Lecture 12

Fundamental Sampling Distributions and Data Distributions II

Lecture Information

Ceng272 *Statistical Computations* at May 17, 2010

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$$\mu_{\bar{X}} = 800, \ \sigma_{\bar{X}} = 40/\sqrt{16} = 10$$

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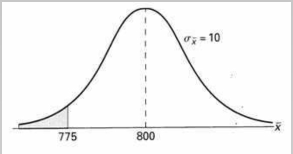

Figure: Area for Example 8.6.

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$$P\left[|(\bar{X} - 5)| \geq 0.027\right]$$

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$$= 2P(Z \geq 2.7) = 2 * 0.0035 = 0.007$$

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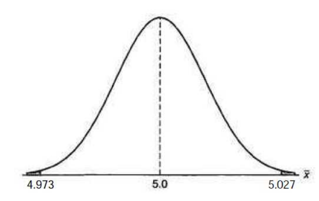

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- How about the sampling distribution
- Solution: Using Theorem 7.11, $\bar{X}_1 - \bar{X}_2$ is approximately normally distributed with mean

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_{\bar{X}_1} - \mu_{\bar{X}_2} = \mu_1 - \mu_2$$

and variance

$$\sigma^2_{\bar{X}_1 - \bar{X}_2} = \sigma^2_{\bar{X}_1} + \sigma^2_{\bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Sampling Distribution of Means X

- **Theorem 8.3:**

If independent sample of size n_1 and n_2 are drawn at random from two populations, discrete or continuous, with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively, then the sampling distribution of the differences of means, $\bar{X}_1 - \bar{X}_2$ is approximately normally distributed with mean and variance given by

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 \text{ and } \sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Hence

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

is approximately a standard normal variable.

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- find $P(\bar{X}_A - \bar{X}_B > 1.0)$ where $\bar{X}_A$ and $\bar{X}_B$ are average drying times for samples of size $n_A = n_B = 18$.

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- Solution:

$$\mu_{\bar{X}_A - \bar{X}_B} = \mu_A - \mu_B = 0$$

$$\sigma^2_{\bar{X}_A - \bar{X}_B} = \frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}$$

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{1 - 0}{\sqrt{1/9}} = 3.0$$

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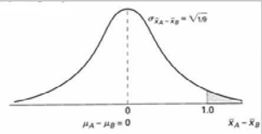

Figure: Area for Example 8.8.

Low probability. Assumption?

Sampling Distribution of Means XII

- **Example 8.9**: The television picture tubes of manufacturer A have a mean lifetime of 6.5 years and a standard deviation of 0.9 year, while those of manufacturer B have a mean lifetime of 6.0 years and a standard deviation of 0.8 year.

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- What is the probability that a random sample of 36 tubes from manufacturer *A* will have a mean lifetime that is at least 1 year more than the mean lifetime of a sample of 49 tubes from manufacturer *B*?

Table: Data for Example 8.9.

Population 1	Population 2
$\mu_1 = 6.5$	$\mu_2 = 6.0$
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$$z = \frac{1 - 0.5}{\sqrt{0.0356}} = 2.65$$

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$$= 0.004$$

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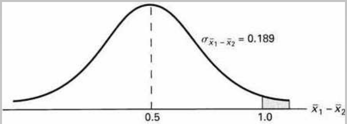

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- Consider the distribution of the statistics $\frac{(n-1)S^2}{\sigma^2}$

$$\sum_{i=1}^{n}(X_i-\mu)^2=\sum_{i=1}^{n}\left[(X_i-\bar{X})+(\bar{X}-\mu)\right]^2$$

$$=\sum_{i=1}^{n}(X_i-\bar{X})^2+\sum_{i=1}^{n}(\bar{X}-\mu)^2+2(\bar{X}-\mu)\sum_{i=1}^{n}(X_i-\bar{X})$$

$$=\sum_{i=1}^{n}(X_i-\bar{X})^2+n(\bar{X}-\mu)^2$$

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Sampling Distribution of S^2 III

- **Theorem 8.4**:

If S^2 is the variance of a random sample of size n taken from a normal population having the variance σ^2, then the statistic

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{\sigma^2}$$

has a chi-squared distribution with $\nu = n - 1$ degrees of freedom.

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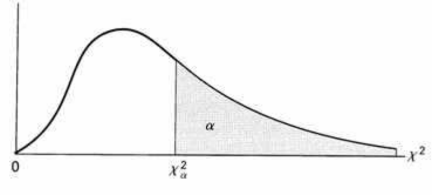

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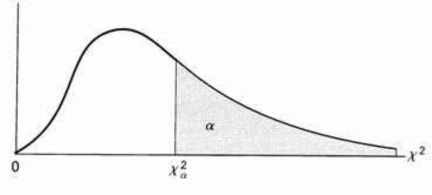

For $\nu = 7, \chi^2_{0.05} = 14.067$, and $\chi^2_{0.95} = 2.1677$ (Table A.5)

Figure: The chi-squared distribution.

Sampling Distribution of S^2 IV

TABLE A.5 Critical Values of the Chi-Squared Distribution

v	α = 0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.0^4393	0.0^3157	0.0^3628	0.0^3982	0.00393	0.0158	0.0642	0.102	0.148	0.455
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342

TABLE A.5 (continued) Critical Values of the Chi-Squared Distribution

v	α = 0.30	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588

Sampling Distribution of S^2 V

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- Assume that the battery lifetime follows a normal distribution.
- Solution:

$$s^2 = \frac{n\sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2}{n(n-1)} = \frac{5 * 48.26 - 15^2}{5 * 4} = 0.815$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{4 * 0.815}{1} = 3.26$$

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- Therefore the manufacturer has no reason to suspect that the standard deviation is other than 1 year.

t-distribution I

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- where $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ and $V = \frac{(n-1)S^2}{\sigma^2}$

t-distribution II

- **Theorem 8.5**:

  Let Z be a standard normal random variable and V a chi-squared random variable with ν degrees of freedom.
  If Z and V are independent, then the distribution of the random variable T, where

  $$T = \frac{Z}{\sqrt{V/\nu}}$$

  is given by the density function

  $$h(t) = \frac{\Gamma\left[(\nu+1)/2\right]}{\Gamma(\nu/2)\sqrt{\pi\nu}}\left(1+\frac{t^2}{\nu}\right)^{-(\nu+1)/2}, -\infty < t < \infty$$

  This is known as the **t-distribution** with ν degrees of freedom.

t-distribution III

- **Corollary 8.1**:

Let $X_1, X_2, \ldots, X_n$ be independent random variables that are all normal with mean μ and standard deviation σ. Let

$$\bar{X} = \sum_{i=1}^{n} \frac{X_i}{n} \text{ and } S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

Then the random variable $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$ has a t-distribution with $\nu = n - 1$ degrees of freedom.

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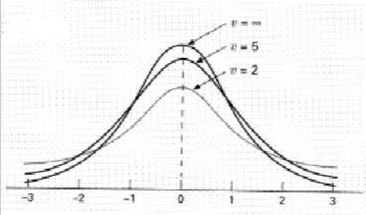

Figure: The t-distribution curves for $\nu = 2, 5$, and ∞

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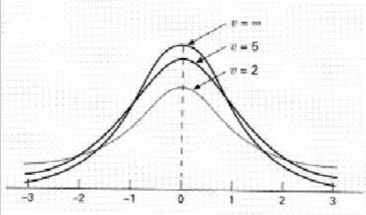

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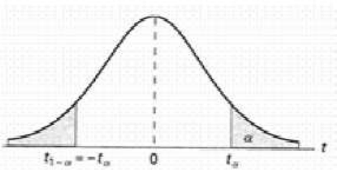

Figure: Symmetry property of the t-distribution.

t-distribution VI

TABLE A.4 Critical Values of the t-Distribution

	α						
v	**0.40**	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131

t-distribution VI

- **Example 8.11**: The t-value with $\nu = 14$ degrees of freedom that leaves an area of 0.025 to the left, and therefore an area of 0.975 to the right, is

$$t_{0.975} = -t_{0.025} = -2.145$$

Look up $t_{0.025}$, and then place a negative sign.

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- **Example 8.12**: Find $P(-t_{0.025} < T < t_{0.05}) = ?$

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  Since $t_{0.05}$ leaves an area of 0.05 to the right, and $-t_{0.025}$ leaves an area of 0.025 to the left, we find a total area of 0.925.

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- Find k such that $P(k < T < -1.761) = 0.045$, for a random sample of size 15 selected from a normal distribution and

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

t-distribution VII

- Solution:

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$$\nu = 15 - 1 = 14$$

From Table A.4, $-t_{0.05} = -1.761$
Let $k = -t_{\alpha}$,
$0.045 = 0.05 - \alpha \Rightarrow \alpha = 0.005$
see Fig. 8

$$k = -t_{0.05} = -2.977 \; (Table\ A.4)$$

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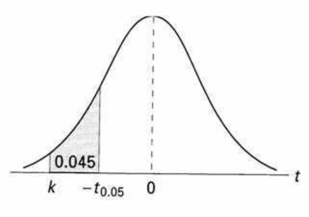

Figure: The t-values for Example 8.13.

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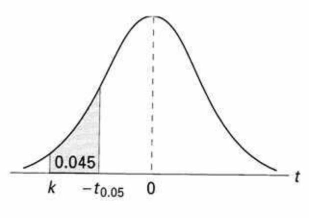

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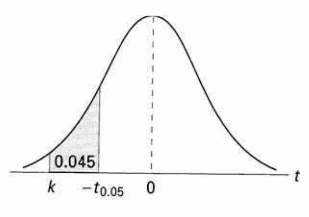

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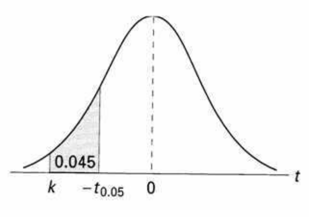

Figure: The t-values for Example 8.13.

- **Example 8.14**: A engineer claims that the population mean of a process is 500 grams. To check this claim he samples 25 batches each month.
- If the computed t-value falls between $-t_{0.05}$ and $t_{0.05}$, he is satisfied with his claim.
- What conclusion should he draw from a sample that has a mean $\bar{x} = 518$ grams and a sample standard deviation $s = 40$ grams? Assume the distribution of yields to be approximately normal.

t-distribution VIII

- Solution: From Table A.4,

$$t_{0.05} = 1.711 \ (\nu = 24)$$

Assumption $\mu = 500 \Rightarrow$

$$t = \frac{518 - 500}{40/\sqrt{25}} = 2.25$$

$$2.25 > 1.711 \rightarrow \textit{error}$$

if $\mu > 500$, t-value would be more reasonable. The process produces a better product than he thought.

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  - Comparative samples (two sample means).
- Use of the t-distribution for the statistic

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

requires that $X_1, X_2, \ldots, X_n$ be normal.

F-distribution I

- The F-distribution finds enormous application in comparing sample variances.

F-distribution I

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- **Theorem 8.6**:

  Let U and V be two independent random variables having chi-squared distribution with ν_1 and ν_2 degrees of freedom, respectively.

  Then the distribution of the random variable $F = \frac{U/\nu_1}{V/\nu_2}$ is given by the density

$$h(f) = \begin{cases} \frac{\Gamma[(\nu_1+\nu_2)/2](\nu_1/\nu_2)^{\nu_1/2}}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)} \frac{f^{\nu_1/2-1}}{(1+\nu_1 f/\nu_2)^{(\nu_1+\nu_2)+1}}, & f > 0 \\ 0, & f \leq 0 \end{cases}$$

  This is known as the ***F*-distribution** with ν_1 and ν_2 degrees of freedom (d.f.).

F-distribution II

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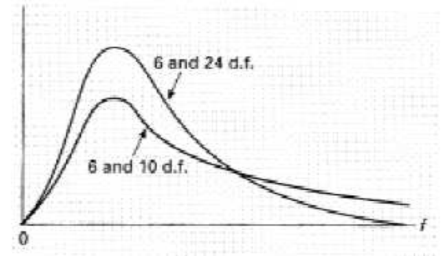

Figure: Typical F-distributions.

F-distribution II

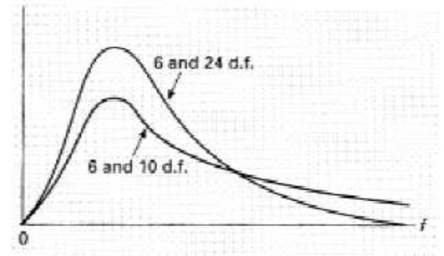

Figure: Typical F-distributions.

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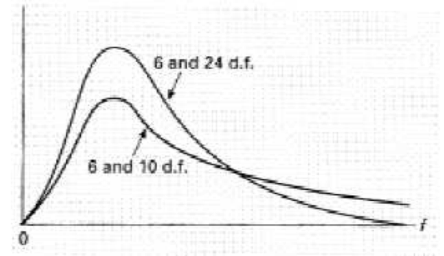

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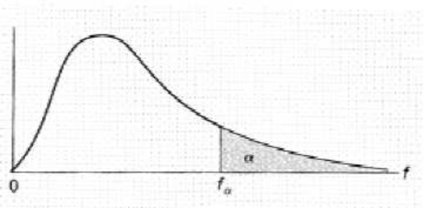

Figure: Illustration of the f_α for the F-distribution.

F-distribution III

- **Theorem 8.7**:

  Writing $f_\alpha(\nu_1, \nu_2)$ for f_α with ν_1 and ν_2 degrees of freedom, we obtain

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- ***F*-distribution with two sample variances.** Suppose that random samples of size n_1 and n_2 are selected from two normal populations with variances σ_1^2 and σ_2^2

  $$X_1^2 = \frac{(n_1 - 1)S_1^2}{\sigma_1^2} \textit{ and } X_2^2 = \frac{(n_2 - 1)S_2^2}{\sigma_2^2} \; (\textit{from Theorem } 8.4)$$

  Let $U = X_1^2$ and $V = X_2^2$ having chi-squared distribution with $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$ degrees of freedom.

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- Using Theorem 8.6, we obtain the following result (theorem:)

F-distribution IV

- **Theorem 8.8**: If S_1^2 and S_2^2 are the variances of independent random samples of size n_1 and n_2 taken from normal populations with variances σ_1^2 and σ_2^2, respectively, then

$$F = \frac{U/\nu_1}{V/\nu_2} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2}$$

  has an F-distribution with $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$ degrees of freedom.

F-distribution V

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- Suppose there are three types of paints to compare. We wish to determine if the population means are equivalent.

Paint	Sample Mean	Sample Variance	Sample Size
A	4.5	0.20	10
B	5.5	0.14	10
C	6.5	0.11	10

F-distribution VI

The notion of the important components of variability is best seen through some simple graphics.

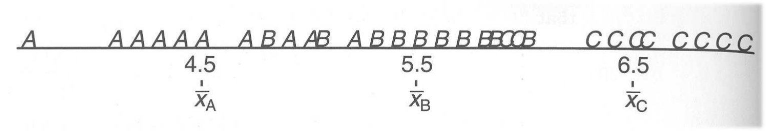

Figure: Data from three distinct samples.

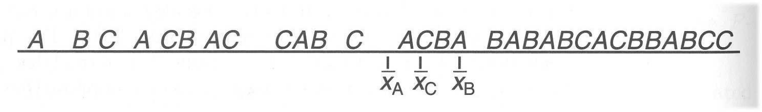

Figure: Data that easily could have come from the same population.

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- The above two sources of variability generate important ratios of sample variances, which are used in conjunction with the F-distribution.
- The general procedure involved is called **analysis of variance.**