Exercise Set III.1

1. (3.5) Determine the value c so that each of the following functions can serve as a probability distribution of the discrete random variable X:
   (a) $f(x) = c(x^2 + 4)$ for $x = 0, 1, 2, 3$
   (b) $f(x) = c \begin{pmatrix} 2 \\ x \end{pmatrix} \begin{pmatrix} 3 \\ 3 - x \end{pmatrix}$ for $x = 0, 1, 2,$
2. (3.6) The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \left\{ \begin{array}{l} \frac{20000}{(x+100)^3}, \quad x > 0 \\ 0, \quad \textit{elsewhere} \end{array} \right\}$$

   Find the probability that a bottle of this medicine will have a shell life of
   (a) at least 200 days;
   (b) anywhere from 80 to 120 days.

Exercise Set III.2

3 (3.12) An investment firm offers its customers municipal bonds that mature after varying numbers of years. Given that the cumulative distribution function of T, the number of years to maturity for a randomly selected bond, is,

$$F(t) = \left\{ \begin{array}{ll} 0, & t < 1 \\ \frac{1}{4}, & 1 \leq t < 3 \\ \frac{1}{2}, & 3 \leq t < 5 \\ \frac{3}{4}, & 5 \leq t < 7 \\ 1, & t \geq 7 \end{array} \right\}$$

Find

(a) $P(T = 5)$
(b) $P(T > 3)$
(c) $P(1.4 < T < 6)$

Exercise Set III.3

4 (3.21) Consider the density function

$$f(x)) = \left\{ \begin{array}{ll} k\sqrt{x}, & 0 < x < 1 \\ 0, & \textit{elsewhere} \end{array} \right\}$$

(a) Evaluate k
(b) Find $F(x)$ and use it to evaluate $P(0.3 < X < 0.6)$

5 (3.38) If the joint probability distribution of X and Y is given by

$$f(x, y) = \frac{x + y}{30}, \ \textit{for } x = 0, 1, 2, 3 \textit{ and } y = 0, 1, 2$$

Find

(a) $P(X \leq 2, Y = 1)$
(b) $P(X > 2, Y \leq 1)$
(c) $P(X > Y)$
(d) $P(X + Y = 4)$