1 Solving Nonlinear Equations

- We have given the following function;

$$f(x) = 3x + sin(x) - e^x$$

- To obtain the true value for the root r, which is needed to compute the actual error. MATLAB is used as:

```
>> solve('3*x + sin(x) - exp(x)')
ans=
.36042170296032440136932951583028
```

- Use the function used in the previous item, and write a MATLAB program for Muller's method:
An algorithm for Muller's method :

Given the points x_2, x_0, x_1 in increasing value,
Evaluate the corresponding function values: f_2, f_0, f_1.
Repeat
(Evaluate the coefficients of the parabola, $ax^2 + bx + c$, determined by the three points.
$(x_2, f_2), (x_0, f_0), (x_l, f_1)$.)
Set $h_l = x_1 - x_0; h_2 = x_0 - x_2; \gamma = h_2/h_1$.
Set $c = f_0$
Set $a = \frac{\gamma f_1 - f_0(1+\gamma) + f_2}{\gamma h_1^2(1+\gamma)}$
Set $b = \frac{f_1 - f_0 - a h_1^2}{h_1}$
(Next, compute the roots of the polynomial.)
Set $root = x_0 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$
Choose root, x_r, closest to x_0 by making the denominator as large as possible; i.e. if
$b > 0$, choose plus; otherwise, choose minus.
If $x_r > x_0$,
Then rearrange to:x_0, x_1, and the root
Else rearrange to: x_0, x_2, and the root
End If.
(In either case, reset subscripts so that x_0, is in the middle.)
Until $|f(x_r)| < Ftol$

- Use the function used in the previous item, and write a MATLAB program for Fixed-point Iteration; $x = g(x)$ Method:

Iteration algorithm with the form $x = g(x)$

To determine a root of $f(x) = 0$, given a value x_1 reasonably close to the root
Rearrange the equation to an equivalent form $x = g(x)$
Repeat
Set $x_2 = x_l$
Set $x_l = g(x_1)$
Until $\lvert x_1 - x_2 \rvert < tolerance\ value$

- Tabulate the actual error values as the following; (See Table 1. The number of iterations is not limited to or defined as 15.)

n	Muller $(x_n - r)$	Fixed-point $(x_n - r)$	Muller $f(x_n)$	Fixed-point $f(x_n)$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
12				
13				
14				
15				

Table 1: The Error Sequences

- Plot the behaviours of the errors (use ratios) for both cases. Compare and discuss the rate of convergence.
- A pair of equations:
  $x^2 + y^2 = 4$
  $e^x + y = 1$
  Solve this system by expanding both functions as a Taylor series (begin with $x_0 = 1, y_0 = -1.7$) and by Iteration (begin with $x = 1$)
- Tabulate the actual error values as the following; (See Table 2. The number of iterations is not limited to or defined as 15.)

n	Expansion $f(x_n)$	Iteration $f(x_n)$
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
12		
13		
14		
15		

Table 2: The Error Sequences