Mcs 331 Numerical Methods
Midterm
Dec 06, 2013 10.40–12.30
Good Luck!

Solve all the questions. Each question is 25 pts.

1.

$$f(x) = 3 * x + sin(x) - e^x$$

This nonlinear equation is solved by using three methods, namely *Bisection*, *Newton's*, *Muller's* methods. Then, the following tables are obtained.

iteration	$(x)_1$	$(x)_2$	$(x)_3$
1	0.50000000000000	0.33333333333333	0.50000000000000
2	0.25000000000000	0.36017071357763	0.35491389049015
3	0.37500000000000	0.36042168047602	0.36046467792776
4	0.31250000000000	0.36042170296032	0.36042169766326
5	0.34375000000000	0.36042170296032	0.36042170296032

iteration	$(f(x))_1$	$(f(x))_2$	$(f(x))_3$
1	3.3070e-01	-1.0000e+00	3.3070e-01
2	-2.8662e-01	-6.8418e-02	-1.3807e-02
3	3.6281e-02	-6.2799e-04	1.0751e-04
4	-1.2190e-01	-5.6252e-08	-1.3252e-08
5	-4.1956e-02	-6.6613e-16	2.2204e-16

i If the exact value is given as 0.36042170296032, fill the following tables (use scientific notation as %12.4e, see the table above);

iteration	$Error_1$	$Error_2$	$Error_3$
1			
2			
3			
4			
5			

$ErrorRatio_1$	$ErrorRatio_2$	$ErrorRatio_3$

ii Analyze the obtained tables. Is the convergence sustained for the each methods? For the sustained ones; at which iteration and why?

iii What can you say about the speed of convergences for each method?

iv By using your answers for the previous items, fill the following table. You should explain your decision.

	$Method_1$	$Method_2$	$Method_3$
Name			

v Which method is the best one? Why?

2. Consider the function:

$$f(x) = 2x - 6log(x)$$

Plot of the function is given at the following figure;

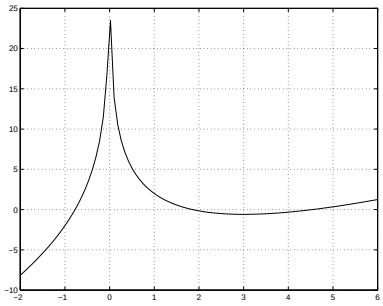

Figure 1: Plot of the function, 2*x-6*log(x)

i Use two iterations of Newton's method to estimate only one of the roots of this function. *Hint:* $\int \frac{1}{x} dx = log(x)$

ii Estimate the error in your answer to part i.

iii Approximately how many iterations of the bisection method would have been required to achieve for the error value of 0.0004? *Hint: Take the interval as ((initial+1)-initial)*

3. Solve this system by Gaussian elimination with pivoting

$$\begin{bmatrix} 1 & -2 & 4 & 6 \\ 8 & -3 & 2 & 2 \\ -1 & 10 & 2 & 4 \end{bmatrix}$$

i How many row interchanges are needed?

ii Repeat without any row interchanges. Do you get the same results?

iii You could have saved the row multipliers and obtained a LU equivalent of the coefficient matrix. Use this LU to solve but with right-hand sides of $[-3, 7, -2]^T$

4. Consider the linear system

$$\begin{aligned} 7x_1 - 3x_2 + 4x_3 &= 6 \\ -3x_1 + 2x_2 + 6x_3 &= 2 \\ 2x_1 + 5x_2 + 3x_3 &= -5 \end{aligned}$$

   i Solve this system with the Jacobi method. First rearrange to make it diagonally dominant if possible. Use $[0, 0, 0]$ as the starting vector.

   ii Repeat with Gauss-Seidel method. Compare with Jacobi method.