLN Fig. 30-16)
 by an inductor L carrying a current i .

$U_E = \frac{q^2}{2C}$: Energy stored in Electric Fields by a capacitor with capacitance C and charge q .

Example Energy stored in Magnetic Fields

$L = 53 \text{ mH}$
 $R = 0.35 \Omega$

i) $\mathcal{E} = 12 \text{ V}$ $U_B = ?$ $i_{\text{eq}} = \frac{\mathcal{E}}{R} = \frac{12 \text{ V}}{0.35 \Omega} = 34.3 \text{ A}$ $U_B = \frac{1}{2} Li_{\text{eq}}^2 = \frac{1}{2} (53 \times 10^{-3} \text{ H}) (34.3 \text{ A})^2 = 31 \text{ J}$

equilibrium current (steady state)

ii) When $U_B \rightarrow \frac{1}{2} U_{B0}$
 $\frac{1}{2} Li^2 \rightarrow \frac{1}{2} (\frac{1}{2} Li_{\text{eq}}^2)$
 $i = \frac{1}{\sqrt{2}} i_{\text{eq}}$
 $i = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L})$
 $\frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) = \frac{\mathcal{E}}{R} \frac{1}{\sqrt{2}}$
 $1 - e^{-t/\tau_L} = \frac{1}{\sqrt{2}} = 0.707$
 $e^{-t/\tau_L} = 1 - 0.707 = 0.293$
 $-\frac{t}{\tau_L} = \ln(0.293)$
 $t = 1.23 \tau_L$

In Solenoid: Area A , length l } volume is Al
 (long, near the middle) $B = \mu_0 n i$

Mutual Induction

Not self-induction. Mutual interaction of two coils: Mutual Induction (only one coil is involved) SLN Fig. 30.2

Fig. 30-19a • Two circular close-packed coils
 • Near each other
 • A common central axis.
 an $\mathcal{E}$, emf appears in coil ② due to changing current in coil ①.
 M : proportionality constant

Mutual Inductance of coil 2 wrt coil 1
 $M_{21} = \frac{N_2 \Phi_{21}}{i_1}$ { $M_{21} = M_{12}$

$\mathcal{E}_2 = -M \frac{di_1}{dt}$ $\mathcal{E}_1 = -M \frac{di_2}{dt}$
 minus sign indicate the direction

[Faint, illegible handwritten text covering the majority of the page]

Induced Electric Fields

A changing magnetic field produces an electric field.

- SLN. Fig. 30-11
- There is an external magnetic field in cylindrical form
 - A copper ring of radius r is placed in that field
 - Change magnetic field $\rightarrow$ Flux with change $\rightarrow$ Faraday's law
 - at a steady rate $\rightarrow$ induced emf
 - $\rightarrow$ induced current (ccw)
- $\Rightarrow$ If there is a current in the ring $\rightarrow$ an electric field must be present along the ring (induced)

* Electric field due to changing magnetic field is induced even if there is no copper ring. Fig. 30-11b

A Reformulation of Faraday's Law

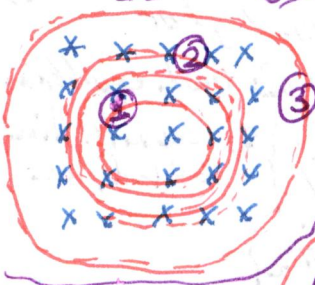
- Fig. 30-11b
- A circular path ($2\pi r$)
 - Work done in one revolution: $W = q_0 \mathcal{E}$ (test charge $\sim$ induced emf)

$$\rightarrow W = \int \vec{F} \cdot d\vec{s} = (q_0 \mathcal{E})(2\pi r) \Rightarrow q_0 \mathcal{E} 2\pi r = q_0 \mathcal{E} \Rightarrow \boxed{\mathcal{E} = 2\pi r \mathcal{E}}$$

$$W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s} = q_0 \mathcal{E} \Rightarrow \boxed{\mathcal{E} = \oint \vec{E} \cdot d\vec{s}} \quad \mathcal{E} = -\frac{d\Phi_B}{dt} \Rightarrow \boxed{\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

closed path $\rightarrow$ changing magnetic field $\rightarrow$ Faraday's law $\rightarrow$ Reformulation

$-\frac{d\Phi_B}{dt} = \oint \vec{E} \cdot d\vec{s}$ produces electric field } This can be applied any closed loop in changing magnetic field



$\mathcal{E}_1 > \mathcal{E}_2$

$\mathcal{E}_3 = ?$ outside of the changing magnetic field

- ① $\vec{E}$ can be produced by static charges. (previous knowledge)
- ② $\vec{E}$ can be produced by changing magnetic flux.

- ① These lines never produce closed loops. From (+) charge to (-) charge
- ② These lines form closed forms

Electric potential has meaning only for electric fields that are produced by static charges. It has no meaning for electric fields that are produced by induction.

$$V_f - V_i = -\int \vec{E} \cdot d\vec{s} = 0 \text{ since } V_f = V_i \text{ in closed loop.}$$

Example: Induced electric field due to changing magnetic field inside and outside

Fig. 30-11b $R = 8.5 \text{ cm}$ & $\frac{dB}{dt} = 0.13 \text{ T/s}$ $\vec{B}$ $\vec{u}$ $r = 12.5 \text{ cm}$ Outside $r > R$

i) inside $r < R$ $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$
 $r = 5.2 \text{ cm}$
 $\rightarrow E(2\pi r) = (\pi r^2) \frac{dB}{dt}$ (drop minus sign)
 $\rightarrow E = \frac{r}{2} \frac{dB}{dt} = \frac{(5.2 \times 10^{-2} \text{ m})}{2} (0.13 \text{ T/s})$
 $E = 0.0034 \text{ V/m} = \boxed{3.4 \text{ mV/m}}$

ii) $r = 12.5 \text{ cm}$ Outside $r > R$
 $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ $\{ E 2\pi r = (\pi R^2) \frac{dB}{dt}$
 $\Rightarrow E = \frac{R^2}{2r} \frac{dB}{dt}$
 $= \frac{(8.5 \times 10^{-2} \text{ m})^2}{2(12.5 \times 10^{-2} \text{ m})} (0.13 \text{ T/s}) = \boxed{3.8 \text{ mV/m}}$ see Fig. 30-12

Inductors and Inductance

Capacitor → to produce electric field

Inductor → to produce magnetic field

reeler
(symbol)

Inductance of a Solenoid

Consider a long solenoid of cross-sectional area A . $L = ? \rightarrow N\Phi_B = ?$

$$N\Phi_B = (nL)BA = (nL)(\mu_0 n i)A \rightarrow L = \frac{(nL)(\mu_0 n i)A}{i} = \mu_0 n^2 LA \Rightarrow \boxed{\frac{L}{i} = \mu_0 n^2 A} \text{ (solenoid)}$$

of turns per unit length

Inductance depends only on the geometry of the device!

For long solenoid, $L \gg r \rightarrow$ uniform magnetic field at the middle.

$$C = \epsilon_0 \frac{A}{d}$$

Self Induction

Two inductors are near each other { current i produces } changing i { An induced emf appears in the first coil as well!
①② (coils) $\Rightarrow$ magnetic flux Φ_B $\Rightarrow$ induced $\mathcal{E}$

$\Rightarrow$ An induced emf $\mathcal{E}_L$ appears in any coil in which the current is changing.

$$\Rightarrow \text{Self-Induction} \Rightarrow \text{Self-induced emf} \Rightarrow \mathcal{E}_L = -\frac{d(N\Phi_B)}{dt} = -L \frac{di}{dt}$$

SLN Fig. 30-13: current changes with time.

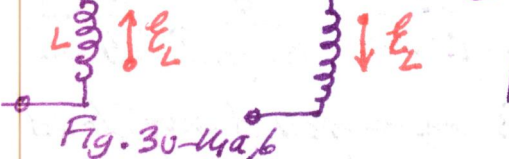


Fig. 30-14a,b

Direction of induced emf: Opposes the change in current i (From Lenz's Law)

- We can define potential difference V_L btw the terminals of inductor
- We can not define potential within the conductor itself
- For an ideal inductor; $V_L \approx \mathcal{E}_L$. If has some resistance, they differ

RL Circuits

RC Circuit { The charge on the capacitor does not build up immediately to its final equilibrium value (steady state).
charge ~ But approaches it an exponential fashion $q = C\mathcal{E}(1 - e^{-t/\tau_C})$ } τ_C : capacitive time constant, RC
discharge ~ Remove $\mathcal{E}$ from circuit. Does not immediately fall to zero.
But approaches zero in an exponential fashion. $q = q_0 e^{-t/\tau_C}$

RL Circuit { If there were no inductor, current will rise suddenly to $\mathcal{E}/R$.
SLN Fig 30-15 { When there is an inductor
• Self-induced emf $\mathcal{E}_L$ appears in the circuit (Lenz's law)
• This opposes the rise of current
• The current will be less than $\mathcal{E}/R$. By time the current increases less rapidly and self-induced emf becomes smaller
• Thus the current in the circuit approaches $\mathcal{E}/R$ asymptotically.

* An inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wires

- When S is on "a" (Fig. 30-15) - current $\rightarrow$ cw

$$\begin{aligned} -iR - L \frac{di}{dt} + \mathcal{E} &= 0 \\ L \frac{di}{dt} + Ri &= \mathcal{E} \end{aligned} \quad \left\{ \begin{array}{l} \text{Rule of current} \\ i = \frac{\mathcal{E}}{R} (1 - e^{-Rt/L}) = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \end{array} \right.$$

τ_L : inductive time constant L/R .
 $t=0 \rightarrow e^0=1 \rightarrow i=0$ sta
 $t=\infty \rightarrow e^\infty=0 \rightarrow i = \frac{\mathcal{E}}{R}$ sta

Chapter 31 - Electromagnetic Oscillations and Alternating Current

Energy is stored (either electric or magnetic energy) to be transferred, then, usage.

Mostly, as a sinusoidally oscillating current, ac. Now, we will study electrically oscillating systems. LC systems

RC $\tau_c = RC$	Charging a capacitor $q = C\mathcal{E}(1 - e^{-t/\tau_c})$ Capacitor $i = \frac{\mathcal{E}}{R} e^{-t/\tau_c}$	Discharging a capacitor $q = q_0 e^{-t/\tau_c}$ capacitor $i = -\frac{q_0}{RC} e^{-t/\tau_c}$	Charge, current potential difference $\Rightarrow$ grow & decay exponentially
RL $\tau_L = \frac{L}{R}$	Rise of current $i = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L})$	Decay of current $i = I_0 e^{-t/\tau_L}$	

τ_c & τ_L are capacitive and inductive time constants

LC Systems: Circuits containing inductance and capacitance } vary sinusoidally (with period and angular frequency ω)

Oscillations $\leftarrow$ Capacitor's electric field, $U_E = \frac{q^2}{2C}$
Inductor's magnetic field, $U_B = \frac{1}{2} Li^2$

Electromagnetic Oscillations SW Fig. 31-1: Eight stages in a single cycle of oscillation of LC circuits

As circuit oscillates, energy shift back and forth from one type of stored energy to the other but total amount of energy is conserved.

V_c : time-varying potential difference across the capacitor, C } Oscillating means that the quantity is varying with time such as sinusoidally - $\sqrt{1/m}$
 V_R : time-varying potential difference across the resistor, R } $V_c = \frac{1}{C} q$
 $V_R = Ri$ SW Fig. 31-1

The Electrical Mechanical Analogy

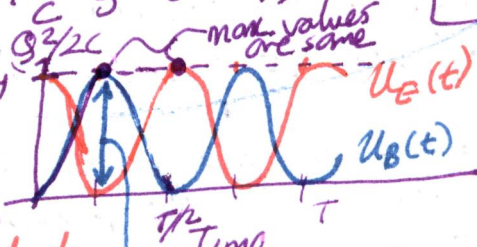
Block-spring system $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \rightarrow x(t) = x_0 \cos(\omega t + \phi)$: Oscillation Sinusoidal!
 $F = ma$, $F = -kx$ ~ battery.
 $m \frac{d^2x}{dt^2} = -kx$ $-L \frac{d^2q}{dt^2} - \frac{q}{C} + \mathcal{E} = 0 \rightarrow q(t) = q_0 \cos(\omega t + \phi)$: LC Oscillator $\omega = 1/\sqrt{LC}$ SW Table 31

SW The LC Oscillator, Quantitatively

$L \frac{d^2q}{dt^2} + \frac{1}{C}q = 0$ charge, $q = Q \cos(\omega t + \phi)$
current, $i = -\omega Q \sin(\omega t + \phi)$
 $i = -I \sin(\omega t + \phi)$
angular frequency $\omega = 1/\sqrt{LC}$
 $-L\omega^2 Q \cos(\omega t + \phi) + \frac{1}{C} Q \cos(\omega t + \phi) = 0 \Rightarrow \omega = \frac{1}{\sqrt{LC}}$
 Q, I : Amplitudes
 ω : angular frequency of EM oscillations
 ϕ : phase constant

Electrical & Magnetic Energy Oscillations

$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C} \cos^2(\omega t + \phi)$
 $U_B = \frac{1}{2} Li^2 = \frac{1}{2} L \omega^2 Q^2 \sin^2(\omega t + \phi) = \frac{Q^2}{2C} \sin^2(\omega t + \phi)$
 $U_E + U_B = \frac{Q^2}{2C}$
Total energy, constant
When $U_E \rightarrow \text{max}$, then $U_B \rightarrow 0$, and conversely

Energy 

Example LC Oscillator, potential charge, rate of current change

$C = 15 \mu\text{F}$
 $V = 57 \text{ V}$
 $L = 12 \text{ mH}$
 LC circuit

i) $V_L(t) = ?$ At time $t=0 \rightarrow V_L(t) = V_C(t)$ maximum (also; charge, $\phi=0$)
 $\Rightarrow \frac{q}{C} = \frac{Q}{C} \cos \omega t \Rightarrow V_C = \frac{Q}{C} \cos \omega t = V_L \Rightarrow \omega = \frac{1}{\sqrt{LC}} \approx 7500 \text{ rad/s}$
 $\Rightarrow V_L = (57 \text{ V}) \cos(7500 \text{ rad/s})t$

ii) $\left(\frac{di}{dt}\right)_{\text{max}} = ?$ $i = \frac{dq}{dt} = -Q \omega \sin \omega t \Rightarrow \frac{di}{dt} = -Q \omega^2 \cos \omega t = -C V_C \omega^2 \cos \omega t = -\frac{V_C \cos \omega t}{L}$
 $\Rightarrow \left(\frac{di}{dt}\right)_{\text{max}} = \frac{V_C}{L} = \frac{57 \text{ V}}{0.012 \text{ H}} \approx 4800 \text{ A/s}$

$q = Q \cos \omega t$
 $q = Q$ (maximum charge)

Alternating Current

Energy should be supplied to the circuit against "damping". It is mostly by AC.
 Alternating Current, AC: Oscillating currents (and emfs) $\leftarrow$ vary sinusoidally with time
 Direct Current, DC: Nonoscillating current (ie from a battery) $\leftarrow$ reversing its direction
 * As the current alternates, so does the magnetic field that surrounds the conductor.
 $\rightarrow$ Possible use of Faraday's Law: Generators, motors, etc. SLN Fig. 31-6

In the loop of Fig. 31-6:

$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$ (induced) (ω_d)
 ω_d : Angular frequency of emf $\equiv$ angular speed of rotation (ω)
 $\omega_d t$: phase of emf
 $\mathcal{E}_m$: amplitude of the emf
 Produces sinusoidally (alternating) current with $i = I \sin(\omega_d t - \phi)$
 $\omega_d \rightarrow$ Driving angular frequency $\Rightarrow i = I \sin(\omega_d t - \phi)$ (needed since current and emf may not be in phase depends on the generator)

Three Simple Circuits

Before RLC circuit, simpler "pure" circuits

① Resistive Load: Fig. 31-8 R & AC generator. Loop Rule $\mathcal{E} - V_R = 0 \rightarrow iR$
 $\rightarrow V_R = \mathcal{E}_m \sin \omega_d t = V_R \sin \omega_d t$ $i_R = \frac{V_R \sin \omega_d t}{R} = I_R \sin(\omega_d t - \phi)$ $\left\{ \begin{array}{l} V_R \\ i_R \end{array} \right\}$ Time varying
 equal $\Rightarrow \phi = 0$ & $V_R = I_R R$
 Fig. 31-9a $\rightarrow$ Do not decay due to "damping" $\rightarrow$ Oscillating
 Fig. 31-9b "Phasor" representation of V_R & i_R

Phasors

Angular Speed: ω_d , Length: Amplitude, Projection: At vertical axis, Rotation Angle: $\omega_d t$
 Voltage & Current are in phase for Resistive Load. SLN Sample Problem

② Capacitive Load: Fig. 31-10 C & AC generator. Loop Rule $\mathcal{E} - V_C = 0$ q/C
 $\rightarrow V_C = V_C \sin \omega_d t$ $C V_C = C V_C \sin \omega_d t = q_C$ $i_C = \frac{dq_C}{dt} = \omega_d C V_C \cos \omega_d t$ $\left\{ \begin{array}{l} V_C \\ i_C \end{array} \right\}$ Time varying
 $\Rightarrow i_C = \frac{V_C}{X_C} \cos \omega_d t = \frac{V_C}{X_C} \sin(\omega_d t + 90^\circ) = I_C \sin(\omega_d t - \phi)$ $\phi = -90^\circ$ pure capacitive load
 $V_C = I_C X_C$ SLN Fig. 31-11a & b Current "leads" the potential difference by 90° .

③ Inductive Load: Fig. 31-12 L & AC generator. Loop Rule $\mathcal{E} - V_L = 0 \quad \mathcal{E} = V_L = L \frac{di_L}{dt}$

$\Rightarrow \boxed{V_L = V_L \sin \omega_d t} \left\{ L \frac{di_L}{dt} = V_L \sin \omega_d t \right\} \frac{di_L}{dt} = \frac{V_L}{L} \sin \omega_d t \left\{ i_L = \int \frac{V_L}{L} \sin \omega_d t dt = \frac{V_L}{L} \int \sin \omega_d t dt \right.$
 $\Rightarrow i_L = -\left(\frac{V_L}{\omega_d L}\right) \cos \omega_d t \left\{ \begin{array}{l} \text{Define} \\ \text{Inductive} \\ \text{Reactance} \end{array} \right. X_L = \omega_d L \left\{ i_L = \frac{V_L}{X_L} \sin(\omega_d t - 90^\circ) \Rightarrow \boxed{i_L = \frac{V_L}{X_L} \sin(\omega_d t - \phi)} \right.$
 $\boxed{I_L = V_L / X_L}$

$\phi = 90^\circ$ pure inductive load

SLN Fig. 31-13a & b Current "lags" the potential difference by 90°

SLN Sample Problem Table 31-2

The Series RLC Circuit

SLN Fig. 31-7

Now, full RLC circuit.
R, L and C are in series.

Applied Alternating Emf
 $\mathcal{E} = \mathcal{E}_m \sin \omega_d t$

in series: same current
 $i = I \sin(\omega_d t - \phi)$
 $I = ? \quad \phi = ?$
by phasor diagrams!

① Current Amplitude, I

Phasor diagrams of current and voltage. SLN Fig 31-14

Fig. 31-14a

$\omega_d t - \phi$ Angle of rotation of phasor
Phase of the current at time t
Length of: the current amplitude I
Phasor

Fig 31-14b

Resistor: current and voltage in phase
Capacitor: voltage is behind current by 90°
Inductor: voltage is ahead of current by 90°
Lengths of: instantaneous voltages V_R, V_L, V_C
phasors at time t.

Fig. 31-14c

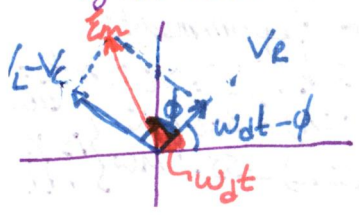
Length of: the emf amplitude $\mathcal{E}_m$
phasor
Loop Rule $\Rightarrow \mathcal{E} = V_R + V_L + V_C$ instantaneous voltages
 $\mathcal{E}_m$ is the vector sum: $\vec{\mathcal{E}}_m = \vec{V}_R + \vec{V}_L + \vec{V}_C$
First combine V_L & $V_C \Rightarrow \vec{V}_L - \vec{V}_C$
Then combine with $V_R \Rightarrow \vec{V}_R + (\vec{V}_L - \vec{V}_C)$

$\mathcal{E}_m^2 = V_R^2 + (V_L - V_C)^2$ Fig. 31-14d
 $= (IR)^2 + (IX_L - IX_C)^2$ Now, see Table 31-2
 $\Rightarrow I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_m}{Z} = I$
 $Z = \sqrt{R^2 + (X_L - X_C)^2}$ impedance
 $X_L = \omega_d L, X_C = \frac{1}{\omega_d C}$
 $I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - \frac{1}{\omega_d C})^2}}$ Current Amplitude

The steady state current that occurs after alternating emf has been applied for some time.

② Phase Constant, ϕ

Fig. 31-14d



$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}$
 $\boxed{\tan \phi = \frac{X_L - X_C}{R}}$ phase constant

$X_L > X_C$: Positive ϕ . Phasor I rotates behind phasor $\mathcal{E}_m$.
 Circuit is more inductive than capacitive.
 Fig. 31-15a & b
 $X_L < X_C$: Negative ϕ . Phasor I rotates ahead of phasor $\mathcal{E}_m$.
 Circuit is more capacitive than inductive.
 Fig. 31-15c & d
 $X_L = X_C$: Zero ϕ . Phasors I and $\mathcal{E}_m$ rotate together.
 The circuit is said to be in resonance.
 Fig. 31-15e & f

Power in Alternating-Current Circuits

AC generator provides energy to RLC $\left\{ \begin{array}{l} \text{stored in the electric field in the capacitor, } C \\ \text{stored in the magnetic field of the inductor, } L \\ \text{dissipated as thermal energy, } R \end{array} \right\}$ In steady-state average stored energy remains constant

The "instantaneous" dissipated energy in the resistor: $P = i^2 R = [I \sin(\omega t - \phi)]^2 R = I^2 R \sin^2(\omega t - \phi)$

What about "average" dissipated energy in the resistor? See Fig. 31-17 a & b

Average over time. One complete cycle $\left\{ \begin{array}{l} \sin \theta \rightarrow 0 \text{ Fig. 31-17a} \\ \sin^2 \theta \rightarrow 1/2 \text{ Fig. 31-17b} \end{array} \right\}$ Average Values

$\Rightarrow P_{avg} = I^2 R \left(\frac{1}{2}\right) = \left(\frac{I}{\sqrt{2}}\right)^2 R$

$\frac{I}{\sqrt{2}}$ is called root-mean-square, or rms value of current i : $I_{rms} = \frac{I}{\sqrt{2}}$

$\Rightarrow P_{avg} = I_{rms}^2 R$ average if we switch to the rms current, we can compute the average rate of energy dissipation for AC circuits just as DC circuits.

$I_{rms} = \frac{I}{\sqrt{2}}$, Similarly $V_{rms} = \frac{V}{\sqrt{2}}$, $\mathcal{E}_{rms} = \frac{\mathcal{E}_m}{\sqrt{2}}$

rms voltage rms emf

Example Driven RLC circuit: power factor and average power

RLC circuit $\left\{ \begin{array}{l} \mathcal{E}_{rms} = 120V \\ f_d = 60.0 \text{ Hz} \\ R = 200\Omega \\ X_L = 80.0\Omega \\ X_C = 150\Omega \end{array} \right.$

i) Power factor. $\cos \phi = ?$ $\phi = ?$

$\cos \phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{200\Omega}{211.90\Omega} \approx 0.944$

$\rightarrow \phi = \pm 19.3^\circ$ which sign is correct?

Since $X_C > X_L \rightarrow$ must be negative!

ii) Average rate $P_{avg} = ?$ in resistance

$I_{rms} = \frac{\mathcal{E}_{rms}}{Z}$ & $P_{avg} = I_{rms}^2 R$

$\Rightarrow P_{avg} = \left(\frac{\mathcal{E}_{rms}}{Z}\right) I_{rms} R = \mathcal{E}_{rms} I_{rms} \frac{R}{Z}$

$\cos \phi = \frac{V_R}{\mathcal{E}_m} = \frac{I R}{I Z} = \frac{R}{Z}$

$\Rightarrow P_{avg} = \mathcal{E}_{rms} I_{rms} \cos \phi$ Average Power

$\cos \phi$: Power Factor

If $\cos \phi \rightarrow 1 \Rightarrow$ maximum P_{avg}

$220V \rightarrow$ rms voltage $\Rightarrow$ maximum voltage

$\rightarrow V = V_{rms} \sqrt{2}$ $V_{rms} = \frac{V}{\sqrt{2}}$

$\approx 311V$

$P_{avg} = \mathcal{E}_{rms} I_{rms} \cos \phi = \mathcal{E}_{rms} \left(\frac{\mathcal{E}_{rms}}{Z}\right) \cos \phi \approx 61.4W$ (OR from $P_{avg} = I_{rms}^2 R$)

iii) P_{avg} is maximum $\rightarrow C_{new} = ?$ $\left\{ \begin{array}{l} \cos \phi = 1 \Rightarrow X_C = X_L = \frac{1}{\omega C} \Rightarrow 80\Omega = \frac{1}{2\pi(60)C_{new}} \\ \Rightarrow C_{new} = 33.2\mu F \Rightarrow P_{avg, max} = 72.0W \end{array} \right.$

Transformers

At home: low voltages

At transmission: low current

$\Rightarrow$ we need transformers

high-voltage transmission

safe voltage at home

See Fig. 31-18 Ideal Transformer

$\mathcal{E} = \mathcal{E}_m \sin \omega t$. AC $\rightarrow$ induction generator at both coils

Energy Transmission Requirements

Example Case: $\mathcal{E} = 735 \text{ kV}$, $I = 500 \text{ A}$

Supply Rate $\Rightarrow P_{avg} = \mathcal{E} I = (735 \text{ kV})(500 \text{ A}) = 368 \text{ MW}$

Dissipated $\Rightarrow P_{avg} = I^2 R = (500 \text{ A})(220\Omega) = 55.0 \text{ MW}$

I_{mag} : magnetization current

$$V_s = V_p \frac{N_s}{N_p}$$

where $V_p = \mathcal{E}_{turn} N_p$

$V_s = \mathcal{E}_{turn} N_s$

$\mathcal{E}_{turn} = \frac{d\Phi}{dt}$ primary secondary

ELECTROMAGNETIC WAVES

- CHAPTER 33 -

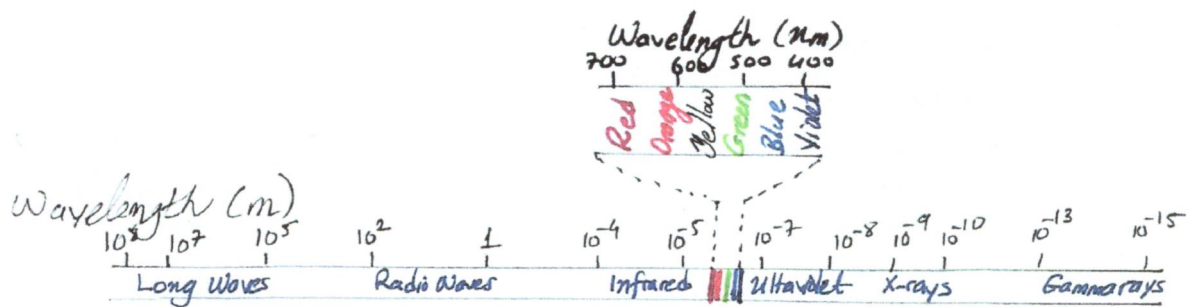
Information age $\longleftrightarrow$ physics of electromagnetic waves $\Rightarrow$ signals

Maxwell's Rainbow

A beam of light is a travelling wave of electric and magnetic fields
an electromagnetic wave (EMW)

Optics, the study of visible light, is a branch of electromagnetism. In Maxwell's time, the visible, infrared and ultraviolet forms of light were the only electromagnetic waves known.

Hertz discovered radio waves $\rightarrow$ same speed as visible light.



Frequency (Hz) Maxwell's Rainbow, Spectrum of EMWs

The Sun: dominant source

Radio and Television signals

Microwaves: Radar and Telephone relay systems

Lightbulbs

Heated engine blocks
of automobiles

X-ray machines

Lightning flashes

Radioactive materials

EMWs

(700 nm - 400 nm)

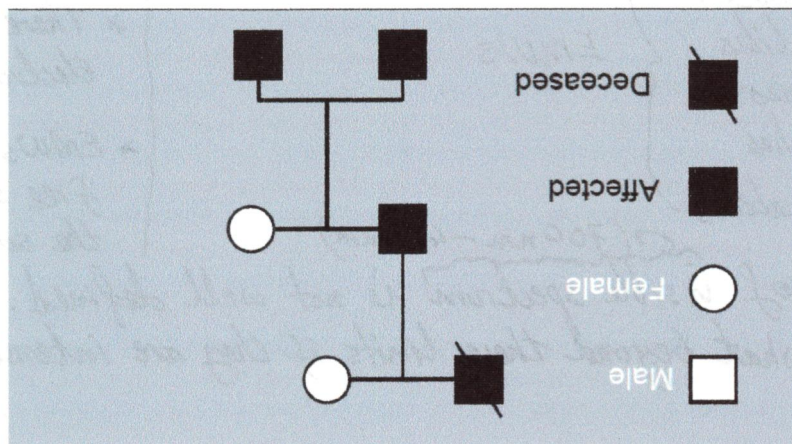
The limits of visible spectrum is not well defined. The eye can detect EMWs somewhat beyond these limits if they are intense enough.

- * scale with a factor of 10
- * scale is open-ended. The wavelengths of EMWs have no inherent upper or lower bounds.

- * There are no gaps in the electromagnetic spectrum

- * EMWs travel through free space (vacuum) with the same speed c .

Pedigree Notation



Traveling EMW

How EMWs are generated.

Restriction: $\lambda \approx 1 \text{ mm} \Rightarrow$

$\Rightarrow$ source of the radiation is both macroscopic and manageable dimensions

NOT, (the sources) are of atomic or nuclear size. But, an antenna.
An LC oscillator with $\omega = \frac{1}{\sqrt{LC}}$

The antenna has the effect of an electric dipole.

* Electric dipole moment varies sinusoidally in magnitude and direction along the antenna.

$\Rightarrow$ produced, $\vec{E}$, varies in magnitude and direction

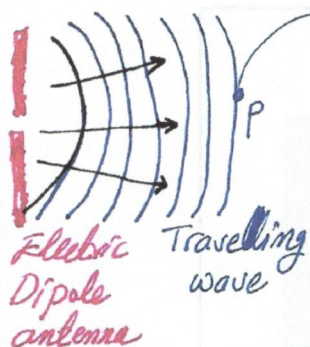
* Also, the current varies in magnitude and direction

$\Rightarrow$ produced, $\vec{B}$, varies

$\rightarrow$ The changing fields form an EMW that travels away from the antenna at speed c .

The angular frequency of this wave is ω , the same as that of the LC oscillator.

A distant point P : Plane wave (curvature of the waves is small enough to neglect)



Key Features:

i) $\vec{E}$ & $\vec{B}$ are always perpendicular to the direction in which the wave is travelling. Transverse wave

ii) $\vec{E} \perp \vec{B}$

iii) $\vec{E} \times \vec{B}$ gives the direction in which the wave travels

iv) The fields always vary sinusoidally

Assume that, P in $+x$,

$\vec{E} \parallel y$ (i) $\vec{B} \parallel z$ (ii)

$\vec{E} \times \vec{B}$ (iii) in $+x$

(iv)
$$\begin{cases} \mathcal{E} = E_m \sin(kx - \omega t) \\ B = B_m \sin(kx - \omega t) \end{cases} \begin{cases} E_m \& B_m: \text{amplitudes} \\ \omega: \text{angular frequency} \\ k: \text{angular wave number} \end{cases}$$

$\downarrow$ Magnetic wave component

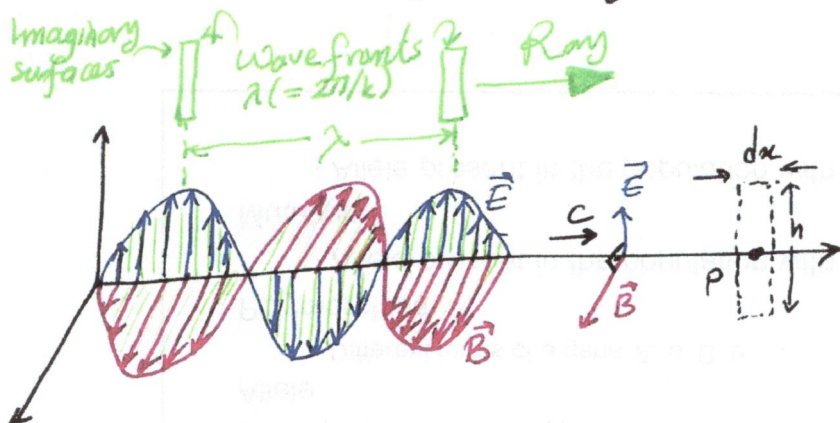
$\downarrow$ Electric wave component

Two fields form the EMW $\Rightarrow$

Two component can not exist independently

$\Rightarrow$ Each also forms its own wave

The speed of the wave is $\frac{\omega}{k}$. In EMW, $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$
 All EMWs, including visible light, have the same speed c in vacuum



$$\frac{E_m}{B_m} = c \quad \text{Amplitude ratio}$$

$$\frac{E}{B} = c \quad \text{Magnitude ratio}$$

First consider $\vec{B}$;
 - varies sinusoidally
 $\Rightarrow$ induces a perpendicular $\vec{E}$
 (Faraday's law of induction)
 Induced $\vec{E}$
 - perpendicular to $\vec{B}$
 - also varies sinusoidally
 $\Rightarrow$ induces a perpendicular $\vec{B}$

Two fields continuously create each other via induction.

Resulting sinusoidal variations in the fields travel as a wave, EMW.

Energy Transport and the Poynting Vector

An EMW can transport energy.

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$: The rate of energy transport per unit area in wave.

$\left(\frac{\text{energy}}{\text{time}} \right)_{\text{area}} = \left(\frac{\text{Power}}{\text{area}} \right)_{\text{int}}$ Poynting vector. Wave's direction and direction of energy transport

$$S = \frac{1}{\mu_0} E B \quad \left\{ \begin{array}{l} \text{dealt with} \\ \vec{B} = \frac{E}{c} \end{array} \right\} \Rightarrow S = \frac{1}{c \mu_0} E^2$$

$$I = S_{\text{avg}} = \left(\frac{\text{Power}}{\text{area}} \right)_{\text{avg}} = \frac{1}{c \mu_0} [E^2]_{\text{avg}} = \frac{1}{c \mu_0} E E_m^2 \sin^2(kx - \omega t)]_{\text{avg}} = \frac{1}{c \mu_0} E_{\text{rms}}^2$$

over a full cycle $\left\{ \begin{array}{l} [\sin^2 \theta]_{\text{avg}} \rightarrow \frac{1}{2} \\ E_{\text{rms}} = \frac{E_m}{\sqrt{2}} \end{array} \right.$

$$u_E = u_B : \text{Energy densities } (= \frac{1}{2} \epsilon_0 E^2)$$

$$u_E = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 (cB)^2$$

$$= \frac{1}{2} \epsilon_0 \frac{1}{\mu_0 \epsilon_0} B^2 = \frac{B^2}{2 \mu_0} = u_B \quad \boxed{u_E = u_B}$$

$$\Rightarrow \boxed{I = \frac{1}{c \mu_0} E_{\text{rms}}^2}$$