Chapter 1- Measurement

Measurements } Science &
Comparisons } Engineering, So, how things are measured and compared
based on $\Rightarrow$ experiments to establish the units for those measurements and comparisons.

measure the quantities : such as length, time, mass, temperature, pressure, electric current

in their own units (by comparison with a standard)

i.e. meter (m) for the quantity length. • standard corresponds to exactly 1.0 units of the quantity

• standard for the length (1.0 m)
$\Rightarrow$ distance traveled by light in a vacuum during a certain fraction of a second.

So many physical quantities !! { not all independent } speed ~ $\frac{\text{length}}{\text{time}}$ ratio } Becomes small number of physical quantities.

$\Downarrow$

(Seven) Base quantities
Base standards

Units for three SI Base Quantities [L] [M] [T]

Quantity	Unit Name	Unit Symbol
length	meter	m
Time	second	s
Mass	kilogram	kg

SI Units ≡ metric system

Derived Units:
Units are defined in terms of base units.

i.e.
SI unit for power, Watt
$1 \text{ Watt} = 1 \text{ W} = 1 \text{ kg} \frac{m^2}{s^3}$

Example: Find the distance that light travels in one year.

$c = 2.998 \times 10^8 \text{ m/s}$ light year, ly

Base Unit: time $1 = \frac{60 \text{ sec}}{1 \text{ min}}$, $1 = \frac{365.25 \text{ day}}{1 \text{ year}}$, $1 \text{ ly} = \frac{(2.998 \times 10^8 \text{ m/s})}{(\text{year})}$

$1 \text{ ly} = (2.998 \times 10^8 \cancel{\text{m/s}}) (\cancel{1 \text{ year}}) \left(\frac{365.25 \cancel{\text{day}}}{\cancel{1 \text{ year}}}\right) \left(\frac{24 \cancel{\text{hours}}}{\cancel{1 \text{ day}}}\right) \left(\frac{60 \cancel{\text{min}}}{\cancel{1 \text{ hour}}}\right) \left(\frac{60 \cancel{\text{sec}}}{\cancel{1 \text{ min}}}\right)$

$= 9.461 \times 10^{15} \text{ m}$

Very large
Very small quantities ; we need scientific notation ⇒ powers of 10

$0.00035 \xrightarrow{2} 3.5 \times 10^{-4}$ or 3.5 E4 ; exponent of ten

~ # of significant figures

$0.000325400 \xrightarrow{6} 3.25400 \times 10^{-4}$

$2500 \xrightarrow{2} 2.5 \times 10^{3}$

$3560000000 \xrightarrow{3} 3.56 \times 10^{9}$

} 10 sf ? Do we know the quantity so accurate? Solution is the scientific notation.

See Table 1.2 for Prefixes for SI Units.

1.27×10^{9} watts = 1.27 gigawatts = 1.27 GW

2.35×10^{-9} s = 2.35 nanosecond = 2.35 ns

See lecture notes for "Changing Units", Chain-link conversion

Example: Uncertainity, How accurate?

Page width? a measure : 1 mm divisions (accuracy)

0.1 cm

21.6 cm

21.6 ± 0.1 cm plus minus 0.1 : Error in measurement

Percentage error in measurement : $\left(\frac{0.1}{21.6}\right) \times 100 = 0.5\ \%$

Page area?

21.6 cm (± 0.1)

27.9 cm (± 0.1)

(0.4%) what is the percentage error in measurement?

} $(21.6\ cm) * (27.6\ cm) = 603\ cm^2$

What about uncertainity in measurement of area?

$(0.4 + 0.5)$ addition $0.9 \rightarrow 0.9\ \%$

$(0.9) * (603\ cm^2) = 5\ cm^2 \Rightarrow 603 \pm 5\ cm^2$

Example: Significant figures. (sf) Physical properties → uncertain

2.00 m (3 sf) < 1.995 m OR 2.005 m ?

2.000 m (4 sf) < 1.9995 m, 2.0005 m

$603\ cm^2$ (3 sf) < $602.5\ cm^2$, $603.5\ cm^2$

0.00035 (2 sf, not 6 sf)

mass of earth 5.98×10^{24} kg (3 sf)

6.0×10^{24} kg (2 sf)

In calculations, in exams!

2sf $\frac{3.0}{11.0}$ 3sf $= 0.27272727\ldots$ with calculator

what should be the answer ?!

⇒ least number of sf !

⇒ 0.27 ✓

002

Chapter 2 - Motion Along a Straight Line

Basic physics of motion. Object moves along a single axis. 1D motion.

The world, and everything in it, moves. Even seemingly stationary things.

Classification
Comparison of motion } Kinematics — Kinema means movement

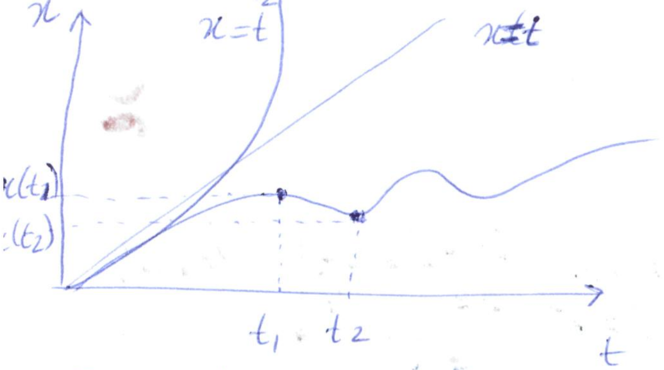
Restrictions in ~~1D~~ motion (only for this chapter): • straight line motion, 1D
• no forces
• point-like objects, particles

we will discuss only the motion itself and changes in the motion. not what causes to these changes.

Mathematical description of motion:
• position
• displacement (Δx)
• time interval (Δt)
• velocity; ~~absolute~~ value: speed
• acceleration

coordinate system is used to position of a particle in space } Relative to some reference point. Origin. Then, we have positive and negative directions

position is a function of time; $x(t)$



position

Δx, Displacement

The change of position of particle from $\underbrace{x(t_1)}_{x_1}$ to $\underbrace{x(t_2)}_{x_2}$

• $\Delta x = x(t_2) - x(t_1)$

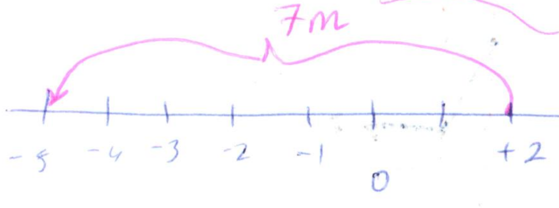
• Displacement is a vector quantity, means that it has both direction and quantity.

Example: $x_1 = +2m$
$x_2 = -5m$

$\rightarrow \Delta x = -5 - 2 = -7\,m$

magnitude of displacement

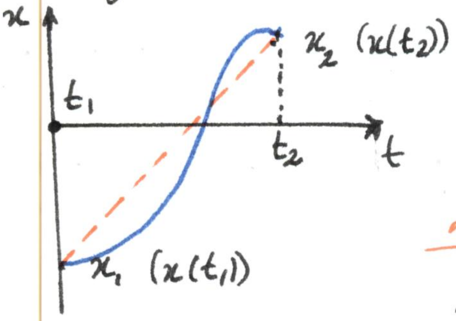
indicates the direction of motion



Average Velocity and Instantenous Velocity

Average velocity : v_{avg}

• The question is "how fast" an object is moving. is the "average velocity" correct answer? (or "average speed")



$$\boxed{v_{avg} = \frac{\overbrace{x_2 - x_1}^{\Delta x}}{\underbrace{t_2 - t_1}_{\Delta t}}}$$

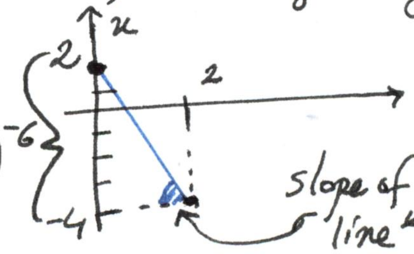
it is simply the slope of the "straight line" connecting two position points.

Δt Also, vector quantity < direction, magnitude

Example

$x_1 = 2m$, $(t_1 = 0)$

$x_2 = -4\,m$, $(t_2 = 2\text{ sec})$



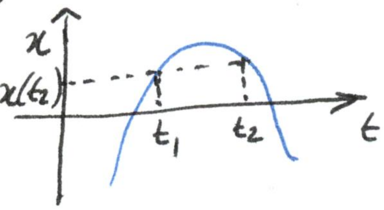
$$v_{avg} = \frac{-4-2}{2-0} = \ominus 3 \text{ m/s}$$

Average speed :

it is also used to describe how fast an object is moving.

$$S_{avg} = \frac{\text{total distance}}{\Delta t}$$ < only magnitude

Sometimes no information is obtained by using average velocity $x(t_1) = x(t_2)$



$\Rightarrow v_{avg} = 0$!!

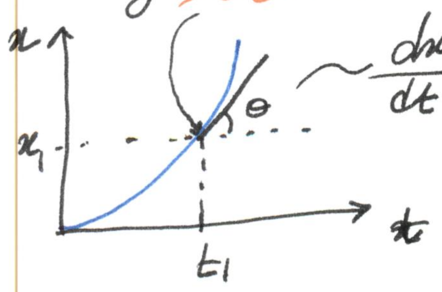
See Lecture notes

Instantenous velocity : v

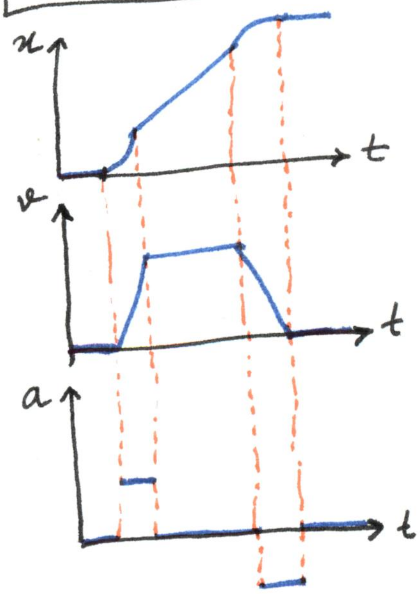
The velocity at any instant is obtained from the average velocity by shrinking the time interval, Δt close and closer to 0.

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$ slope!

That is the slope of $x(t)$ curve at any instant



$$\frac{dx}{dt} = \tan\theta$$

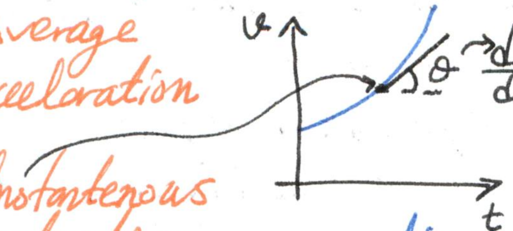


Average Acceleration and Instantenous Acceleration

when a particle's velocity changes, the particle is said to undergo **acceleration**

$$\boxed{a_{avg} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}}$$: Average acceleration

$$\boxed{a = \frac{dv}{dt} = \frac{d^2x}{dt^2}}$$: Instantenous acceleration OR acceleration (vector quantity)



$\frac{dv}{dt} = \tan\theta$

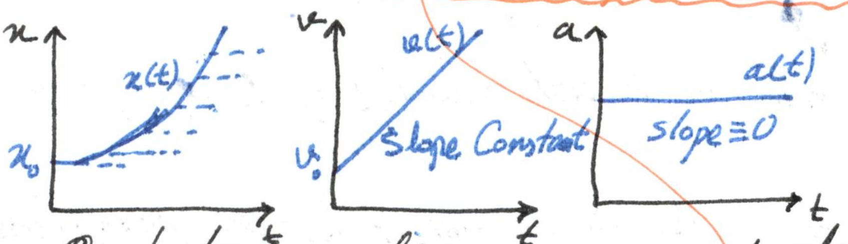
acceleration: direction, magnitude

{the sign of acceleration indicates a direction

not whether object's velocity is increasing or decreasing

See lecture notes

Constant Acceleration: A special Case



Quadratic (i.e. $x(t) \propto t^2$)   linear (i.e $v(t) \sim t$)   constant

In many types of motion, the acceleration is either constant or approximately constant.

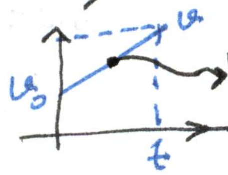
$\Rightarrow$ Average & Instantenous accelerations are equal

Basic Equations

$$a = a_{avg} = \frac{v - v_0}{t - \cancel{t_0}} = \frac{v - v_0}{t} \Rightarrow \boxed{v = v_0 + at} \quad (1)$$

$$\boxed{x - x_0 = v_0 t + \frac{1}{2}at^2} \quad (2)$$

$$v_{avg} = \frac{x - x_0}{t - \cancel{t_0}} = \frac{x - x_0}{t} \Rightarrow x - x_0 = v_{avg}\, t$$



$$v_{avg} = \frac{v_0 + v}{2}$$

$$x - x_0 = \left(\frac{v_0 + v}{2}\right)t$$

$$x - x_0 = \left(\frac{v_0 + v_0 + at}{2}\right)t$$

Five quantities

$x - x_0$, v, t, a, v_0

(1) (2) (3) (4) (5) missing in eqns

Derived Equations

eliminate t in (1) & (2) $\rightarrow$ $v^2 = v_0^2 + 2a(x - x_0)$ (3)

eliminate a in (1) & (2) $\rightarrow$ $x - x_0 = \frac{1}{2}(v_0 + v)t$ (4)

eliminate v_0 in (1) & (2) $\rightarrow$ $x - x_0 = vt - \frac{1}{2}at^2$ (5)

Basic & Derived Equations $\Rightarrow$ Equations for motion with constant acceleration.

Another look at Constant Acceleration:

See lecture notes

$$a = \frac{dv}{dt} \rightarrow \int dv = \int a\,dt \rightarrow v = at + c \left\{\begin{matrix} t=0 \\ c = v_0 \end{matrix}\right\} v = v_0 + at$$

$$v = \frac{dx}{dt} \rightarrow \int dx = \int v\,dt = \int (v_0 + at)\,dt = v_0 t + \frac{1}{2}at^2 + c \left\{\begin{matrix} t=0 \\ c = x_0 \end{matrix}\right\} x - x_0 = v_0 t + \frac{1}{2}at^2$$

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Free Fall Acceleration

If an object is thrown and release from a height, the object accelerates downward at a certain constant rate. That rate is called *free fall acceleration*.

- magnitude represented by g
- same for all objects (independent of object's characteristics)
- value of g varies slightly with latitude and with elevation

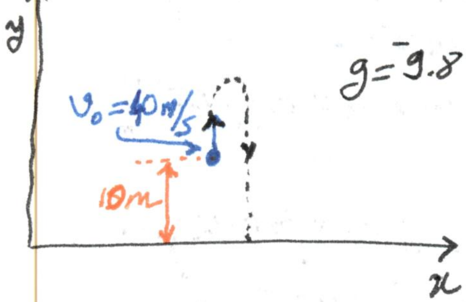
- $a = -g = -9.8\ m/s^2$ Free fall acceleration is negative
  $a \Rightarrow -g$
- motion is along y-axis
  $x - x_0 \Rightarrow y - y_0$

$$\left.\begin{aligned} v &= v_0 - gt \\ y - y_0 &= v_0 t - \tfrac{1}{2}gt^2 \\ v^2 &= v_0^2 - 2g(y - y_0) \\ y - y_0 &= \tfrac{1}{2}(v + v_0)t \\ y - y_0 &= vt + \tfrac{1}{2}gt^2 \end{aligned}\right\}$$ The eqns of motion for constant acceleration apply!

Example: A ball is thrown with an initial velocity of 40 m/s from a height of 10 m.

i) Determine the maximum height of the ball

ii) When will it hit the ground? And also determine the velocity of ball at the time of hit?



$g = -9.8\ m/s$

maximum height $\Rightarrow v = 0$

$v = v_0 - gt \rightarrow 0 = 40\ m/s - 9.8\ m/s^2\ t$

$\Rightarrow \boxed{t = 4.08\ sec}$

$y - y_0 = v_0 t - \frac{1}{2}gt^2$

$y - 10m = 40\ m/s(4.08s) - \frac{1}{2}\,9.8\ m/s^2 (4.08s)^2$

$\Rightarrow \boxed{y = \cancel{80.1\ m}\ 91.63\ m}$

t	0	4.08	8.16	8.40
v	40 m/s	0	40 m/s	42.38 m/s

$y - y_0 = v_0 t - \frac{1}{2}gt^2$

$0 - 10m = (40\ m/s)t - \frac{1}{2}(9.8\ m/s^2)t^2$

$4.9t^2 - 40t - 10 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{40 \pm \sqrt{1600 - 4(4.9)(-10)}}{2 \times (4.9)}$

~~3.05 sec~~

$\boxed{t = 8.40\ sec}$

-0.24 sec not physical

$v^2 = v_0^2 - 2g(y - y_0)$

$v^2 = (40\ m/s)^2 - 2(9.8\ m/s^2)(0 - 10m)$

$\boxed{v = 42.38\ m/s}$

Chapter 3 – Vectors

* Language of vectors to describe physical quantities
* Vectors follow certain rules of combination
* Motion along a straight line: ∓ sign is enough to indicate the direction
* Motion in three dimensions: ∓ sign is not enough to indicate the direction of motion ⟹ $\boxed{\text{use vectors}}$

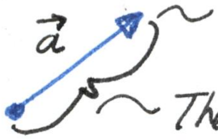
{ position, displacement, velocity, acceleration } All defined by means of vectors

{ time, speed, temperature } Quantities which only indicate magnitude

vector
· magnitude
· direction

scalar
· magnitude

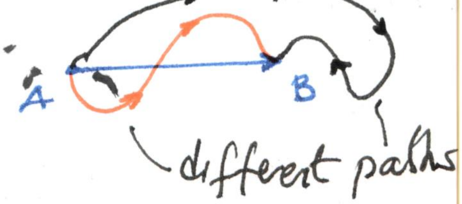
Vectors: shown by arrows



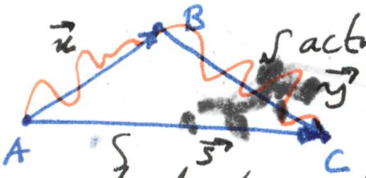
~ The head of arrow signifies direction.

~ The length of arrow signifies magnitude, $|\vec{a}|$ or a

Displacement vector: Change of position. Does not tell us the actual path that particle takes.



Adding Vectors Geometrically



$$\vec{s} = \vec{x} + \vec{y}$$

net displacement is the $\overset{\text{resultant}}{\text{vector sum}}$. not the usual algebraic sum.

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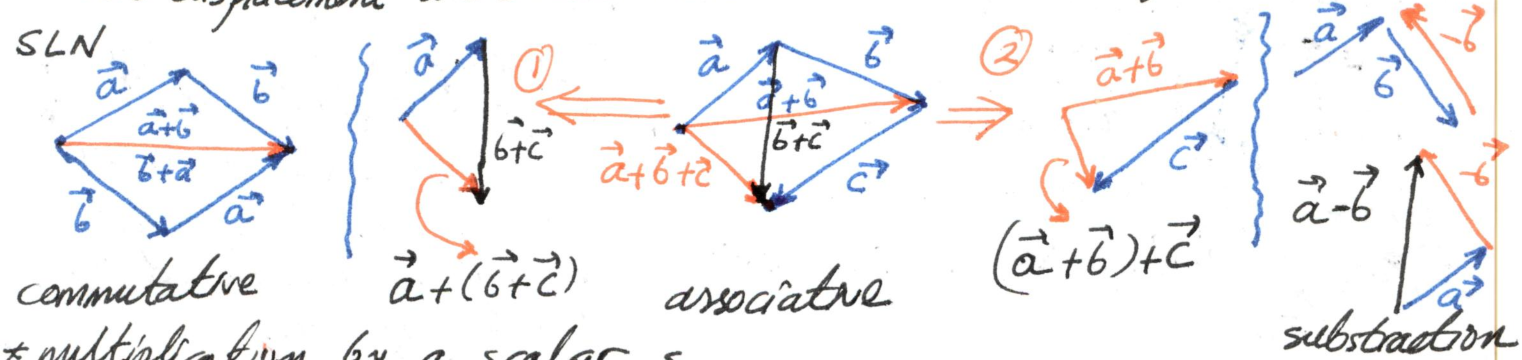
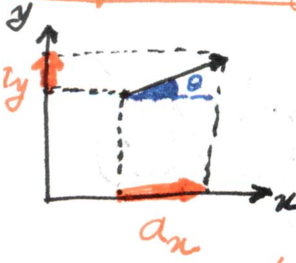


* multiplication by a scalar, s

$s(\vec{a}+\vec{b}) = s\vec{a} + s\vec{b}$ Distrubutive low

Component of vectors



The component of a vector along an axis is the projection of the vector onto that axis.

$a_x = |\vec{a}| \cos\theta$

$a_y = |\vec{a}| \sin\theta$

a_x, a_y : scalar quantity

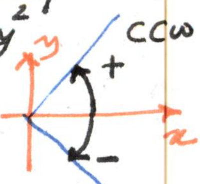
Construction of a vector from its components

Suppose that components are a_x & a_y

Magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

Direction (angle) $\tan\theta = \dfrac{a_y}{a_x}$

$\theta = \tan^{-1} \dfrac{a_y}{a_x}$



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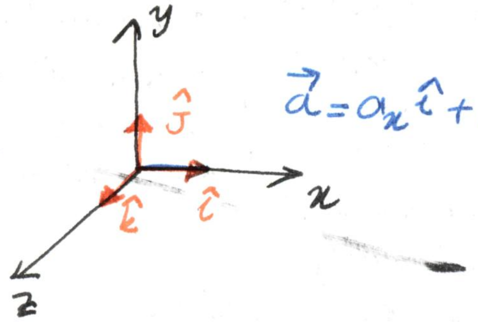
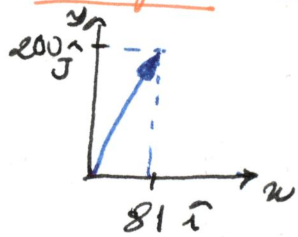
Unit Vectors

A unit vector is a vector that has a magnitude of exactly 1 and points in a particular direction.



$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Example: $\vec{a} = 81\hat{i} + 200\hat{j}$



$|\vec{a}| = \sqrt{81^2 + 200^2} \cong 215$

$\tan\theta = \frac{a_y}{a_x} = \frac{200}{81}$

$\Rightarrow \theta \sim 68°$

Adding vectors by components

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$
$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ $\Big\}$ $\vec{r} = \vec{a} + \vec{b}$ $\Big\}$ $\vec{r} = \underset{r_x}{(a_x+b_x)}\hat{i} + \underset{r_y}{(a_y+b_y)}\hat{j} + \underset{r_z}{(a_z+b_z)}\hat{k}$

Example:

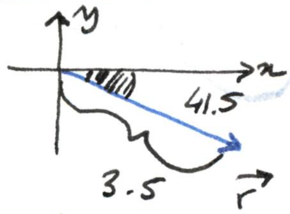
$\vec{a} = 4.2\hat{i} - 1.5\hat{j}$
$\vec{b} = -1.6\hat{i} + 2.9\hat{j}$
$c = -3.7\hat{j}$

$\vec{r} = \vec{a} + \vec{b} + \vec{c}$
magnitude & angle

$\vec{r} = (4.2 - 1.6)\hat{i} + (-1.5 + 2.9 - 3.7)\hat{j} + (0+0+0)\hat{k}$

$\boxed{\vec{r} = 2.6\hat{i} + (-2.3)\hat{j}}$

$|\vec{r}| = \sqrt{(2.6)^2 + (-2.3)^2}$
$\cong 3.5$



$\theta = \tan^{-1}\frac{-2.3}{2.6} = -41.5°$

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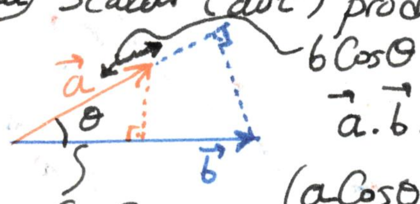
Multiplying Vectors

① Multiplying or dividing vector by a scalar. $\vec{a}(s) = s\vec{a}$

$\vec{a} = 3\hat{i} + 5\hat{j}$
$2\vec{a} = 6\hat{i} + 10\hat{j}$

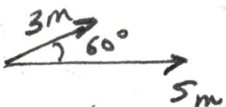
② " " " " vector

(2a) Scalar (dot) product



$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$(a\cos\theta)b = a(b\cos\theta)$



$\vec{a}.\vec{b} = (3(\cos 60))5 = 3(5\cos 60°)$

properties

$\vec{a}.\vec{b} = \vec{b}.\vec{a}$
$c\vec{a}.\vec{b} = c(\vec{a}.\vec{b})$
$(\vec{a}+\vec{b}).\vec{c} = \vec{a}.\vec{c} + \vec{b}.\vec{c}$

in cartesian coor.

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
$\hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = 0$

$\hat{i}.\hat{i} = 1\,1\cos 0$
$\hat{i}.\hat{j} = 1\,1\cos 90°$

Example: $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

$\vec{a}.\vec{b} = a_xb_x + a_yb_y + a_zb_z$

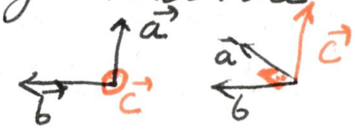
produces a scalar

(2b) Vector (cross) product

Produces a new vector

$|\vec{a}\times\vec{b}| = \underbrace{|\vec{a}||\vec{b}|\sin\theta}_{\text{magnitude of vector}}$, $\vec{a}\times\vec{b} = \vec{c}$

* The direction of third vector is determined by right hand rule



properties

$\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$
$c(\vec{a}\times\vec{b}) = c\vec{a}\times\vec{b} = \vec{a}\times c\vec{b}$
$(\vec{a}+\vec{b})\times\vec{c} = (\vec{a}\times\vec{c}) + (\vec{b}\times\vec{c})$

$\hat{i}\times\hat{j} = \hat{k}$ | $\hat{i}\times\hat{i} = 0$
$\hat{j}\times\hat{k} = \hat{i}$ | $\hat{j}\times\hat{j} = 0$
$\hat{k}\times\hat{i} = \hat{j}$ | $\hat{k}\times\hat{k} = 0$

$i \rightarrow j \rightarrow k$

otherwise minus

(26) Components of cross product

$$\left.\begin{aligned}\vec{a} &= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\\ \vec{b} &= b_x\hat{i} + b_y\hat{j} + b_z\hat{k}\end{aligned}\right\}\ \vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} \sim \text{determinant}$$

$$\begin{aligned}\vec{a}\times\vec{b} = &\left(\cancel{a_x b_x\, \hat{i}\times\hat{i}}^{\,0} + a_x b_y\, \hat{i}\times\hat{j} + a_x b_z\, \hat{i}\times\hat{k}\right) +\\ &\left(a_y b_x\, \hat{j}\times\hat{i} + \cancel{a_y b_y\, \hat{j}\times\hat{j}}^{\,0} + a_y b_z\, \hat{j}\times\hat{k}\right) +\\ &\left(a_z b_x\, \hat{k}\times\hat{i} + a_z b_y\, \hat{k}\times\hat{j} + \cancel{a_z b_z\, \hat{k}\times\hat{k}}^{\,0}\right)\end{aligned}$$

$i \rightarrow j \rightarrow k$

otherwise minus sign

$\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{i} = -\hat{k}$

$$= a_x b_y\,\hat{k} + a_x b_z(-\hat{j}) + a_y b_x(-\hat{k}) + a_y b_z\,\hat{i} + a_z b_x\,\hat{j} + a_z b_y(-\hat{i})$$

$$\boxed{\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\,\hat{i} + (a_z b_x - a_x b_z)\,\hat{j} + (a_x b_y - a_y b_x)\,\hat{k}}$$

Components

Chapter 4 - Motion in Two and Three Dimensions

Now, the motion can be in two or three dimensions. Starting point is revisiting position and displacement in 2 & 3D.

Locating a particle-like object : position vector, $\vec{r}$ < from origin to particle

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ — x, y, z ; scalar components — $x\hat{i}, y\hat{j}, z\hat{k}$; vector components

Motion ⇒ position vector changes

⇓

particle's displacement, $\Delta\vec{r}$ $\left\{ \Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \underbrace{(x_2-x_1)}_{\Delta x}\hat{i} + \underbrace{(y_2-y_1)}_{\Delta y}\hat{j} + \underbrace{(z_2-z_1)}_{\Delta z}\hat{k} \right.$

SLN Example

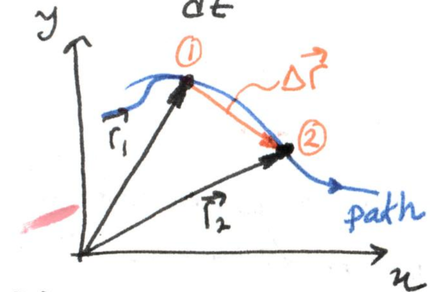
Average Velocity and Instantenous Velocity:

How fast? now, in vector notation

$$v_{avg} = \frac{\Delta x}{\Delta t} \xrightarrow{\text{now becomes}} \vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

shrink Δt to zero

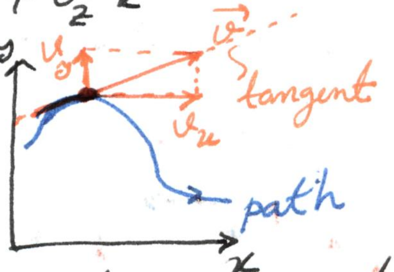
$$v = \frac{dx}{dt} \longrightarrow \vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$



$\vec{r}_1$: position vector at time t_1

2: " " " " t_2

$\Delta\vec{r}$: displacement vector



$\vec{r}$: extends from one point to another point.

$\vec{v}$: shows instantenous direction of travel

The direction of the instantenous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

SLN Example

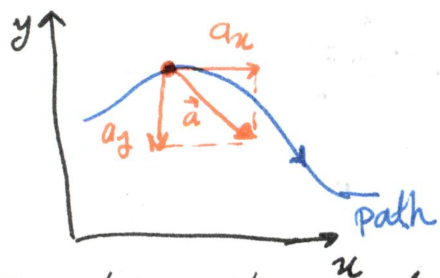
Average Acceleration and Instantenous Acceleration:

$$a_{avg} = \frac{\Delta v}{\Delta t} \xrightarrow{\text{now becomes}} \vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

shrink Δt to zero

$$a = \frac{dv}{dt} \longrightarrow \vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

If velocity changes in either magnitude or direction (or both), the particle must have an acceleration.



$\vec{a}$: shows the direction of acceleration

SLN. Example

at $t = 15$

$x = -0.31t^2 + 7.2t + 28$ | $\vec{r} = (66\,m)\hat{i} - (57\,m)\hat{j}$

$y = 0.22t^2 - 9.1t + 30$ | $|\vec{r}| = 87\,m \quad \theta = -41°$

$v_x = \frac{dx}{dt} = -0.62t + 7.2$ | $\vec{v} = (-2.1\,m/s)\hat{i} + (-2.5\,m/s)\hat{j}$

$v_y = \frac{dy}{dt} = 0.44t - 9.1$ | $|\vec{v}| = 3.3\,m/s \quad \theta = -130°$

$a_x = \frac{dv_x}{dt} = -0.62\,m/s^2 \quad a_y = 0.44$ | $\vec{a} = (-0.62\,m/s^2)\hat{i} + (0.44\,m/s^2)\hat{j}$

$|\vec{a}| = 0.76\,m/s^2 \quad \theta = -35°$

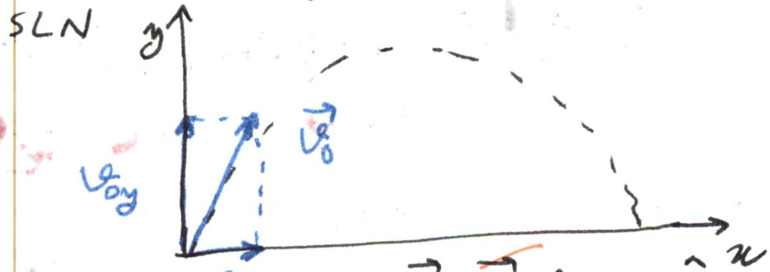
Projectile Motion

A special case of 2D motion:

- A particle moves in a vertical plane with some initial velocity $\vec{v}_0$
- Its acceleration is always the free fall acceleration $\vec{g}$, which is downward
- Assumption: Air has no effect on the projectile

Such a particle is called a projectile. Its motion is called projectile motion.

SLN



$$\vec{v}_0 = \vec{v}_{ox}\hat{\imath} + v_{oy}\hat{\jmath}$$

During motion

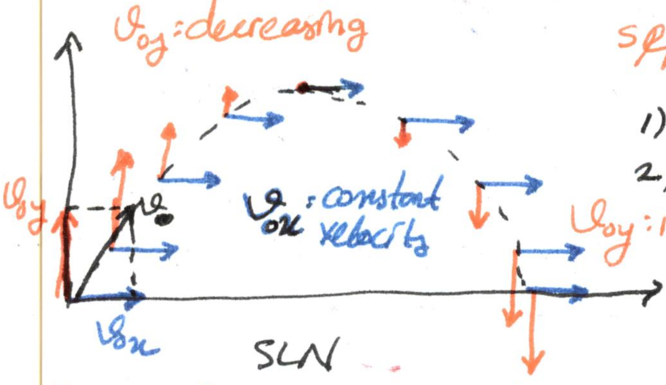
- the projectile's position vector $\vec{r}$ and velocity vector $\vec{v}$ change continuously
- But, its acceleration vector $\vec{a}$ is constant and always directed downward. No horizontal acceleration

$\Rightarrow$ Horizontal Motion, Vertical Motion $\}$ are independent. SLN

$\Downarrow$

Break up 2D motion problem into two separate and easier 1D motion problems.

1) Horizontal motion (zero acceleration)
2) Vertical motion (constant downward ")



$a_x = 0$, $a_y = -g$

$\Downarrow$ constant velocity

$\Downarrow$ increasing / decreasing velocity

Projectile Motion Analyzed

The Horizontal Motion:

$v_x = v_{ox}$ $\}$ • no acceleration • velocity remains unchanged

$$x - x_0 = v_{ox}t + \cancel{at}^{\,0}$$

$$\boxed{x - x_0 = (v_0 \cos\theta)t} \quad (1)$$

t	0	t	$2t$	...
v_x	v_{ox}	v_{ox}	v_{ox}	...

The Vertical Motion:

is the free fall motion that we have discussed before. $a \to -g$

$$y - y_0 = v_{oy}t - \frac{1}{2}gt^2$$

$$\boxed{y - y_0 = (v_0 \sin\theta)t - \frac{1}{2}gt^2} \quad (2)$$

$$\boxed{v_y = v_{oy} - gt = v_0 \sin\theta - gt}$$

$$\boxed{v_y^2 = (v_0 \sin\theta)^2 - 2g(y - y_0)}$$

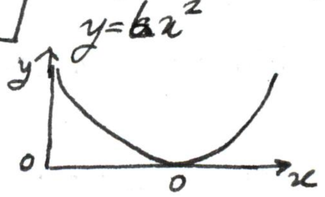
t	0	t ?	$2t$	...
v_y	v_{oy}	0	v_{oy} ?	...

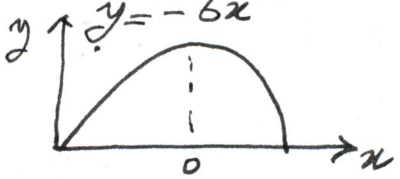
The Equation of the Path:

by using ① & ②; eliminating t in the eqns. (also $x_0 = y_0 = 0$)

$$\boxed{y = (\tan\theta)x - \frac{gx^2}{2(v_0\cos\theta)^2}}$$: Equation of path, trajectory

$y = ax - bx^2$
parabolic eqn





The Horizontal Range:

horizontal distance the projectile has travelled when it returns the initial height } The horizontal range, R

$\Rightarrow x - x_0 = R$
$y - y_0 = 0$ by using ① & ② again

$\Rightarrow R = (v_0\cos\theta)t$
$0 = (v_0\sin\theta)t - \frac{1}{2}gt^2$ } eliminating t in the eqns { $R = (v_0\cos\theta)t$, $(v_0\sin\theta)\cancel{t} = \frac{1}{2}gt^{\cancel{2}}$

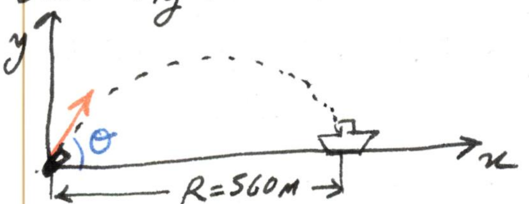
$$R = (v_0\cos\theta)\frac{2(v_0\sin\theta)}{g} = \frac{v_0^2}{g}\underbrace{2\sin\theta\cos\theta}_{\sin 2\theta}$$

$$\boxed{R = \frac{v_0^2 \cancel{2}\sin 2\theta}{g}}$$ < final height = initial height; maximum at launch angle of 45° $\left(\sin 2\times45° = \sin 90° = 1\right)$ R_{max}

SLN

Example: Cannonball to pirate ship i) at what angle?

SLN Fig 4-15



$v_0 = 82\,m/s$
$x - x_0 = R = 560m$ { $R = \frac{v_0^2}{g}\sin 2\theta$ | $\theta = \frac{1}{2}\sin^{-1}\frac{gR}{v_0^2}$

$\theta = \frac{1}{2}\sin^{-1}\frac{(9.8\,m/s^2)(560m)}{(82\,m/s)^2}$

$\theta = \frac{1}{2}\sin^{-1} 0.816$

$\theta = 27.3\cancel{42}°$ & $\theta = 62.7°$

since $\sin(90-\theta) = \sin\theta$

ii) what is the maximum range?

$R_{max} = \frac{v_0^2}{g}\sin 90° = 686\,m$

Example Projectile dropped from airplane

SLN Fig 4-14

$v_0 = v_{ox} = 198\,km/h = 55\,m/s$; $v_{oy} = 0$

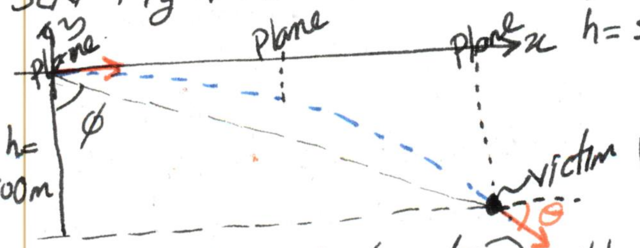
$h = 500m$ ii) what is the distance btw plane & victim?

$y - y_0 = v_{oy}t - \frac{1}{2}gt^2 \rightarrow -500 - 0 = \cancel{55\,m/s}\,t - 4.9\,m/s^2$ (0, v_{oy})

$\Rightarrow t = 10.1s \Rightarrow x - x_0 = v_{ox}t \rightarrow x - 0 = 55\,m/s\,(10.1s)$

$x = 555.5\,m$

i) when the object hits the victim!



Dropped: Plane & object moves together at x-direction } with same velocity $v_{x} = v_{ox}$

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iii) what is the sight angle, ϕ?

$\tan\phi = \frac{\cancel{y}\,x}{\cancel{x}\,y} = \frac{555.5}{500} \Rightarrow \phi = \tan^{-1}\frac{555.5}{500} = 48°$

if your calculator works in radian, $\tan^{-1}\frac{555.5}{500} = 0.8379$

$\Rightarrow \frac{0.8379 \times 180}{3.14} = 48°$

iv) what is the velocity of the object at arrival?

$v_x = v_{0x}$
$v_y = v_{0y} - gt$

$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$

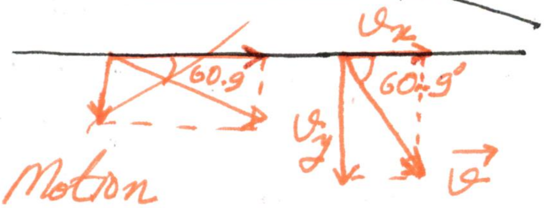
$v_x = 55\ m/s$

$v_y = \underbrace{v_0 \sin\theta°}_{0} - 9.8(10.1\ \text{sec}) = -99.0\ m/s^2$

$\boxed{\vec{v} = 55\ m/s\ \hat{\imath} + (99.0\ m/s)(-\hat{\jmath})}$

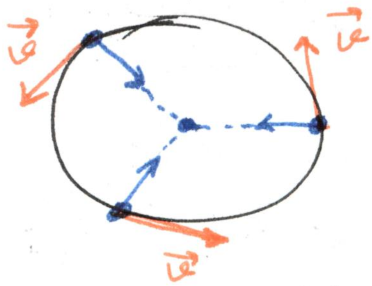
$|\vec{v}| = 113\ m/s$ $\quad \sqrt{55^2 + 99^2}$

$\theta = \tan^{-1}\frac{-90}{55} = -60.9°$



Uniform Circular Motion

A particle is in uniform circular motion if it travels ~~around a~~ circle or a circular arc at constant (uniform) speed.
Although the speed does not vary, the particle is accelerating because the velocity changes its direction. SLN



$\vec{v}$: tangential
$\vec{a}$: towards the center. (radially inward)

$a = \underbrace{\frac{v^2}{r}}_{\text{magnitude.}}$: centripetal acceleration (center-seeking)

direction

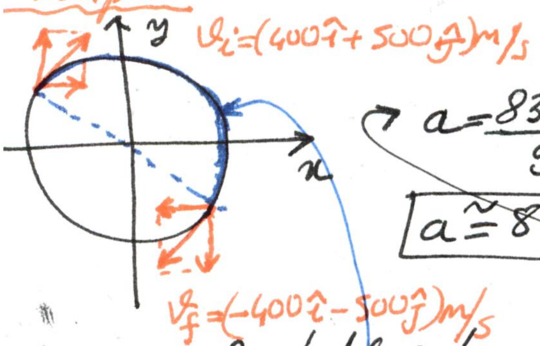
v: speed of the particle

r: radius of the circle

$T = \frac{2\pi r}{v}$: Period of Revolution (OR Period)

SLN proof of $a = \frac{v^2}{r}$!

Example



$a = \frac{83.81\ m/s^2}{9.8\ m/s^2}$

$\boxed{a \cong 8.6g}$

$t = 24\ s$ for half circle

$\Rightarrow T = 48\ s$: period

$a = \frac{v^2}{r}$, $T = \frac{2\pi r}{v}$ $\Big\}$ $\frac{v^2}{a} = \frac{2\pi}{T} \Rightarrow \frac{vT}{2\pi}$ $\Big|$ $a = \frac{2\pi v}{T}$?

$v = \sqrt{(400\ m/s)^2 + (500\ m/s)^2}$
$= 640.31\ m/s$

$\Rightarrow a = \frac{2\pi(640.31\ m/s)}{48\ s} = 83.81\ m/s^2$

Relative Motion in 1D

The velocity of a particle depends on the reference frame of whoever is observing or measuring the velocity.

reference frame — e.g. ground
— stationary or moving

* Observers on different frames of reference that move at constant velocity relative to each other will measure the same accelleration for a moving particle. $\boxed{a_{PA} = a_{PB}}$

SLN Fig. 4-18

Chapter 5 - Force and Motion I

Study of motion: including acceleration (changes in velocities)

Study of what can cause an object to accelerate.

⇓ Force (said to act on the object to change its velocity)

Force, Acceleration } A relation btw them ⇒ Newtonian Mechanics ⇒ Three laws of motion

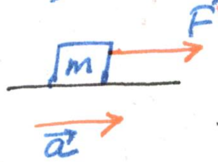
If the interacting bodies; SLN

- speeds are very large ⇒ Einstein's Special Theory of Relativity
- scale of atomic structure ⇒ Quantum Mechanics

} Newtonian mechanics special case of

Force } (Gravitational force, Weight, Normal Force, Friction ...)

Unit of force : in terms of acceleration $\left\{ \begin{array}{l} \text{mass}: 1\,kg \\ \text{acceleration}: 1\,m/s^2 \end{array} \right.$

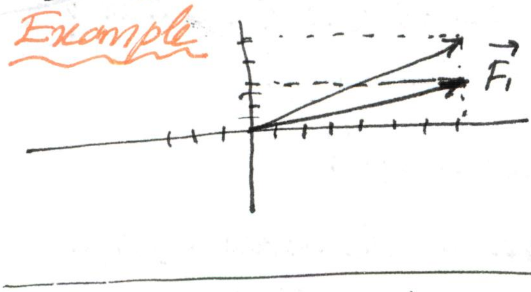


} in Newton (SI) $1\,N \equiv 1\,kg\,m/s^2$

Magnitude of force : mass × magnitude of acceleration
Direction of force : same as the acceleration's direction } Force is a vector quantity

Superposition Principle : A single force that has the magnitude and direction of the net force has the same effect on the body as all the individual forces together. ($\vec{F}_{net}$)

Example



$\vec{F}_1 = 70N\,\hat{i} + 20N\,\hat{j}$
$\vec{F}_2 = -30N\,\hat{i} + 40N\,\hat{j}$

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = (F_{1x}+F_{2x})\hat{i} + (F_{1y}+F_{2y})\hat{j}$

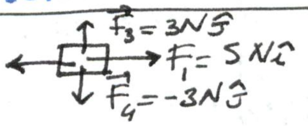
$\boxed{\vec{F}_{net} = 40\,\hat{i} + 60\,\hat{j}}$ vector

Magnitude: $|\vec{F}| = F = \sqrt{40^2+60^2} = 72\,N$

Direction: $\tan\theta = \frac{F_y}{F_x} = \frac{60}{40} \Rightarrow \boxed{\theta = 63°}$ Angle made by positive x-axis

> If no net force acts on a body ($\vec{F}_{net} = 0$), the body's velocity can not change; that is, the body can not accelerate. Newton's 1st law SLN

$\vec{F}_{net} = 0 \Rightarrow \vec{v} = \text{constant}$ Example



$\vec{F}_{net} = (\sum_i F_{x_i})\hat{i} + (\sum_i F_{y_i})\hat{j}$
$\vec{F}_{net} = 0$

Inertial Reference Frames : SLN

Mass Proportionality constant: $F \propto a$ Measure how hard to change motion. (acceleration)
Intrinsic characteristics of a body. Moment of inertia!!

> The net force on a body is equal to the product of the body's mass and acceleration. Newton's 2nd law $\vec{F}_{net} = m\vec{a}$ SLN

$\vec{F}_{net} = m\vec{a} = m(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) = \underbrace{ma_x}_{F_{net,x}}\hat{i} + \underbrace{ma_y}_{F_{net,y}}\hat{j} + \underbrace{ma_z}_{F_{net,z}}\hat{k}$

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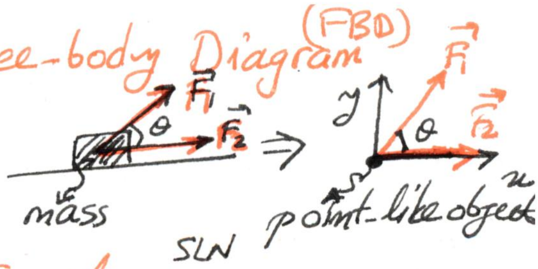
To Solve Problems with Newton's 2nd law

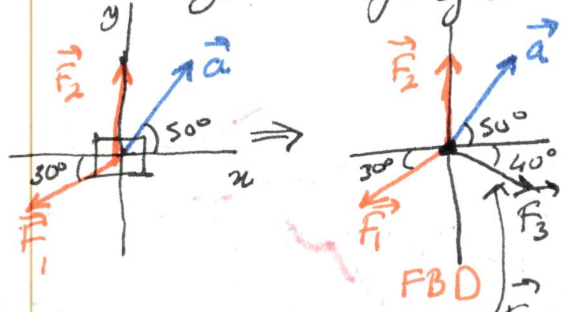
- Treat each object as a point like object
- Identify all forces on body and draw free-body diagram (only external forces, do not include internal forces)
- Apply Newton's 2nd law
- Find relevant constraint eqns
- Solve system of eqns

Free-body Diagram (FBD)



Example

A 2 kg object is accelerated at 3 m/s² on a frictionless table by three forces. $\vec{F}_1$ and $\vec{F}_2$ vectors are shown. ($F_1 = 10\,N$, $F_2 = 20\,N$). Find third force vector, $\vec{F}_3$ in unit vector notation, magnitude and its angle with respect to +x-axis



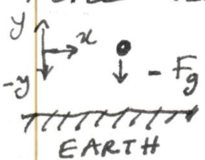
$$\vec{F}_{net} = m\vec{a}$$
$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a}$$

$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2 = F_{3,x}\hat{i} + F_{3,y}\hat{j} + F_{3,z}\hat{k}$$
$$\Rightarrow F_{3,x} = ma_x - F_{1,x} - F_{2,x}$$
$$= m|\vec{a}|\cos 50^\circ - F_1 \cos 210 - F_2 \cancel{\cos 90^\circ}_{0}$$
$$\boxed{F_{3,x} = 12.5\,N}$$
$$F_{3,y} = ma_y - F_{1,y} - F_{2,y}$$
$$= 2(3 \times \sin 50^\circ) - 10\sin 210 - 20 \cancel{\sin 90}_{1}$$
$$\boxed{F_{3,y} = 10.4\,N}$$

$$\vec{F}_3 = 12.5\,N\,\hat{i} - 10.4\,N\,\hat{j}$$
$$|\vec{F}_3| = F_3 = \sqrt{12.5^2 + 10.4^2} \simeq 16.3\,N$$
$$\tan\theta = \frac{F_{3,y}}{F_{3,x}} \Rightarrow \theta = \tan^{-1}\frac{-10.4}{12.5} = -40^\circ$$

Some Particular Forces

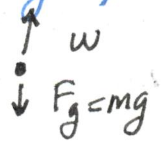
① **Gravitational Force, $\vec{F}_g$**: everytime present even the object is stationary. Force that pulls the object toward the center of Earth.



$$\boxed{\vec{F}_g = -F_g\hat{j} = -mg\hat{j}}$$

magnitude: mg
direction: always downward!

② **Weight, w**: The weight of a body is the magnitude of the upward force needed to balance gravitational force on the body.

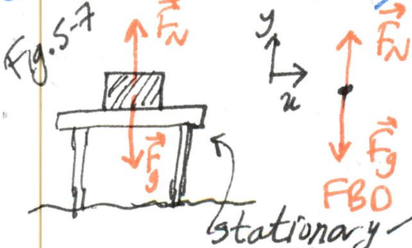


$$\boxed{w = F_g = mg}$$

mass: $m_{moon} = m_{earth}$
weight: $w_{moon} = \frac{1}{6} w_{earth}$ (Since $g_{moon} = \frac{1}{6} g_{earth}$)

③ **The Normal Force, $\vec{F}_N$**: The force on a body from a surface against which the body presses. Always perpendicular to the surface

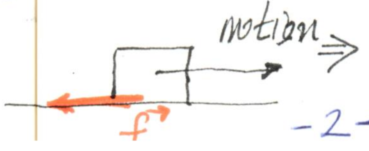
Fig. 5-7



$$F_N - F_g = ma_y \quad \Big\} \quad F_N = ma_y + mg = m(a_y + g) \quad \Big\{ \quad \boxed{F_N = mg}$$
$$\boxed{a_y = 0}$$

SLN

④ **Friction, $\vec{f}$**: if we attempt to slide a body over a surface, the motion is resisted by a bonding btw the body and the surface.



Resistance: frictional force, $\vec{f}$
Always parallel to surface. opposite in motion direction

⑤ **Tension, $\vec{T}$**: when a cord (cable, rope, ..) which is attached to a body is pulled, the cord pulls on the body with a force $\vec{T}$ directed away from the body.

Tension force

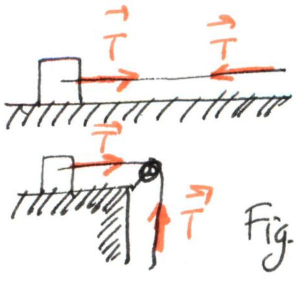
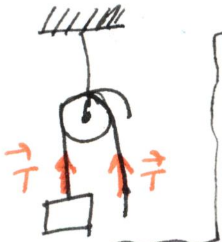


Fig. 5-9

The cord pulls on both bodies with the same magnitude T (even if the bodies and the cord are accelerating and even if the cord runs around a massless, frictionless pulley)

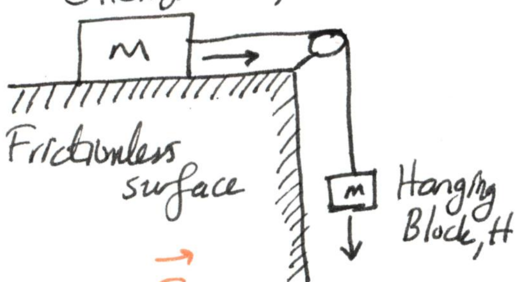


To every action there is always opposed an equal reaction. The mutual action of two bodies upon each other are always equal, and directed to the other.

⇒ when two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.
Newton's 3rd law   SLN Fig. 5-10

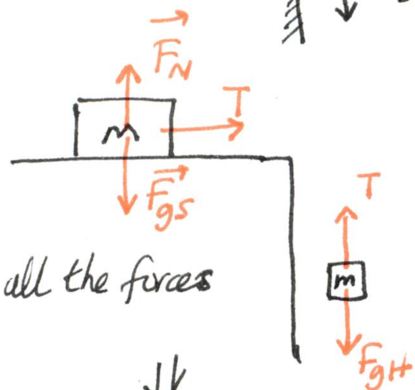
- Forces btw two interacting bodies are called Third Law Force Pair.
- Third law is also valid when the interacting objects are moving and accelerating

Example



$M = 3.3$ kg
$m = 2.1$ kg

Find
i) acceleration of block, S
ii) " " " , H
iii) the tension in the cord



M: $F_N - F_{gS} = Ma_y$, $T = Ma_x$

m: $T - F_{gH} = ma_y$

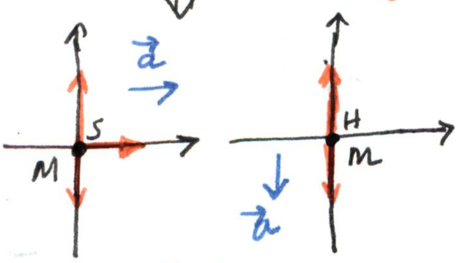
Simultaneous equations. Good one: a_x, a_y are common for all the masses.

① $F_N - Mg = 0$ ③ $T - mg = -ma$

② $T = Ma$

②&③ $Ma - mg = -ma$ $\left\{ \boxed{a = \frac{mg}{M+m} = 3.8\,m/s^2} \right.$ i, ii

$\boxed{T = \frac{Mm}{M+m} g = 13\,N}$ iii



FBDs

check
$2.1\,kg \times 9.8\,m/s^2 > 13\,N$
⇒ Downward motion ✓

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Chapter 6 – Force and Motion II

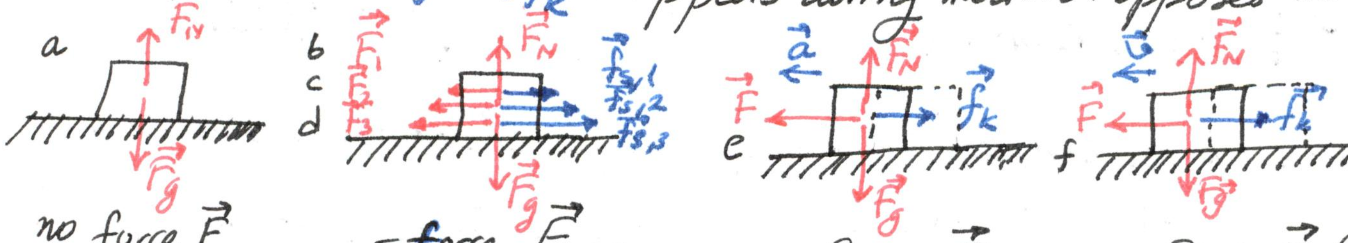
Three common types of force: Frictional, Drag, Centripetal

1) Friction

Two types of frictional force

i) Static frictional force, $\vec{f}_s$: exists when the body is stationary

ii) Kinetic frictional force, $\vec{f}_k$: appears during motion. opposes motion

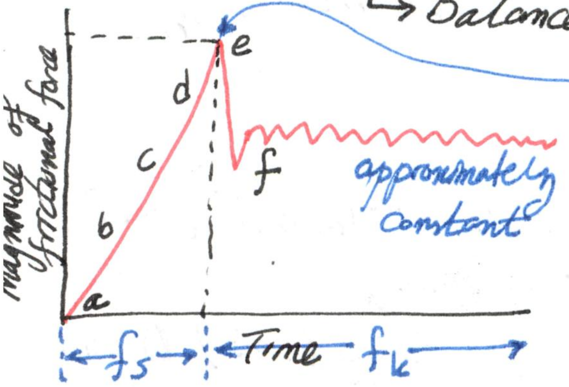


no force, $\vec{F}$
no motion
no friction, $\vec{f}_s = 0$

force, $\vec{F}_{1,2,3}$
no motion
frictional force, $\vec{f}_{s,1,2,3}$
→ Balanced

force, $\vec{F}$
motion starts
frictional force, $\vec{f}_k$ (weak)
→ $\vec{F} > \vec{f}_{s,max}$

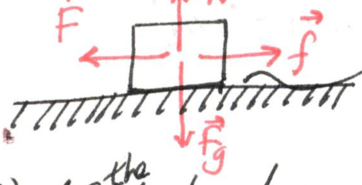
force, $\vec{F}$ (weakens)
speed is maintained
frictional force, $\vec{f}_k$
→ Balanced!



SCW Fig. 6-2

* When $|\vec{F}|$ reaches to $|\vec{f}_{s,max}|$, the block starts motion
* Generally $f_k < f_{s,max}$
* If we want the block to move at a constant speed, we should decrease the magnitude of force once the block begins to move.

Properties of friction



dry, unlubricated (nonslippery) body. Three properties

i) If the body does not move → $\vec{F} = -\vec{f}_s$ → $|\vec{F}| = |\vec{f}_s|$ magnitude same

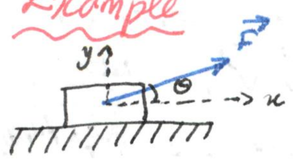
ii) Maximum value of $\vec{f}_s$: magnitude $\boxed{f_{s,max} = \mu_s F_N}$ μ_s: coefficient of static friction; F_N: magnitude of the normal force

If $F_{applied} > f_{s,max} \Rightarrow$ MOTION

iii) If body moves along the surface, then frictional force becomes $\vec{f}_k$

magnitude $\boxed{f_k = \mu_k F_N}$ μ_k: coefficient of kinetic friction

Example



$f_k = \mu_k F_N$

m = 3.0 kg
slides
$|\vec{F}| = 12\,N$
$\mu_k = 0.40$
$\theta = 0 \leftrightarrow 90°$
$a_{max}(\theta = ?)$

Newton's 2nd law

y-component
$F_N - F_g + F\sin\theta = m\cancel{a_y}^{0}$

x-component
$F_x - f_k = ma_x$

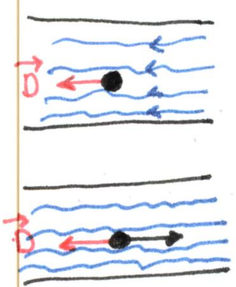
$F_N = -F\sin\theta + mg$

$a_x = \frac{F\cos\theta}{m} - \frac{\mu_k}{m}(-F\sin\theta + mg)$

$\frac{da_x}{d\theta} = 0 \rightarrow \tan\theta = \mu_k$ $\boxed{\theta = \sim 22°}$

2) The Drag Force and Terminal Speed

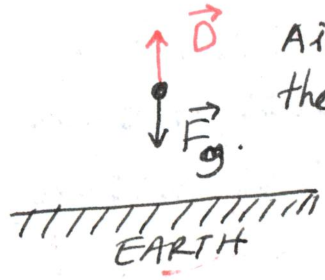
A fluid is anything (gas, liquid, ...) that can flow.



Fluid is flowing
Object is stationary

Fluid is stationary
Object is moving

In both cases, object experiences a drag force $\vec{D}$.



Air is the fluid.

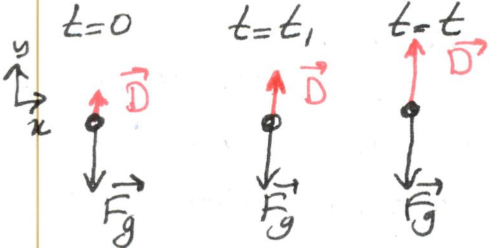
Drag force is related to the (relative) speed of the object in the fluid.

$$\boxed{D = \frac{1}{2} C \rho A v^2}$$

where C: drag coefficient (experimentally obtained)
ρ: density of fluid
A: effective cross-sectional ($A \perp v$) area. Area should be minimized $\Rightarrow$ less drag force
v = speed

- Terminal Speed

Consider a falling object in air



$D \propto v^2$
$\vec{D}$ opposes $\vec{F_g}$
- $D < F_g$?
- $D = F_g$?
- $D > F_g$?

Newton's 2nd law ($\vec{F}_{net} = m\vec{a}$)
$D - mg = ma_y$
At some time ($t=t$) D will be equal to $F_g \Rightarrow a_y = 0$!
- The body then falls at a constant speed, terminal speed

During falling as the speed of the object increases the magnitude of the drag force increases.

$$D - F_g = 0 \rightarrow \frac{1}{2} C \rho A v_t^2 = F_g$$

$$\Rightarrow \boxed{v_t = \sqrt{\frac{2F_g}{C\rho A}}}$$

Example Terminal Speed of Falling Raindrop

A raindrop with radius $R = 1.5$ mm
$h = 1200$ m SLN
$C = 0.60$
Assumption: The drop is spherical
$\rho_{water} = 1000$ kg/m^3, $\rho_{air} = 1.2$ kg/m^3

i) What is the terminal speed?

at balance
$a = 0$
$v = v_t$

$\vec{D} - \vec{F_g} = ma = 0$

$\frac{1}{2} C \rho_{air} A v_t^2 = |\vec{F_g}|$ — ?

$F_g = mg$ $\left\{ m = ? \quad \rho_{water} = \frac{m}{V} \right\}$ $m = \rho_{water} V$

$V = \frac{4}{3}\pi R^3$ & $A = \pi R^2$

$\frac{1}{2} C \rho_{air} (\pi R^2) v_t^2 = \rho_{water} \frac{4}{3}\pi R^3 g$

$\rightarrow v_t = \sqrt{\frac{8 R \rho_{water} g}{3 C \rho_{air}}} = \frac{8(1.5\times10^{-3} m)(1000 kg/m^3) 9.8 m/...}{3(0.60)(1.2 kg/m^3)}$

$\boxed{v_t = 7.4 m/... \sim 27 km/h}$

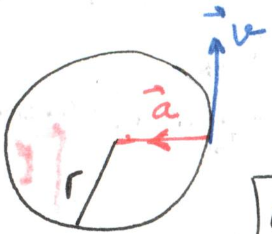
ii) No drag force! What is the speed?
$\Rightarrow$ free-fall with g acceleration

~~$y - y_0 (=h) = 0 - 1200m = v_{oy} t - \frac{1}{2} g t^2$~~

$v^2 = v_{oy}^2{}^{0} - 2g(y - y_0) = -2g(0 - 1200m)$

$v = \sqrt{2g(1200m)}$
$v = 153$ m/s
≈ 550 km/h

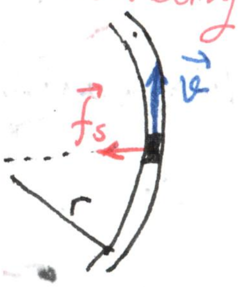
③ Uniform Circular Motion



- A body moves in a circle at constant speed, v
- The body has a centripetal acceleration of constant magnitude
  - directed toward the center of the circle
  - due to change in direction of velocity

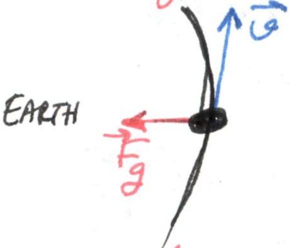
$$\boxed{a = \frac{v^2}{r}}$$

- Rounding a curve in a car



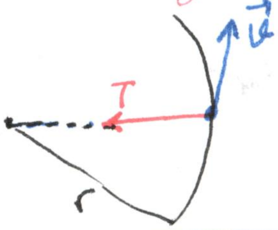
- Car will have an acceleration toward center.
- A force must cause this acceleration. Newton's 2^{nd} law
- This force is called centripetal force.
- In this case, it is frictional force on the tires from the road.

- Orbitting Earth in a space shuttle



- In this case, it is Earth's gravitational pull.

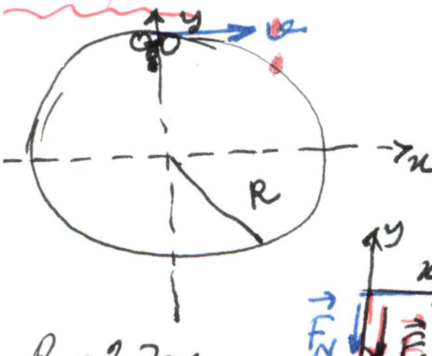
- Whirling an object connected to a string



- In this case, it is the tension at the string.

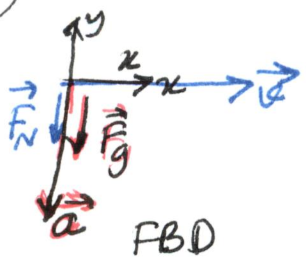
A centripetal force accelerates a body by changing the direction of the body's velocity without changing its speed.	$F = m\frac{v^2}{r}$

Example SLN



R = 2.7 m

Consider a man on bicycle turning around a circular wall. At the top of the wall, what should be his speed to remain in contact with the wall?



FBD

Newton's 2^{nd} law $\quad F_{net,y} = ma$

$-F_N - F_g = m(-a) = -m\frac{v^2}{R}$

remain in contact → just about loosing contact $F_N = 0$

$0 - mg = -m\frac{v^2}{R} \Rightarrow v = \sqrt{gR} = 5.1\ m/s$

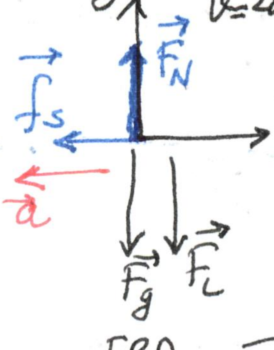
Sample Problem — Car in flat circular turn

$M = 600\,kg \quad R = 100\,m$

SLN Fig. 6-10a $\quad \mu_s = 0.75$

- A centripetal force must act which is frictional force
- Car is Not Sliding $\underset{\text{in radial direction}}{\text{not motion}}$ a static frictional force
- Just about sliding $\Rightarrow f_s \to f_{s,max} = \mu_s F_N$

i) $F_L = ?$ when $v = 28.6\,m/s$ $\Big\}$ Newton's 2nd law



FBD

x: $-f_s = m(-a) = m\left(-\frac{v^2}{r}\right)$

y: $F_N - F_g - F_L = m a_y = 0$

$$\Rightarrow \left.\begin{array}{l} \mu_s F_N = m\frac{v^2}{r} \quad ① \\ F_N = mg + F_L \quad ② \end{array}\right\} F_L = \frac{m}{\mu_s}\frac{v^2}{r} - mg = (600\,kg)\left(\frac{(28.6\,m/s)^2}{(0.75)(100\,m)} - 9.8\,m/s^2\right) = 663.\ N$$

$F_L \propto v^2$ as in Drag Force

ii) $F_L = ?$ when $v = 90\,m/s$ $\quad F_L$ is proportional to v^2

$$\frac{F_{L,90}}{F_L} = \frac{(90\,m/s)^2}{(28.6\,m/s)^2} \to F_{L,90} = 6572\,N$$

$\Big\{$ Notice that $F_g = (600\,kg)(9.8\,m/s^2) = 5880\,N$

$F_{L,90} > F_g \Rightarrow$ upside down motion!!

$v = 90\,m/s \approx 324\,km/h$

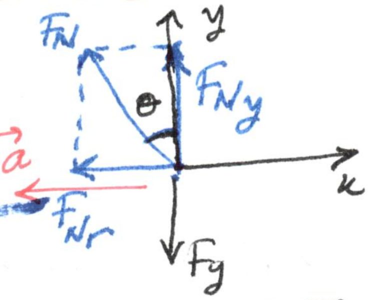
Sample Problem — Car in banked circular turn

SLN Fig. 6-11a

$v = 20\,m/s$

$R = 190\,m$

$\theta = ?$ without sliding



Newton's 2nd law

x: $-F_N \sin\theta = m(-a) = m\left(-\frac{v^2}{r}\right)$

$F_N = \frac{1}{\sin\theta}\frac{m v^2}{r}$ ①

y: $F_N \cos\theta - F_g = m a_y = 0$

$F_N = \frac{F_g}{\cos\theta}$ ②

① = ②

$$\frac{1}{\sin\theta}\frac{m v^2}{r} = \frac{F_g}{\cos\theta} \sim mg \Rightarrow \frac{v^2}{gr} = \tan\theta$$

$$\Rightarrow \theta = \tan^{-1}\frac{(20\,m/s)^2}{(9.8\,m/s^2)(190\,m)} \Rightarrow \boxed{\theta = 12°}$$

Chapter 7 - Kinetic Energy and Work

what is energy? — Technically, a scalar quantity associated with the state
— OR, a number that we associate with a system of one more objects.

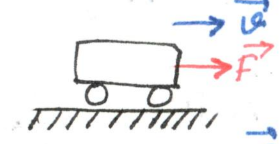
Energy can be — tranformed from one type to another (Kinetic ⇄ Potential)
— transfered from one object to another (Work), but total amount remains constant → Conservation of Energy

unit: Joule $\equiv 1\,Nm \equiv 1\,kg\,m^2/s^2$

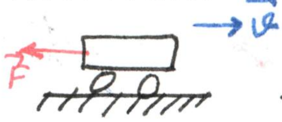
· **Kinetic Energy**: Associated with the state of motion of an object.

$$\boxed{K = \frac{1}{2}\underbrace{m}_{\text{mass}} v^2}$$ ~ speed; faster the object moves ⇒ greater KE.

· **Work**: Transferred Energy (W)



: acceleration ← velocity is increased → KE Increases



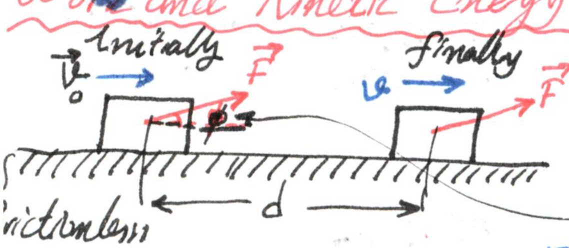
: deceleration ← velocity is decreased → KE decreases

} Changes in KE ⇓ $\vec{F}$ transfered energy to / from the object.

Transfer of energy via a force, → Work is said to be done on the object by the force

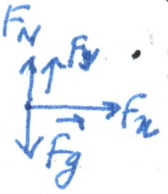
· If energy is transfered to the object, work is positive.
· If energy is transfered from the object, work is negative.
: Doing work is the act of tranferring the energy.

Work and Kinetic Energy



· Object was initially moving with $\vec{v}_0$
· Force $(\vec{F})$ applied → acceleration of the object.
· Constant Force ⇒ constant acceleration
· Velocity; $v_0 \to v$
· Object is moved a distance of d

$\Rightarrow v^2 = v_0^2 + 2a_x(x - x_0)$



· $F_{net,y} = 0$ & $F_{net,x} = F_x = ma_x$ SLN

$$ma_x = m\frac{v^2 - v_0^2}{2(x - x_0)}$$

$$F_x = \frac{1}{2}m(v^2 - v_0^2)\frac{1}{d}$$

$F_x d = \frac{1}{2}m(v^2 - v_0^2)$ ~ Change in Kinetic Energy

Work done on the object, W

$W = F_x d = F d \cos\phi$

$$\boxed{W = \vec{F} \cdot \vec{d}}$$

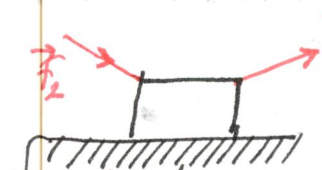
· Constant Force
· Rigid Object

- $\vec{F} \parallel \vec{d} \Rightarrow (+)$ work
- $\vec{F}$ antiparallel $\vec{d} \Rightarrow (-)$ work
- No force at that direction ⇒ No work
- More than one force ⇒ F_{net}

- Work-Kinetic Energy Theorem:

Change in KE of the particle $\equiv$ net work done on the system $\Big\}$ $\Delta K = W = \underbrace{K_f}_{\text{final KE}} - \underbrace{K_i}_{\text{initial KE}}$ $\Big\}$ $+W \rightarrow KE\uparrow$, $-W \rightarrow KE\downarrow$

Example



i) What is the work done on the object by forces $\vec{F_1}, \vec{F_2}$ during the displacement? What is the final speed of the object?

$m = 225\ kg$

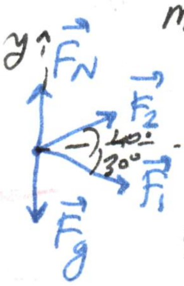
$|\vec{F_1}| = 12.0N$

$|\vec{F_2}| = 10.0N$

$d = 8.5m$, $v_0 = 0$

forces are constant.

frictionless — magnitudes and directions of the forces do not change



$\Big\{ W = \vec{F}\cdot\vec{d}$, $W = \Delta K$

$W_1 = F_1 d \cos\phi_1 = (12N)(8.5m)(\cos 30°) = 88.33J$

$W_2 = F_2 d \cos\phi_2 = (10N)(8.5m)(\cos 40°) = 65.11J$

$W_{net} = W_1 + W_2 \boxed{\simeq 153J} = K_f - \cancel{K_i}^{0} = \frac{1}{2} m v_f^2$

$\Rightarrow \boxed{v_f = 1.17\ m/s}$

ii) What is the work done on the object by the gravitational force $\vec{F_g}$?

iii) the normal force, $\vec{F_N}$?

$W_g = mgd \cos 90 = 0$

$W_N = F_N d \cos 90 = 0$

- Work Done by the Gravitational Force

An object is thrown upward. W_g SLN

Force and displacement are in the same direction / opposite

During Rising

$\uparrow\vec{v}$ $\downarrow\vec{F_g}$

$W_g = Fd\cos 180$

$= -Fd$

$\boxed{W_g = -mgd}$

During Falling

$\downarrow\vec{v}$ $\downarrow\vec{F_g}$

$W_g = Fd\cos 0$

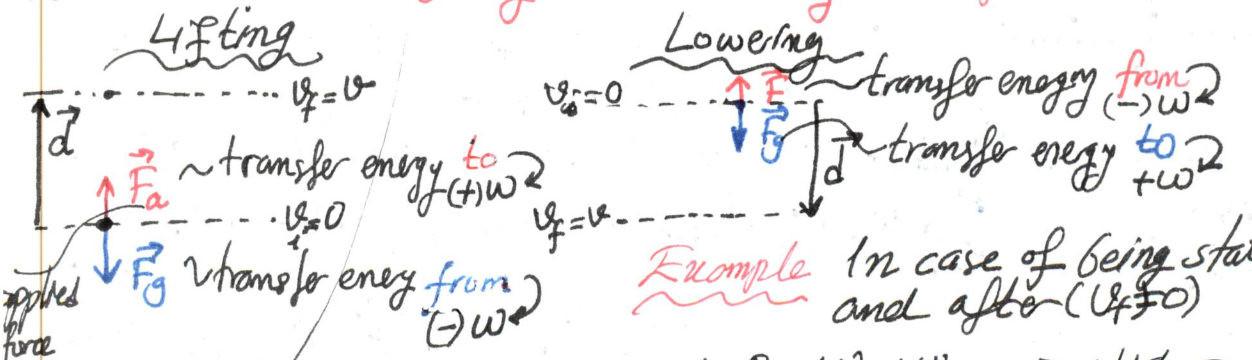
$= Fd$

$\boxed{W_g = mgd}$

(+) : Gravitational force transfers energy to KE of the object

(−) : from

- Work Done in Lifting and Lowering an Object



Example

In case of being stationary before ($v_i = 0$) and after ($v_f = 0$)

Work-KE theorem:

$\Delta K = K_f - K_i = W_a + W_g$

$\Delta K = 0 = W_a + W_g \rightarrow W_a = -W_g$

Lifting

$W_a = -Fd\cos 180 = mgd$

(+)W Energy transferred to obj

Lowering

$W_a = -F_g d\cos 0 = -mgd$

(−)W Energy transferred from

Example Work done on an accelerating elevator cab

SLN Fig. 7-8 $m = 500\,kg$, $v_i = 4.0\,m/s$

i) $W_g = mgd\cos\theta = (500\,kg)(9.8\,m/s^2)(12\,m)(1) = 5.88\times10^4\,J \sim 59\,kJ$

ii) $T - mg = ma_y \mid T = m(g+a) \left\{ \underbrace{Td\cos\theta}_{W_T} = md\cos\theta(g+a) \right\} \left\{ \theta = 180°,\ a = g/5\,(-\hat{j}) \right.$

$\rightarrow W_T = m\left(g - \frac{g}{5}\right)d\cos\theta = \frac{4}{5}(500\,kg)(9.8\,m/s^2)(12\,m)(-1) = -4.70\times10^4\,J \sim -47\,kJ$

iii) $W = W_g + W_T \simeq 12\,kJ$

iv) $K_f - K_i = W \rightarrow K_f = K_i + W = \frac{1}{2}(500\,kg)(4.0\,m/s)^2 + 1.18\times10^4\,J = 1.58\times10^4\,J \sim 16$

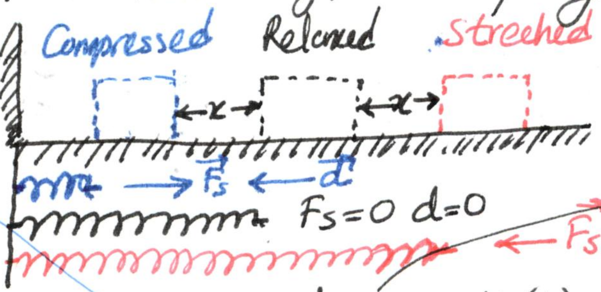
Work Done by a Spring Force

Variable force! Particular type is spring force. A common form.

SLN Fig. 7-9

$F_s = -kx$

$F(x)$: variable force



F_s: Restoring force

$\boxed{\vec{F}_s = -k\vec{d}}$ Hooke's Law

k: spring constant (stiffness of the spring) $k\uparrow$ stiffness $\uparrow$

Minus sign shows: $x(+)\ F(-)$; $x(-)\ F(+)$

Work

Assumptions
1. Massless spring
2. Ideal spring (Obeys Hooke's law)
3. Frictionless surface

$\left\{ W_s = Fd\cos\theta \right.$ does not work! Since, variable force

Divide the distance into smaller parts: Δx & J segments

$W_s = \sum -F_{x_j}\Delta x$ $\left\{ \text{limit } \Delta x \rightarrow 0 \right.$

$\Rightarrow W_s = -\int_{x_i}^{x_f} F_x\,dx$

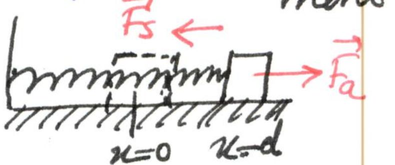
$\Rightarrow W_s = -\int_{x_i}^{x_f} -kx\,dx = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2 \rightarrow \boxed{W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2}$ work done by a spring force

SLN $\boxed{W_s = -\frac{1}{2}kx^2}$ $\Leftarrow$ if $x_i = 0$ (relaxed state)

- The work Done by an Applied Force: Apply a force $\vec{F}_a$ during displacement

Work–KE Theorem:

$\Delta K = K_f - K_i = W_a + W_s$ if the block stationary before and after displacement $\Big\}\ K_f = K_i \Big\}$ $W_a = -W_s$



Example Work done by spring to change KE

SLN Fig. 7-10

$m = 0.40\,kg$

$v_i = 0.50\,m/s$

$k = 750\,N/m$

when momentarily stopped, $d = ?$ $v_f = 0$

$K_f - K_i = W$

$0 - \frac{1}{2}mv_i^2 = -\frac{1}{2}kd^2$

$d = v\sqrt{\frac{m}{k}} = (0.50\,m/s)\sqrt{\frac{0.40\,kg}{750\,N/m}} = 1.2\times10^{-2}\,m = 1.2\,cm$

$W = \int_{x_i}^{x_f} F(x)\,dx$: Work done by a general variable force SLN

$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$: Three dimensional analysis SLN

$dW = \vec{F} \cdot d\vec{r}$

Work-KE Theorem also holds for variable forces. SLN

$\underbrace{\text{Work-KE Theorem}}_{\Delta K = W}$

$F(x) \Rightarrow m\,a(x) = m\,a(x(t))$

$a \to \frac{d\,v(x(t))}{dt} \to \frac{dv}{dx}\frac{dx}{dt} \to \frac{dv}{dx} v$

Power

the time rate at which work is done by a force is said to be power

$\underbrace{\text{the time rate}}_{\Delta t}$, $\underbrace{\text{work is done by a force}}_{W}$, $\underbrace{\text{power}}_{P}$

$P_{avg} = \frac{W}{\Delta t}$, $P = \frac{dW}{dt}$ instantaneous power

$P = \frac{dW}{dt} = \frac{F \cos\theta\, dx}{dt} = F v \cos\theta$

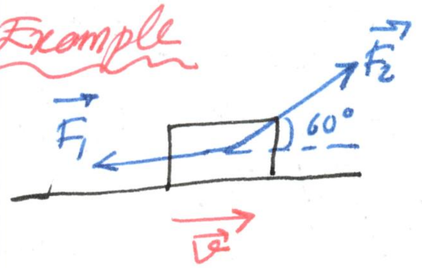
$\Rightarrow \boxed{P = \vec{F} \cdot \vec{v}}$

Unit: 1 Watt = 1 Joule/second

1 Hp = 746 W

1 kWh = 10^3 W 3600 s = 3.6 MJ

Example



$|\vec{F_1}| = 2N$

$|\vec{F_2}| = 4N$

$\vec{v} = 3 m/s$

$P_{net} = ?$

$P_{net} = P_1 + P_2$

$= F_1 v \cos\theta_1 + F_2 v \cos\theta_2$

$= (2N)(3 m/s)\cos 180° + (4N)(3 m/s)\cos 60°$

$= -6 W + 6W = 0 W$!

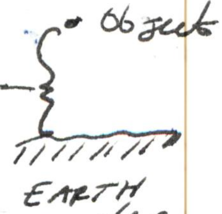
OR $P = \vec{F}_{net} \cdot \vec{v}$ where $F_{net} = 0$

$\Rightarrow P = 0$

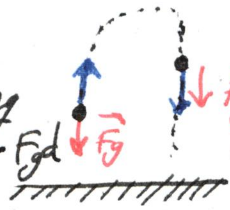
$P_{net} = 0 \to$ net rate of transfer of energy to or from the box is zero

$\Rightarrow$ KE is not changing $\Rightarrow$ speed will remain constant

Chapter 8 - Potential Energy and Conservation of Energy



KE : state : velocity
PE : state : seperation (Gravitational Potential Energy, u)



Work done is negative, Energy is taken from KE and transfered to PE: $W_g = \vec{F}\cdot\vec{d} = -F_g d$

Falling: $W_g = F_g d$. Work done is positive, Energy is taken from PE and transfered to KE.

In both cases; $\Delta U = -W$

Similarly ; Elastic Potential Energy . spring-mass system

Compression : Taken from KE, Transfered to PE — KE → PE (decreased, increased)

Strecthing : Taken from PE, Transfered to KE — PE → KE

• Conservative and Nonconservative Forces

SLN

Consider
- W_1 Rising: KE → PE : F_g transfers some energy from object, and does work W_1
- $-W_1$ Falling: PE → KE : F_g transfers some energy from object, and does work W_2

⇒ Gravitational force is conservative force since $W_1 = -W_2$

* But, frictional force is nonconservative force

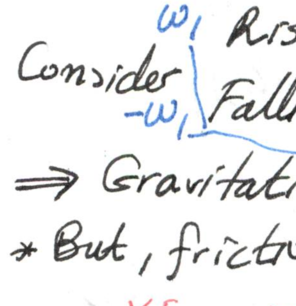
KE → Thermal Energy (acting force is frictional force)

Thermal Energy $\not\to$ KE Thermal energy can not be transformed into KE by frictional force.

⇒ Nonconservative force — Frictional force, Drag force

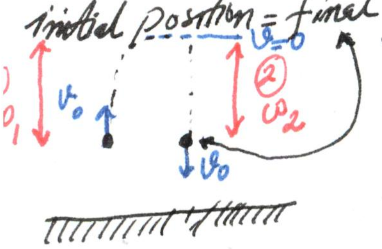
* Only, conservative forces act on a particle-like object → great simplification of the problem (i.e. path independence)

Path Independence of Conservative Forces



The net work done by a conservative force on a particle moving around any closed path is zero.

initial position = final position



Gravitational force :

Initial $KE = \frac{1}{2}mv_0^2$

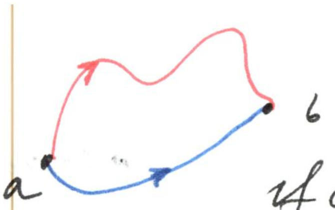
Final $KE = \frac{1}{2}mv_0^2$

$W_{net} = W_1 + W_2$
$= W_1 + (-W_1) = 0$

The net work done by gravitational force is zero ⇒ F_g is conservative

⇓

This brings us path independence.



Moving of a particle from point a to b by following path 1 and path 2.

If only conservative forces acts $\Rightarrow$ $\boxed{W_{ab}(\text{path }1) = W_{ab}(\text{path }2)}$

Example Equivalent paths for calculating work, slippery cheese

SLN Fig. 8-5
$m = 2.0$ kg
track : 2.0 m
vertical distance : 0.80 m
$W_g = ?$

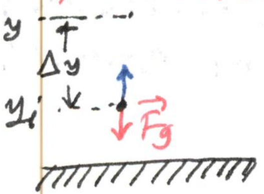
$W_g = \vec{F}\cdot\vec{d} = Fd\cos\phi$ we can not calculate since ϕ changes during the motion.
But, F_g is conservative ; $W_g(\text{path }1) = W_g(\text{path }2)$

$$W_g(\text{path }2) = \underbrace{F_g d\cos 0}_{\text{vertical}} + \underbrace{F_g d\cos 90}_{\text{horizontal}} = (2.0\,kg)(9.8\,m/s^2)(0.80\,m)(1) + 0 = \boxed{15.7\,J}$$

Determining Potential Energy Values

When the work is done on a particle-like object by a conservative force the change in potential energy, ΔU. $\Delta U = -W \rightarrow \Delta U = -\int_{x_i}^{x_f} F(x)\,dx$

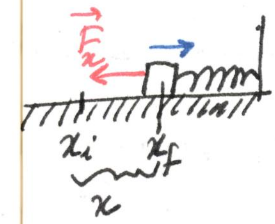
1) Gravitational PE



$$\Delta U = -\int_{y_i}^{y} F_g\,dy = -\int_{y_i}^{y}(-mg)\,dy \Rightarrow \boxed{U - U_i = \Delta U = mg\Delta y}$$

if $y_i = 0$ (reference point) $\rightarrow U_i = 0 \rightarrow \boxed{U = mgy}$

2) Elastic PE



As block moves spring force (F_x) does work on the block

$$\Delta U = -\int_{x_i}^{x_f} F_x\,dx \Rightarrow \Delta U = \int_{x_i}^{x_f} kx\,dx \Rightarrow \boxed{\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2}$$

if $x_i = 0$ (relaxed state) $\rightarrow \boxed{U = \frac{1}{2}kx^2}$

Conservation of Mechanical Energy

$E_{mech} = PE + KE$

- All forces are conservative
- There is no external force
- Isolated system

$\left.\begin{matrix}\Delta K = W \\ \Delta U = -W\end{matrix}\right\}$ $\Delta K = -\Delta U$ $\quad K_2 - K_1 = U_1 - U_2$ $\quad \boxed{K_2 + U_2 = U_1 + K_1}$

KE & PE can change but their sum E_{mech} can not change

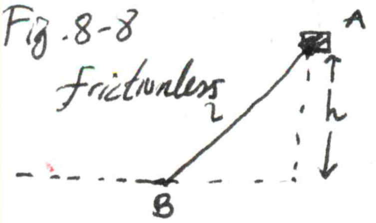
Conservation of Mechanical Energy

$\boxed{\Delta E_{mech} = \Delta K + \Delta U = 0}$

SLN Fig. 8-7

Example Conservation of mechanical energy, water slide

Fig. 8-8



Speed of the block at point B?

Conservation of Energy $\left\{\begin{matrix} K_A + U_A = K_B + U_B \\ 0 + U_A = K_B + 0 \end{matrix}\right\}$ $mgh = \frac{1}{2}mv_B^2$

$\boxed{v_B = \sqrt{2gh}}$

Work Done on a System by an External Force

$W(+) \Rightarrow$ system

work is enegy transfered to / from system by means of external force(s)

$W(-) \Leftarrow$ system

- **No friction involved:** Applied external force $\Rightarrow$ work is added to system

SLN System is Ball-Earth system. $\boxed{W = \Delta U + \Delta K = \Delta E_{mech}}$

- **Friction involved:** SLN Applied $F_{ext} \Rightarrow W(+)$

$F - f_k = ma_x = m\left(\frac{v^2 - v_0^2}{2d}\right)$ Frictional force $f_k \Rightarrow W(-)$ Transfers some enegy to thermal energy

$Fd - f_k d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$Fd = \Delta K + f_k d \xrightarrow[\text{also add } \Delta U]{\text{in general,}} Fd = \Delta E_{mech} + \underbrace{f_k d}_{\text{thermal enegy}} \rightarrow \boxed{W = \Delta E_{mech} + \Delta E_{th}}$

Now, the system is Block-floor system.

Conservation of Enegy:

- Enegy can not be created or destroyed, it only changes form.
- Enegy of the system can be changed by adding or taking enegy.

$\boxed{W = \Delta E = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int}}$ internal enegy!! Enegy transfer to $W(+)$ / from $W(-)$

- **Isolated System:** No enegy transfer btw system and its surroundings

$W = 0 = \Delta E_{sys}$: Total enegy can not change SLN

- **Power:** The rate at which work is done by a force. $P_{avg} = \frac{\Delta E}{\Delta t}$: Average

The rate at which enegy is transfered by a force from one type to another. $P = \frac{dE}{dt}$: Instantaneous

Example,

$m = 2.0$ kg slides

$v_1 = 4.0$ m/s

compress spring

momentarily stops

initially frictionless

$f_k = 15$ N, $k = 10000$ N/m } Compression

$d = ?$

System is isolated !!

$W = 0 = \Delta U + \Delta K + \Delta E_{th} + \cancel{\Delta E_{int}}$

$= U_2 - U_1 + K_2 - K_1 + f_k d$

$= \frac{1}{2}kd^2 - 0 + 0 - \frac{1}{2}mv_1^2 + f_k d$

$-\frac{1}{2}kd^2 = -\frac{1}{2}mv_1^2 + f_k d$

$5000\, d^2 - 15d - 16 = 0$

$\boxed{d = 0.055 \text{ m}}$

Reading a Potential Energy Curve

$\Delta U(x) = -W$ · Spring: Elastic PE $U(x) = \frac{1}{2}kx^2 \rightarrow F(x) = -kx$

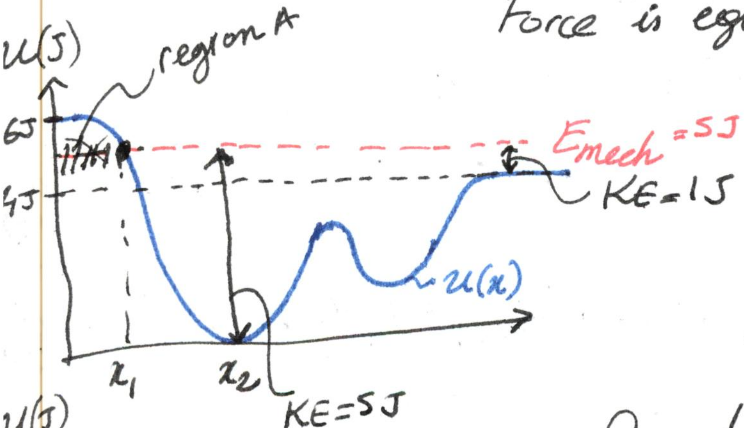
$\int dU(x) = -\int F(x)\,dx$ · Gravitational PE $U(x) = mgx \rightarrow F_g = -mg$

$\Rightarrow \boxed{F(x) = -\frac{dU(x)}{dx}}$

SLN Fig. 8-9

decreasing increasing

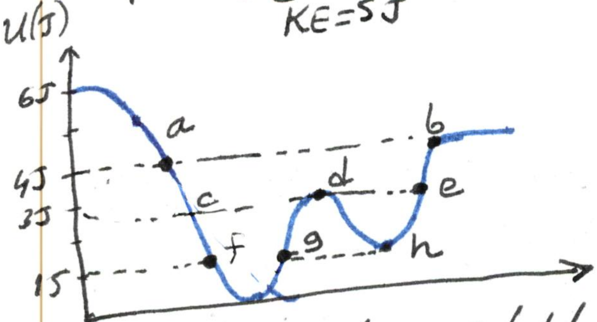
Force is equal to the negative of the slope of the $U(x)$ plot



Region A: $K(x) = E_{mech} - U(x) < 0$ X

$\rightarrow$ can not move to the left side of x_1

x_1: $KE = 0 \rightarrow E_{mech} = U$



Consider 3 different Emech (4J, 3J, 1J)

① $E_{mech} = 4J$ $x = a$: Turning point
$x > b$: $K = 0$, $U = E_{mech}$

neutral equilibrium $\Leftarrow$ stationary, no force acting

② $E_{mech} = 3J$ $x = c$, $x = e$: Turning point

unstable equilibrium — $x = d$ The force on the particle is zero. small force $\rightarrow$ it can move

③ $E_{mech} = 1J$ $x = f$, $x = g$: Turning points

stable equilibrium — $x = h$ small force $\rightarrow$ returns back

Example Reading a Potential Energy graph

SLN Fig. 8-10

$m = 2.00$ kg

Conservative force btw

$U(x)$: plotted $x = 0$ & $x = 7.0$ m

$x = 6.5$ m $\rightarrow v_0 = (-4.00\,m/s)\hat{i}$

$\rightarrow KE = \frac{1}{2}mv^2 = 9J \Rightarrow \boxed{v = 3m/s}$

Conservative force $\rightarrow E_{mech} = U + K$

i) $x = 4.5$ m $E_{mech} = 16J = U(7J) + KE$

ii) Turning point: Force momentarily stops and reverses particle's motion

$\Rightarrow KE = 0$ see Fig. 8-10 b $\frac{20-7}{4-1} = \frac{16-7}{d} \rightarrow d = \boxed{1.9m \cong x}$

iii) $\Delta x = (4.0 - 1.9)$m $F(x) = -\frac{dU}{dx} \cong -\frac{\Delta U}{\Delta x} = -\frac{(7-16)J}{(4.0-1.9)m} \boxed{= 4.3N}$

Initially: leftward-moving particle

Turning point ($x = 1.9$m): stopped by the force and then sent rightward

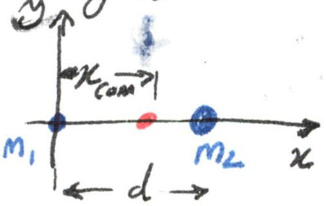
Chapter 9 - Center of Mass and Linear Momentum

To predict the possible motion of the system → The center of mass (com)
i) All the system's mass were concentrated there
ii) All external forces were applied there

SLN Fig. 9-1

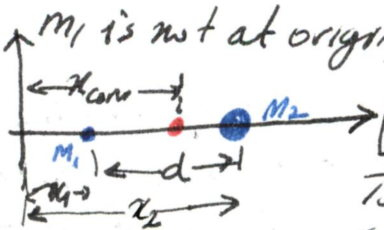
· System of particles

Two particles system. m_1 is at origin.



$$\boxed{x_{com} = \frac{m_2}{m_1+m_2} d}$$

m_1 is not at origin.



$$\boxed{x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$$

Total mass of the system, M

⇒ General Equation

$$\boxed{x_{com} = \frac{m_1 x_1 + \cdots + m_n x_n}{M} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i}$$

y_{com} … y_i

z_{com} … z_i

$$\boxed{\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i}$$

where $\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j} + z_{com}\hat{k}$

· Solid Bodies

So many particles → continuous distribution
⇒ mass → dm

$$\boxed{x_{com} = \frac{1}{M}\int x\,dm \quad y \quad z} \qquad \boxed{x_{com} = \frac{1}{V}\int x\,dV \quad y \quad z}$$

y_{com}, z_{com}

since considering uniform density

$$\rho = \frac{dm}{dV} = \frac{M}{V} \Rightarrow dm = \frac{M}{V}dV$$

Example com of three particles

SLN Fig. 9-4

$m_1 = 1.2\text{ kg}$, $m_2 = 2.5\text{ kg}$, $m_3 = 3.4\text{ kg}$ Equilateral triangle, $a = 1.4\text{ m} = 140\text{ cm}$

$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3} = 83\text{ cm}$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = 58\text{ cm}$$

$$\Rightarrow \vec{r}_{com} = (83\text{ cm})\hat{i} + (58\text{ cm})\hat{j}$$

· Newton's Second Law for a System of Particles

n-particles with different masses moving

$$\boxed{\vec{F}_{net} = M\vec{a}_{com}}$$

center of mass acceleration

Total Mass

→ moves like a particle whose mass is equal to the total mass
↳ velocity
↳ position
↳ acceleration } assign to com

Net force of all external forces (internal forces are not included)
↑ 3rd law force pair cancel each other

SLN Fig. 9-5 Firework explodes
- Initially, com → a parabolic path
- Before explosion $\vec{F}_{net_1} \to \vec{F}_g$ (net external force)
- After explosion $\vec{F}_{net} \to \vec{F}_g$

} $\vec{F}_{net}$ does not change } com of fragments after explosion follows the same parabolic trajectory } $\vec{a}_{com} \equiv \vec{g}$

SLN Proof of final Result

Example Motion of the com of three particles - Fig. 9-7

$F_1 = 6.0\text{ N}$, $F_2 = 12\text{ N}$, $F_3 = 14\text{ N}$

$\vec{a}_{com} = ?$

$\vec{F}_{net} = M\vec{a}_{com}$

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = M\vec{a}_{com}$

$$a_{com,x} = \frac{-6.0\text{N} + (12\text{N})\cos 45 + 14\text{N}}{16\text{ kg}} = 1.03\text{ m/s}^2$$

$$a_{com,y} = \frac{0 + (12\text{N})\sin 45 + 0}{16\text{ kg}} = 0.530\text{ m/s}^2$$

$$a_{com} = \sqrt{(a_{com,x})^2 + (a_{com,y})^2} = 1.16\text{ m/s}^2$$

$$\theta = \tan^{-1}\frac{a_{com,y}}{a_{com,x}} = 27^\circ$$

Linear Momentum

- The linear momentum of a particle is a vector quantity, $\vec{p}$; $\boxed{\vec{p} = m\vec{v}}$ $\quad \vec{p} \parallel \vec{v}$ $\quad$ kg m/s

$\boxed{\vec{F}_{net} = \dfrac{d\vec{p}}{dt}}$. Change of momentum of a particle with respect to time } $\vec{F}_{net}$ acting on the particle
. if $\vec{F}_{net} = 0$, $\vec{p}$ can not change

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

- The linear momentum of a system of particles

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n \rightarrow \boxed{\vec{P} = M\vec{v}_{com}} \quad (\vec{F}_{net} = M\vec{a}_{com})$$

Collision and Impulse

An external force acts on body $\rightarrow \vec{F}_{net} = \dfrac{d\vec{p}}{dt}$

. Single Collision . Collision of a moving particle and other body SLN Fig. 9.8

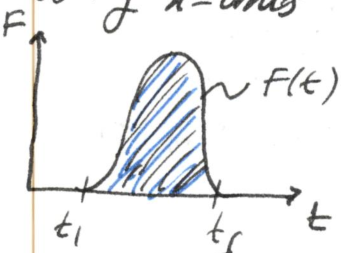
$$\vec{F} = \frac{d\vec{p}}{dt} \rightarrow d\vec{p} = \vec{F}dt \rightarrow \underbrace{\int_{t_i}^{t_f} d\vec{p}}_{\Delta p} = \underbrace{\int_{t_i}^{t_f} \vec{F}(t)dt}_{\text{Impulse } \vec{J} \text{ of the collision}}$$

} t_i : time just before collision
t_f : time just after collision

$\Rightarrow \boxed{\Delta\vec{p} = \vec{J}}$ linear momentum - impulse theorem

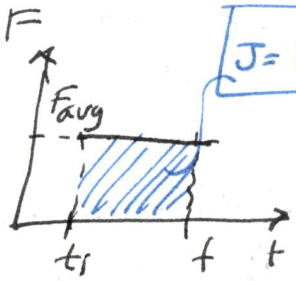
~ measure of both the magnitude and the duration of the collision force

along x-axis $\quad p_{fx} - p_{ix} = \int_{t_i}^{t_f} F_x dt$



Area under F(t)-t curve gives J
$J = \int_{t_i}^{t_f} F(t)dt$

} If F(t) is not known, average of F usually taken $\rightarrow$



$\boxed{J = F_{avg}\Delta t}$

. Series of Collisions SLN Fig. 9-10

$J = F_{avg}\Delta t = -n\Delta p$: The total change in linear momentum of n-particles in time interval Δt is $n\Delta p$ but in opposite directions. J and Δp in opposite directions

$$\Rightarrow F_{avg} = -n\frac{\Delta p}{\Delta t} = -n\frac{m\Delta v}{\Delta t} = -nm\frac{\Delta v}{\Delta t} \boxed{= -\frac{\Delta m}{\Delta t}\Delta v}$$

Δm : amount of mass collides

Example Two-dimensional impulse, race-car-wall collision. Fig. 9-11

$v_i = 70$ m/s
$\theta = 30°$
$v_f = -50$ m/s
$\theta = 10°$
$M = 80$ kg

$$\vec{J} = \vec{p}_f - \vec{p}_i = M(\vec{v}_f - \vec{v}_i)$$

$$J_x = m(v_{fx} - v_{ix}) = (80\text{ kg})[50\text{ m/s}\,(\cos(-10°) - 70\text{ m/s}\cos 30°] = -910\text{ kgm/s}$$

$$J_y = m(v_{fy} - v_{iy}) = (80\text{ kg})[50\text{ m/s}\,(\sin(-10)° - 70\text{ m/s}\sin 10°] = -3495\text{ kg m/s}$$

$J = \sqrt{J_x^2 + J_y^2} = 3616$ kgm/s $\quad \theta = -105°$ } $F_{avg} = \dfrac{J}{\Delta t} = \dfrac{3616\text{ kgm/s}}{0.014\text{ s}} = 2.583 \times 10^3$ N

Conservation of Linear Momentum

$\vec{F}_{net} = \dfrac{d\vec{P}}{dt}$; system of particles; ← total linear momentum

If ① $\vec{F}_{net} = 0$ External Forces $\equiv 0$ $\Rightarrow J \equiv 0$
② No particles leave or enter the system

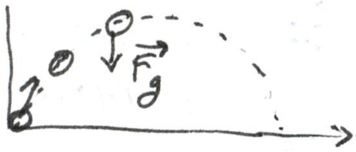
$\vec{P}$ of the system cannot change if there is no external force on it

$\boxed{\vec{P} = \text{constant (closed, isolated system)}}$

Law of conservation of linear momentum

$\boxed{\vec{P}_i = \vec{P}_f}$

Example Projectile Motion



- no horizontal external force
- but in vertical direction, $\vec{F}_g$

} linear momentum along horizontal can not change
- whereas vertical one changes

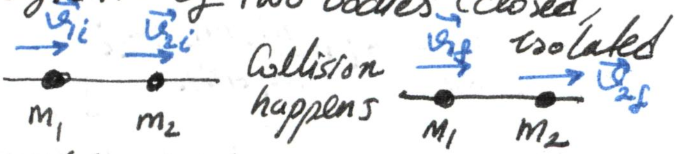
Momentum and Kinetic Energy in Collisions

Collision of two bodies
- If $KE_{initial} = KE_{final}$ → Elastic Collision
- If $KE_i \neq KE_f$ → Inelastic Collision
  Some KE is lost
- Completely Inelastic Collision
  - Bodies stick together after collision
  - Maximum KE loss

Inelastic Collisions in 1D ($KE_i \neq KE_f$)

Inelastic Collision

System of two bodies (closed, isolated)



Total momentum before collision = Total momentum after collision

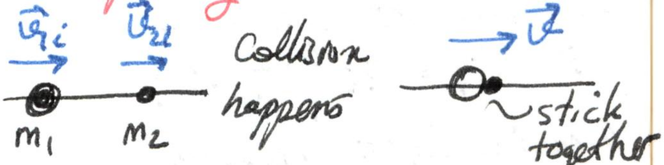
$$\vec{P}_i = \vec{P}_f$$

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

1D motion → omit vectors

$$\boxed{(m_1 v_1)_i + (m_2 v_2)_i = (m_1 v_1)_f + (m_2 v_2)_f}$$

Completely Inelastic Collision



say $v_{2i} = 0$ (stationary object)

$$\vec{P}_i = \vec{P}_f$$

$$(m_1 v_1)_i = (m_1 + m_2) v$$

} $v < v_{1i}$

Velocity of the COM

The velocity of the COM of the closed, isolated system can not be changed by collision since there is no net external force. See Fig. 9-16

$$\vec{P} = M \vec{v}_{com} = (m_1 + m_2)\vec{v}_{com}$$

$\left\{ \vec{P} = \vec{P}_{1i} + \vec{P}_{2i} \text{ OR } \vec{P} = \vec{P}_{1f} + \vec{P}_{2f} \right\}$

The COM moves at the same velocity even after the bodies stick together.

$$(m_1 + m_2)\vec{v}_{com} = \vec{P}_{1i} + \vec{P}_{2i}$$

$$\vec{v}_{com} = \frac{\vec{P}_i}{(m_1 + m_2)} \left(\text{OR } \frac{\vec{P}_f}{(m_1 + m_2)} \right)$$

Example Conservation of momentum, ballistic pendulum. S&W Fig. 9-17

$M = 5.4$ kg initially at rest
$m = 9.5$ g
$h = 6.3$ cm
momentarily stops $(M+m)$
$v = ?$

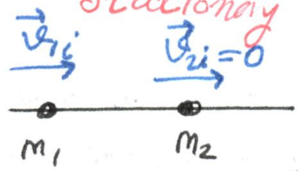
① $\vec{P}_i = \vec{P}_f \;\}\; mv = (M+m)V$

ⓘⓘ $W = \Delta E_{mech} = 0 = \Delta K + \Delta U = \cancel{K_f} - K_i + U_f - \cancel{U_i}$ isolated system $\frac{1}{2}(M+m)V^2$, 0, $(M+m)gh$ $\}\; \frac{1}{2}(M+m)V^2 = (M+m)gh$

① & ⓘⓘ $mv = (M+m)\sqrt{2gh} \rightarrow v = \frac{(M+m)}{m}\sqrt{2gh} \boxed{= 630\ m/s}$

Elastic Collision in 1D $(KE_i = KE_f)$

Stationary Target



Two body system (closed, isolated)

① Linear momentum is conserved

$$(m_1 v_1)_i = (m_1 v_1)_f + (m_2 v_2)_f$$

ⓘⓘ Elastic Collision → KE is conserved

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

① & ⓘⓘ (some algebra, see your book)

$$\boxed{v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}}$$

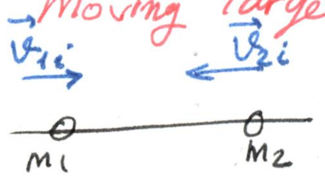
$$\boxed{v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}}$$

1) Equal masses $(m_1 = m_2)$
$\Rightarrow v_{2f} = v_{1i} \;\{\; v_{1f} = 0\,!$

2) Massive Target $(m_2 \gg m_1)$
$\Rightarrow v_{1f} \sim -v_{1i}$ & $v_{2f} \simeq \left(\frac{2m_1}{m_2}\right) v_{1i}$
bounces back; very low velocity

3) Massive Projectile $(m_1 \gg m_2)$
$\Rightarrow v_{1f} \sim v_{1i}$ & $v_{2f} \sim 2 v_{1i}$

Moving Target



head-on collision

ⓘ Conservation of linear momentum

$$(m_1 v_{1i}) + (m_2 v_{2i}) = (m_1 v_{1f}) + (m_2 v_{2f})$$

ⓘⓘ Also KE is conserved

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

ⓘ & ⓘⓘ $m_1(v_{1i} - v_{1f}) = -m_2(v_{2i} - v_{2f})$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = -m_2(v_{2i} - v_{2f})(v_{2i} + v_{2f})$$

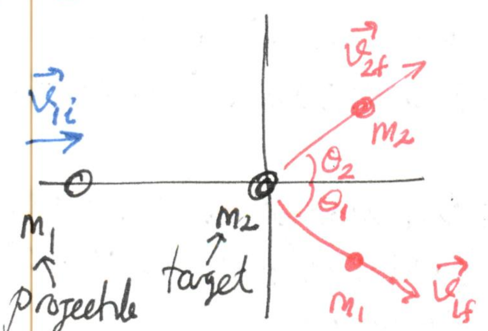
$$\Rightarrow \boxed{\begin{aligned} v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \\ v_{2f} &= \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \end{aligned}}$$

Example Elastic Collision, Two Pendulums S&W Fig. 9-20

$m_1 = 30$ g
$m_2 = 75$ g
$h_1 = 8.0$ cm
released

Elastic Collision → KE is conserved
$K_f - \cancel{K_i} = U_i - \cancel{U_f}$; 0, 0 $\{$ E_{mech} is conserved
$\frac{1}{2} m_1 v_{1f}^2 = m g h_1 \rightarrow v_{1f} = \sqrt{2gh_1} = 1.252\ m/s$
Stationary Target $v_{1i} \rightarrow v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \simeq -0.54\ m/s$
minus sign tells us sphere 1 moves to left

Collision in 2D



Closed, isolated system

- Collision is not head-on
- Total linear momentum conserved $\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$
- Elastic, Total KE is conserved $KE_{1i} + KE_{2i} = KE_{1f} + KE_{2f}$
- 2D:

along x-axis

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$$

along y-axis

$$0 = -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2$$

Chapter 10 - Rotation

Motion of translation → along a straight line

Motion of rotation → (a rigid body) turns around an axis (about COM!) SLN

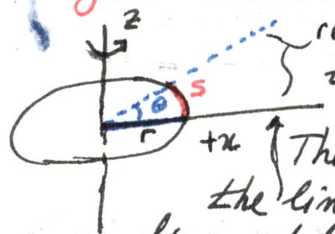
- → rotational acceleration (constant or not!)
- → torque (instead of force)
- → inertia (instead of mass)

- Rotational Variables

A rigid body about a fixed axis SLN Fig. 10-2

1) Angular Position

$\theta(t)$ time dependence



reference line, perpendicular to rotation axis

The angular position of the line is the angle of the line relative to a fixed direction. (zero angular position)

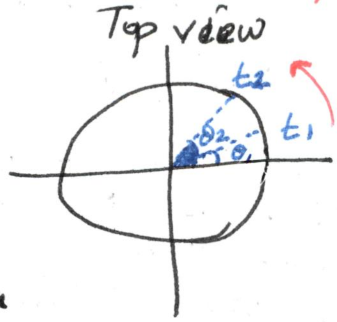
$$\boxed{\theta = \frac{s}{r}}$$

circular arc length, radius of circle

- radian measure
- angle

$360° = 2\pi$ rad $\equiv$ 1 revolution

2) Angular Displacement $\Delta\theta$



$$\boxed{\Delta\theta = \theta_2 - \theta_1}$$

CCW → (+)

CW → (−)

3) Angular Velocity ω (rad/s or rev/s)

average

$$\boxed{\omega_{avg} = \frac{\theta_2-\theta_1}{t_2-t_1} = \frac{\Delta\theta}{\Delta t}}$$

instantaneous

$$\boxed{\omega = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$

- CCW → (+)
- CW → (−)
- magnitude is called angular speed, ω

→ $\theta = \int \omega\, dt$ $\qquad \omega = \int \alpha\, dt$

4) Angular Acceleration α (rad/s² or rev/s²)

$$\boxed{\alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2-\omega_1}{\Delta t}}$$

$$\boxed{\alpha = \lim_{\Delta t\to 0}\frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}}$$

Example Angular velocity derived from angular position

Disk is rotating as $\theta(t) = -1.00 - 0.600t + 0.250t^2$ SLN Fig. 10-5a

i) Angular position of reference line at $t = -2.0$ s, 0 s, 4 s, $\theta = 0$ points

$t=-2 \rightarrow \theta(t=-2) = -1 - 0.6(-2) + 0.25(-2)^2 = 1.2$ rad $\Rightarrow$ 2π rad — 360°, 1.2 rad — x $\Big\}$ $\theta = 69°$ ccw

SLN Fig. 10-5b for the rest $\Big\{$ $t=0 \rightarrow \theta = -1.00$ rad $\rightarrow -57°$ cw; $t=4 \rightarrow \theta = 0.60$ rad $\rightarrow 34°$ ccw

$\theta=0$ points: reference line is aligned with zero angular position (s_2, s_4)

ii) $t_{min} = ?$ $\xrightarrow{\text{that makes}}$ $\theta(t)$ minimum.

SLN. Fig. 10-5c → What about angular acceleration!

To have a minimum $\left.\frac{d\theta}{dt}\right|_{t=t_{min}} = 0 \rightarrow -0.6 + 0.5t = 0 \rightarrow t = 1.20$ s (see Fig. 10-5b)

$\theta(t=1.20\text{s}) = -1.36$ rad $\approx -77.9°$ maximum cw rotation!

iii) $t=0 \rightarrow \omega(0) = -0.6$ rad/s

$t=1 \rightarrow \omega(1) = -0.1$ "

$t=2 \rightarrow \omega(2) = 0.4$ "

$\Big\}$ $\frac{d\theta}{dt} = \omega = -0.6 + 0.5t$

Example Angular velocity derived from angular acceleration

$\alpha = 5t^3 - 4t$

$t=0 \begin{cases} \omega = 5 \text{ rad/s} \\ \theta = 2 \text{ rad} \end{cases}$

i) $\omega(t) = ?$ $\int d\omega = \int \alpha dt \rightarrow \omega = \int (5t^3 - 4t) dt = \frac{5}{4}t^4 - \frac{4}{2}t^2 + C$

$\omega(t=0) = 5 = \frac{5}{4}0^4 - \frac{4}{2}0^2 + C \Rightarrow \omega(t) = \frac{5}{4}t^4 - 2t^2 + 5$

ii) $\theta(t) = ?$ $\int d\theta = \int \omega dt \rightarrow \theta = \int (\frac{5}{4}t^4 - 2t^2 + 5) dt = \frac{1}{4}t^5 - \frac{2}{3}t^2 + 5t + C$

$\theta(t=0) = 2 \rightarrow \theta(t) = \frac{t^5}{4} - \frac{2}{3}t^2 + t + 2$

- Are Angular Quantities vectors?

Angular Displacement, $\Delta\theta \rightarrow$ Can not be treated as vectors. Does not obey to vector arithmetics.

Angular Velocity, ω } Can be treated as vectors
Angular Acceleration, α }

SLN Fig. 10-6 Fig. 10-7 Directions of vector and motion are different

ω and α can be represented by $\pm$ sign. ccw (+) cw (−)

SLN Rotation with Constant Angular Acceleration Table 10-1

Example Constant angular acceleration, grindstone

$\alpha = 0.35 \text{ rad/s}^2$

$\omega_0 = -4.6 \text{ rad/s}$

$\theta_0 = 0$ (reference line)

SLN Fig. 10-8

i) $t = ?$ at $\theta = 5$ rev $\theta - \theta_0$ $\quad \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$ $\quad 5 \times 2\pi \text{ rad} = -4.6 \text{ rad/s}\, t + 0.35 \text{ rad/s}^2\, t^2$

$\Rightarrow t = 32 \text{ s}$

ii) $\alpha \rightarrow$ positive, $\omega_0 \rightarrow$ negative } initially slows down, momentarily stops, rotates again
cw, ccw since α (+) θ (+)

iii) $t = ?$ at $\omega = 0$ $\quad \omega = \omega_0 + \alpha t \rightarrow +4.6 \text{ rad/s} = 0.35 \text{ rad/s}^2\, t$

$\Rightarrow t = 13 \text{ s}$

Example Constant angular acceleration, riding a Rotor

$\omega_0 = 3.4 \text{ rad/s}$

$\omega = 2.0 \text{ rad/s}$

$\theta - \theta_0 = 20.0 \text{ rev} \left(\times 2\pi \frac{\text{rad}}{\text{rev}}\right)$

constant angular acceleration

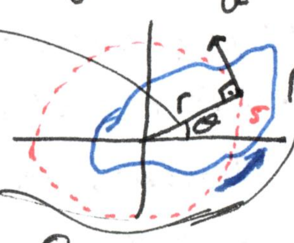
i) $\alpha = ?$ $\quad \omega = \omega_0 + \alpha t$, $\quad \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$ } $\theta - \theta_0 = \omega_0\left(\frac{\omega - \omega_0}{\alpha}\right) + \frac{1}{2}\alpha\left(\frac{\omega - \omega_0}{\alpha}\right)^2 \Rightarrow \alpha = -0.0301 \text{ rad}$

slowing down

ii) $t = \frac{\omega - \omega_0}{\alpha} = \frac{2.0 \text{ rad/s} - 3.4 \text{ rad/s}}{-0.0301 \text{ rad/s}^2} = 46.5 \text{ s}$

Relating the Linear and Angular Variables

Fig. 10-9a



s, v, a } can be related to angular counterparts } θ, ω, α by r: the perpendicular distance of the point from the rotation axis

Point P makes a rotation. velocity v, distance s
Object makes a rotation about a fixed axis. ω } $s = \theta r$, $v = \omega r$ angular speed, linear speed

$\Rightarrow$ linear speed v depend on the "point"'s location

angular speed ω is same at every "point"

$T = \frac{2\pi r}{v} \rightarrow \boxed{T = \frac{2\pi}{\omega}}$ $\quad 2\pi r \leftrightarrow \theta r$: distance travelled

$s = \theta r$

$\frac{ds}{dt} = \frac{d\theta}{dt} r \rightarrow v = \omega r$

$\frac{dv}{dt} = \frac{d\omega}{dt} r \rightarrow a = \alpha r$

SLN Fig. 10-9b

$a \Rightarrow a_t$: tangential component

Remember

$a_r = \frac{v^2}{r} = \omega^2 r$

radially inward (for changes in the direction of linear velocity)

a_t is present when $\alpha \neq 0$

a_r is present when $\omega \neq 0$

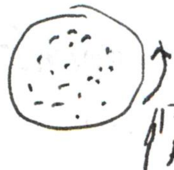
Kinetic Energy of Rotation

$\uparrow KE = \cancel{\frac{1}{2} M v^2}$

since $v_{com} = 0$

M

Instead



Suppose that the body is composed of many particles. Then $K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots + \frac{1}{2} m_n v_n^2$

$$\boxed{K = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2}$$

$$\underbrace{v = \omega r}_{\text{same for all particles}} \Rightarrow K = \frac{1}{2} \sum_{i=1}^{n} m_i \omega^2 r_i^2$$

$$\Rightarrow \boxed{K = \frac{1}{2}\Big(\underbrace{\sum_{i=1}^{n} m_i r_i^2}_{I\text{: rotational inertia}}\Big)\omega^2 = \frac{1}{2} I \omega^2}$$

Kinetic Energy of a rigid body in pure rotation

- Tells us how the mass of rotating body is distributed about its axis of rotation.
- It is specified with respect to rotation axis. S.L.N Fig. 10-11
- $kg\,m^2$
- Smaller I means easier rotation

  Mass distribution is close to rotation axis.

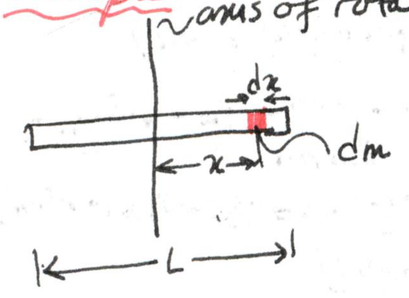
Kinetic Energy of the body in pure translation $\rightarrow K = \frac{1}{2} M v_{com}^2$

Calculating the Rotational Inertia

A rigid body consists

- of a few particles $\rightarrow I = \sum m_i r_i^2$ perpendicular distance from rotation axis
- of a great many adjacent particles $\rightarrow I = \int r^2 dm$: continuous body

Example.



$$\frac{M}{L} = \lambda = \frac{dm}{dx}$$

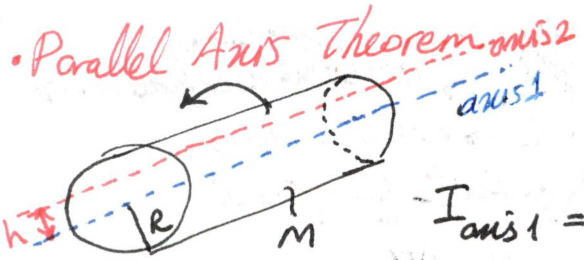
$$dm = \lambda dx = \frac{M}{L} dx$$

$$I = \int r^2 dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx = \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx = \frac{M}{L}\left(\frac{x^3}{3}\right)\Bigg|_{-L/2}^{L/2}$$

$$\boxed{I = \frac{1}{12} M L^2}$$

for thin rod about axis through center perpendicular to length (see Table 10-2e)

- **Parallel Axis Theorem**



If we know I about an axis (com axis), then we can calculate I about an another axis parallel to first one.

$$I_{axis\,1} = \frac{1}{2} M R^2 = I_{com} \Rightarrow \boxed{I = \frac{1}{2} M R^2 + M h^2}$$

$$I_{com} + M h^2$$

Parallel axis theorem

Example Rotational Inertia of a two particle system. Fig. 10-13a

i) Rotational axis $\rightarrow$ com axis $\quad I = \sum_{i=1}^{2} m_i r_i^2 = m_1\left(\frac{L}{2}\right)^2 + m_2\left(\frac{L}{2}\right)^2 = \frac{mL^2}{4} = \boxed{\frac{mL^2}{2}}$ if $m_1 + m_2$, $m_1 = m_2$

ii) Rotational axis $\rightarrow$ at left end $\quad I = I_{com} + M h^2$

$$= \underbrace{\frac{mL^2}{2} + 2m\left(\frac{L}{2}\right)^2}_{\text{by parallel axis theorem}} \boxed{= mL^2}$$

or $I = \sum_{i=1}^{2} m_i r_i^2$

$$= m_1 (0)^2 + m_2 L^2 \boxed{= mL^2}$$

Torque, τ: (To twist)   Does not cause rotation

Resolve $\underbrace{\text{applied force}}_{\vec{F}}$ for rotation into two components $<$ $\vec{F}_r$: radial component; $\vec{F}_t$: tangential "

Does cause rotation $|\vec{F}|\sin\phi = F_t$

SLN Fig. 10-16 $\tau = rF_t = rF\sin\phi \rightarrow$ Fig. 10-16b

SI Unit: N.m $\tau = (r\sin\phi)F = r_\perp F \rightarrow$ Fig. 10-16c

(Be aware that torque is not work! 1 J = 1 N.m)

· Rotation around an axis $\rightarrow$ in 1D $\Rightarrow$ Sign of torque $<$ (+) ccw, (−) cw

When several forces acting $\rightarrow$ several torques $\Rightarrow$ net torque (τ_{net}) is obtained by super-position principle.

Newton's 2nd law for Rotation   SLN Fig. 10-17

Net torque causes an angular acceleration, α. $\boxed{\tau_{net} = I\alpha}$ Newton's 2nd law of rotation

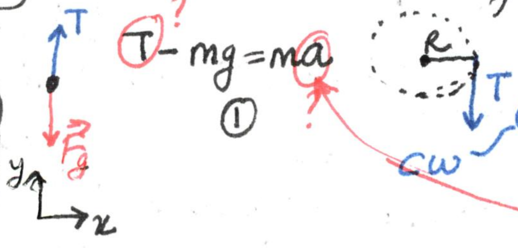
α radian measure

Proof: F_t creates a_t $\{ F_t = ma_t \} \; F_t r = ma_t r \} \; \tau = m(\alpha r)r \} \; \tau = (mr^2)\alpha \} \; \tau = I\alpha$

Example. Newton's 2nd Law in Rotational Motion

SLN Fig. 10-18 | i) a = ? Acceleration of falling block

$M = 2.5$ kg
$R = 0.2$ m
$m = 1.2$ kg
$a = ?$, $\alpha = ?$
$T = ?$



$T - mg = ma$ ①

$\tau = I\alpha$

$-RT = I\alpha$

$-RT = \frac{1}{2}MR^2(a/r)$

$\Rightarrow T = -\frac{1}{2}Ma$ ②

Combining ① & ②

$-\frac{1}{2}Ma - mg = ma$

$a(m + \frac{1}{2}M) = -mg$

$a = -\frac{2m}{2m+M}g = \boxed{-4.8 \text{ m/s}^2}$

ii) $\alpha = ?$ $\alpha = \frac{a}{r} = \frac{-4.8 \text{ m/s}^2}{0.20} = \boxed{-24 \text{ rad/s}^2}$

iii) $T = -\frac{1}{2}Ma = -\frac{1}{2}(2.5\text{kg})(-4.8\text{m/s}^2)$

$\boxed{T = 6.0\text{N}}$

Work and Rotational Kinetic Energy

Translational	Rotational
F on a rigid body (m) $\rightarrow$ acceleration (motion) $\rightarrow$ does work $\rightarrow$ KE can change	τ on a rigid body $\rightarrow$ rotational acceleration (motion) $\rightarrow$ does work $\rightarrow$ KE can change
$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W$	$\Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W$
$W = \int_{x_i}^{x_f} F\,dx$; $P = \frac{dW}{dt} = Fv$	$W = \int_{\theta_i}^{\theta_f} \tau\,d\theta$; $P = \frac{dW}{dt} = \tau\omega$

KE-Work Theorem   SLN Table 10-3

Example Work, Rotational KE, torque, disk   SLN Fig. 10-18

$t = 0 \rightarrow \omega = 0$
$T = 6.0$ N
$\alpha = -24$ rad/s²
KE = ? at $t = 2.5$ s
$M = 2.5$ kg
$R = 0.20$ m

$KE = \frac{1}{2}I\omega^2$?

$I = \frac{1}{2}MR^2$, $\omega = \omega_0 + \alpha t$

$\omega = (-24 \text{ rad/s}^2)(2.5\text{s})$

$KE = \frac{1}{4}(2.5\text{kg})(0.20\text{m})^2[(-24 \text{ rad/s}^2)(2.5\text{s})]^2$

$\boxed{KE = 90\text{J}}$

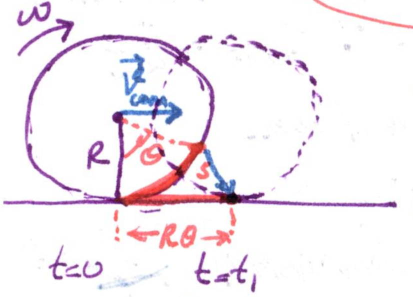
$W = \tau(\theta_f - \theta_i) = \tau(\omega_0 t + \frac{1}{2}\alpha t^2) = (-TR)(\frac{1}{2}\alpha t^2) = -\frac{1}{2}TR\alpha t^2 = 90\text{J}$

Chapter 11 - Rolling, Torque, and Angular Momentum

Rolling motion of wheels → Study of Rotation & Translation

about a fixed axis — along a straight line

Pure Translation Motion + Pure Rotation Motion   SLN Fig. 11-2



Arc length $s = R\theta$ → wheel moves that distance from $t=0$ to $t=t_1$

$v_{com} = \frac{ds}{dt} = \omega R$ → smooth rolling motion (without slipping or bouncing on the surface)

$a_{com} = \frac{dv_{com}}{dt} = \alpha R$

SLN Fig 11-4

Pure Rotation: Every point on the wheel rotates about the center with
(Fig.11-4a) angular speed, ω. Every point at outmost part of the wheel
has linear speed v_{com} ($v_{com} = \omega R$)

+

Pure Translation: Think as if the wheel does not rotate. Every point on the
↓ (Fig.11-4b) wheel moves with v_{com}.

Rolling: * At the bottom of the wheel (Point P), the portion of the wheel is
(Fig.11-4c) stationary.
* The portion at the top (Point T) moving at a speed of $2v_{com}$.
* At points B & D, the speeds are smaller than the point T.

Rolling as Pure Rotation Another way to look at the rolling motion of a wheel.

SLN Fig.11-6 → Pure rotation (with angular speed, ω) about an axis (contact point P)

$v_P = 0 = \omega R = \omega(0)$ (point P)
$v_T = \omega 2R = 2v_{com}$ (point T)

$I_P = I_{com} + MR^2$ — perpendicular distance to com
Parallel Axis Theorem

?? we need Parallel Axis Theorem

Kinetic Energy of Rolling

$K = \frac{1}{2}mv^2 \xrightarrow[\text{becomes}]{\text{Rolling}} K = \frac{1}{2}I_P\omega^2$ } I_P: Rotational Inertia of "Rotation Axis at P"; ω: angular speed of the wheel

$\Rightarrow I_P = I_{com} + MR^2 \rightarrow K = \frac{1}{2}(I_{com} + MR^2)\omega^2 = \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}M\underbrace{(\omega R)^2}_{v_{com}^2}$

$\frac{1}{2}I_P\omega^2 = \boxed{\underbrace{\frac{1}{2}I_{com}\omega^2}_{\text{Rotational Kinetic Energy}} + \underbrace{\frac{1}{2}Mv_{com}^2}_{\text{Translational Kinetic Energy}}}$ ⇒ Object's KE at Rolling

KE; when a "pure" rotation about an axis P

The Forces of Rolling: Friction and Rolling   SLN Fig.11-7

{ • Constant Speed
{ • Constant Acceleration
• sliding
• without sliding

Whell rolls at a constant speed (v_{com})
No sliding
No frictional force

There is a net force acting on the whell → speed up or down ⇒ acceleration a_{com} ⇒ frictional force ↓ oppose sliding

① Does not slide
• static frictional force ✓
• smooth rolling

② Does slide
• Kinetic frictional force
• not smoot rolling
No discuss

Sample Problem Angular Momentum

The Forces of Rolling: Rolling Down a Ramp   SLN Fig. 11-8

Uniform body of mass M
Radius R
Rolling (smoothly)
Ramp at angle θ
$a_{com,x} = ?$
Down the ramp

Newton's 2nd law for linear & angular motions.

Linear motion: along x-axis. $F_{net,x} = Ma_x$ (using not $(-)a_x$!)

$$f_s - Mg\sin\theta = Ma_{com,x} \quad (1)$$

• f_s is not at its maximum value.
• without sliding → smooth rolling

Rotational motion: Rotation axis at point P. $\tau_{net} = I_{com}\alpha$

$\tau = r_\perp F$

$$Rf_s = I_{com}\alpha \quad (2)$$

• only f_s contributes
• F_N & F_g have zero moment arms

Smooth Rolling: • ccw → (+) torque → (+) α

$$a_{com,x} = \alpha R \quad (3)$$

Combining (2) & (3)

$$f_s = \frac{I_{com}}{R}\left(\frac{a_{com,x}}{R}\right) = (-)\frac{I_{com}\,a_{com,x}}{R^2}$$

Combining (1) & (2) & (3)

$$-\frac{I_{com}a_{com,x}}{R^2} - Mg\sin\theta = Ma_{com,x}$$

$$\Rightarrow a_{com,x}\left(M + \frac{I_{com}}{R^2}\right) = -Mg\sin\theta \rightarrow \boxed{a_{com,x} = \frac{-g\sin\theta}{1+\frac{I_{com}}{MR^2}}}$$

Linear acceleration of the body rolling at ramp

SLN Energy Considerations

Example. Ball rolling down a ramp

$M = 6.00\,kg$
R
Smooth Rolling
Down the ramp
$\theta = 30°$

i) $h = 1.20\,m$. speed at the bottom? Mechanical Energy is conserved

• F_g: conservative force
• F_N: does zero work $F_N \perp \Delta x$
• f_s: does not slide (smooth rolling) ⇓ no energy dissipation as thermal energy

$$\Rightarrow K_f + U_f = K_i + U_i \rightarrow Mgh$$

$$\left(\frac{1}{2}I_{com}\omega^2 + \frac{1}{2}Mv_{com}^2\right) + 0 = 0 + Mgh$$

$I_{com} = \frac{2}{5}MR^2$: from Table $\omega = \frac{v_{com}}{R}$: Smooth Rolling

$$\frac{1}{5}MR^2\frac{v_{com}^2}{R^2} + \frac{1}{2}Mv_{com}^2 = Mgh \quad \Big\} \quad \frac{7}{10}v_{com}^2 = gh \quad \Big\} \quad v_{com} = \sqrt{\frac{10}{7}(9.8\,m/s^2)(1.20\,m)} = \boxed{4.10\,m/s}$$

ii) $f_s = ?$ in magnitude and direction (no M or R needed)

$$a_{com,x} = \frac{-g\sin\theta}{1+\frac{I_{com}}{MR^2}} = \frac{-(9.8\,m/s^2)(\sin 30°)}{1+\frac{2/5\,MR^2}{MR^2}} = \boxed{-3.50\,m/s^2} \Leftrightarrow f_s = -\frac{I_{com}a_{com,x}}{R^2} = -\frac{\frac{2}{5}MR^2(-3.50\,m/s^2)}{R^2}$$

Direction: upward at the ramp

$$f_s = \frac{2}{5}(6.0\,kg)(3.5\,m/s^2) = \boxed{8.40\,N}$$

(M is needed)

Torque   SLN Fig. 11-10

$\vec{\tau} = \vec{r}\times\vec{F}$: Torque vector points in a direction perpendicular to plane of $\vec{r}$ and $\vec{F}$ (RHR)

Sample Problem: Torque on a particle due to a force

Angular Momentum   SLN Fig. 11-12

Angular momentum ($\vec{\ell}$) of the particle with respect to origin: $\boxed{\vec{\ell} = \vec{r}\times\vec{p}} \rightarrow \boxed{\vec{\ell} = m\,\vec{r}\times\vec{v}}$

$\vec{r}$: position vector wrt origin.

* To have angular momentum, particle does not itself has to rotate
* It has meaning only wrt a specified origin
* Its direction is perpendicular to plane of $\vec{r}$ and $\vec{p}$.

* Magnitude of $\vec{\ell}$

$$\ell = mrv\sin\theta \; \boxed{\begin{aligned} &= rp_\perp = rmv_\perp \\ &= r_\perp p = r_\perp mv \end{aligned}}$$

component   component

Newton's 2nd Law in Angular Form

$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\,m\vec{v}}{dt} = \boxed{\frac{d\vec{p}}{dt}}$ Single Particle } Angular Form { $\boxed{\vec{\tau}_{net} = \frac{d\vec{l}}{dt}}$ where ($\vec{l} = m\vec{r}\times\vec{v}$) Single Particle

Relation btw force and linear momentum

Relation btw torque and angular momentum

SLN Sample Problem

ith particle

The Angular Momentum of a System of Particles ($\vec{L}$)

System of Particles $\Rightarrow$ Total Angular Momentum $\vec{L} \rightarrow \vec{L} = \vec{l}_1 + \vec{l}_2 + \dots + \vec{l}_n = \sum_{i=1}^{n} \vec{l}_i$

$\vec{\tau}_{net,i} = \frac{d\vec{l}_i}{dt}$

$\vec{\tau}_{net} = \sum_{i=1}^{n} \vec{\tau}_{net,i}$

$\boxed{\vec{\tau}_{net} = \sum_{i=1}^{n} \frac{d\vec{l}_i}{dt} = \frac{d\vec{L}}{dt}}$

System of particles

Only external torques (due to external forces) are considered since internal forces cancel out each other (Newton's 3rd law, internal torques becomes zero). So that $\vec{\tau}_{net}$ is the net external torque.

The Angular Momentum of a Rigid Body Rotating About a Fixed Axis

System of particles $\xrightarrow{\text{form}}$ a rigid body (Which is rotating). SLN Fig. 11-15a

Fixed axis of rotation $\rightarrow$ z-axis

Rotates with constant angular speed $\rightarrow \omega$

$L_z = ?$ (about z-axis)

mass element i: $\underbrace{\Delta m_i \;\&\; r_i}_{\text{known}} \rightarrow l_i = ?$ angular momentum of ith mass element

$\vec{l}_i = \vec{r}_i \times \vec{p}_i$

$l_i = r_i p_i \sin 90° = r_i \cdot \Delta m_i v_i$. we need l_{iz}

$\Rightarrow l_{iz} = l_i \cdot \sin\theta = \underbrace{(r_i \cdot \sin\theta)}_{} (\Delta m_i) v_i = r_{\perp i} \Delta m_i v_i$

Fig. 11-15b

$r_{\perp i}$: perpendicular distance btw element i and z-axis

$\Rightarrow$ we have n elements in the rigid body

$L_z = \sum_{i=1}^{n} l_{iz} = \sum_{i=1}^{n} \Delta m_i v_i r_{\perp i} = \sum_{i=1}^{n} \Delta m_i (\omega r_{\perp i}) r_{\perp i} = \omega \underbrace{\sum_{i=1}^{n} \Delta m_i r_{\perp i}^2}_{} \boxed{= I\omega} \leftrightarrow L$

rotational inertia of the body about a fixed axis, I

Now, we have found angular momentum (L) about the fixed rotation axis (z) for a rigid body. $L_z \rightarrow L$

See Table 11-1 SLN Last Page

Conservation of Angular Momentum

Conservation of Energy
Conservation of Linear Momentum
Conservation of Angular Momentum

If $\vec{\tau}_{net} = 0$ (OR $\vec{F}_{net} = 0$) $\rightarrow \vec{\tau}_{net} = 0 = \frac{d\vec{L}}{dt} \Rightarrow \boxed{\vec{L}_{initial} = \vec{L}_{final}}$ Law of Conservation of Angular Momentum

$\vec{L} \equiv$ a constant $\rightarrow$ Isolated System

$\boxed{I_i \omega_i = I_f \omega_f}$

* Depending on the torques acting on the system ($\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$), the angular momentum of the system might be conserved in one or two directions but not in all directions.

* If net external force along an axis is zero $\rightarrow L$ along that axis is constant.

SLN Examples